Large Spin Atoms

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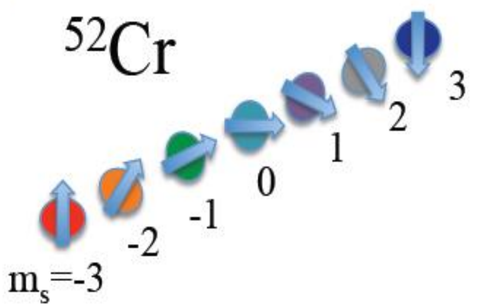
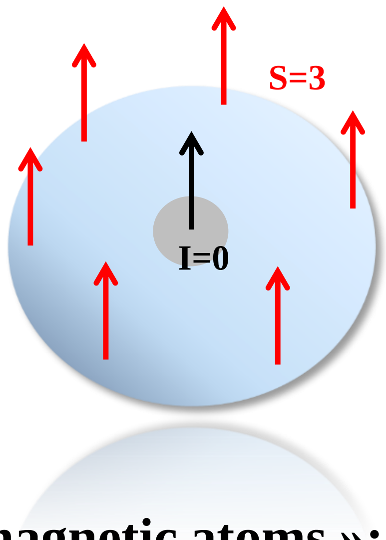
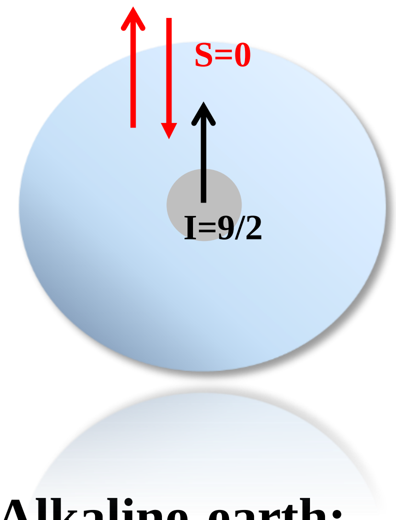
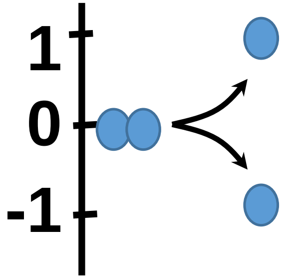




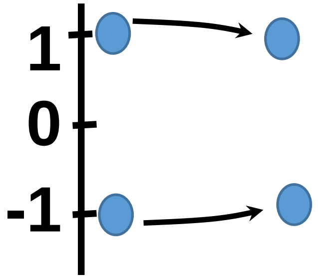


« magnetic atoms »:
spin is purely electronic
Strong dipole-dipole
long-range interaction

Alkaline-earth:
spin is purely nuclear
Spin-independent
contact interactions



« Spinor »



« Mixture »

Investigating quantum magnetism with large spin (s>1/2) particles

Our two experimental platforms at LPL

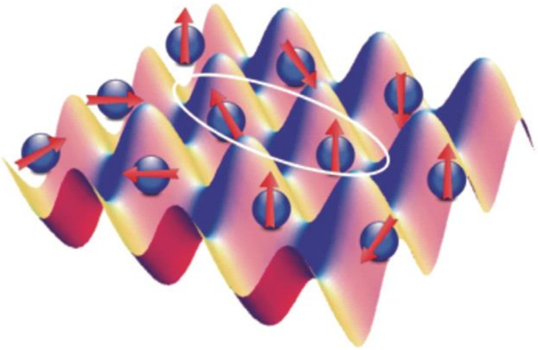
Investigating quantum magnetism with large spin (s>1/2) particles:

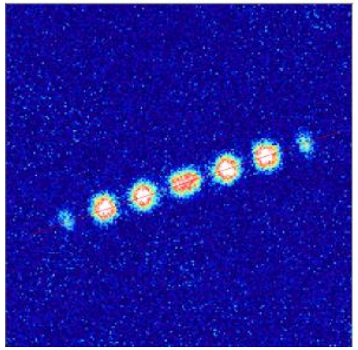
A Chromium Bose-Einstein condensate in a 3D optical lattice

A SU(10) Strontium Fermi gas in a 3D optical lattice

Spin-dependent dipolar interactions
S=3, 7 spin states

Spin-independent contact interactions
F=9/2, 10 spin states





… using collective variables …

How to characterize a quantum many-body system

Density matrix ρ

$$\begin{bmatrix} \rho_{11} & \cdots & \rho_{1k} \\ \vdots & \ddots & \vdots \\ \rho_{1k} & \cdots & \rho_{kk} \end{bmatrix} \quad \updownarrow \; k \sim 2^N$$

Full state tomography impractical

Partial information : trace over subsystem

Partial information : collective measurement

How to characterize a quantum many-body system

Partial information : trace over subsystem

Example for a pure state:

$\rho = |\Psi\rangle\langle\Psi|$

$\mathrm{Tr}(\rho^2)=1$

« pure »

$\mathrm{Tr}((\rho|_A)^2)\neq 1$

« locally mixed »

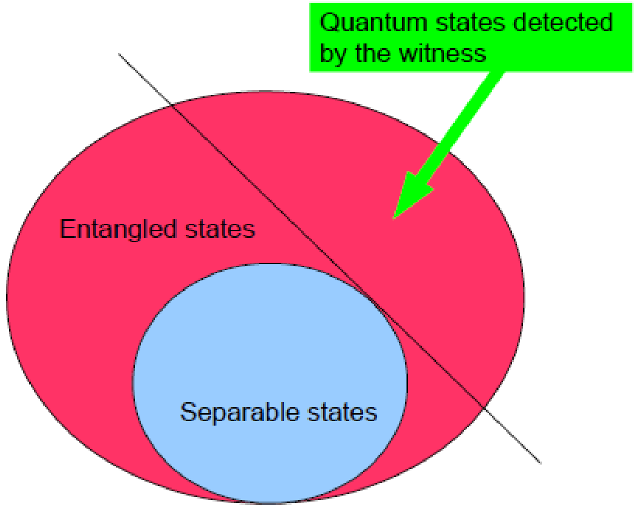
Measures the **entropy associated to entanglement**

See Greiner 2016
Klempt; Treutlein; Oberthaler 2018

(or more simply measure correlations between two sub-systems)

How to characterize a quantum many-body system

Partial information : collective measurement $\sum_{i=1}^{N} \hat{s}_{z,i}$



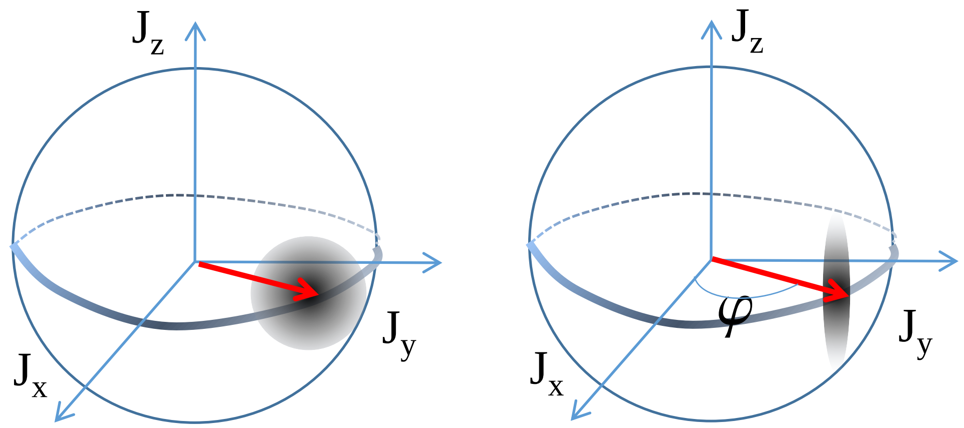
Entanglement witness

(e.g. $(\Delta S_x)^2 + (\Delta S_y)^2 + (\Delta S_z)^2 \geq N/2$ for any mixture of separable states)

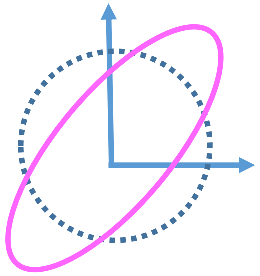
Squeezing (Ueda)

Extreme spin squeezing (Sorensen/Molmer - Klempt) or Fischer information (Oberthaler):

$\rightarrow$ k-particle entanglement



$$\xi = \frac{\sqrt{2J}\,\Delta J_x}{|\langle J_z \rangle|}$$



Meets quantum metrology
Connects to quantum optics

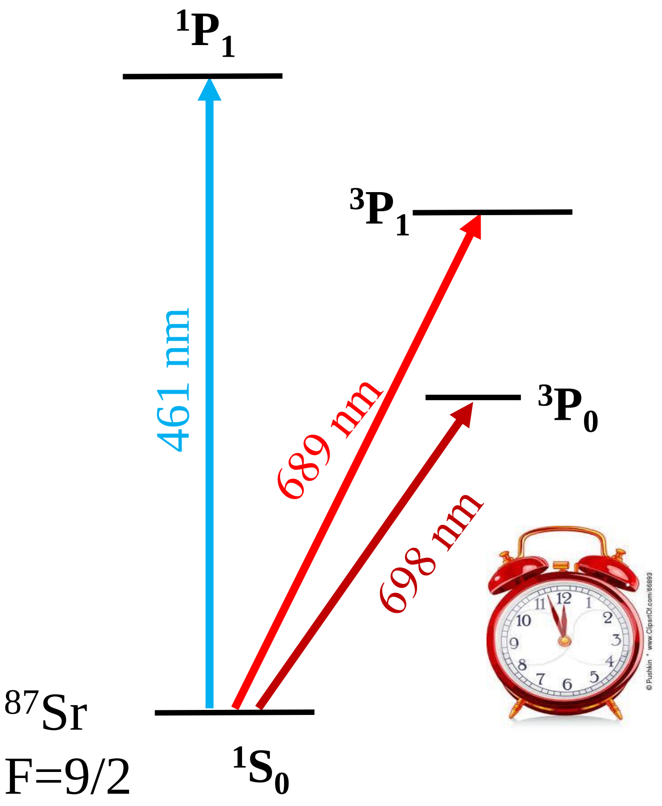
!!! Needs access to coherences !!!

!!! Beware of large spin systems !!!
(i.e. squeezing is not an EW) – See G. Toth

Fluctuations give access to correlations (here for S>1/2)

PART I : Strontium

Introduction to alkaline-earth atoms



Zero electronic spin: no magnetic field sensisivity

Narrow-line laser cooling

Clock transition

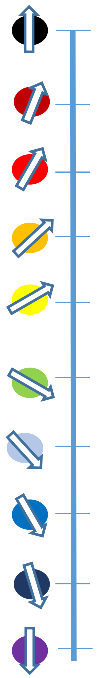
Possibility of a Q-bit in the THz regime

Spin-independent interactions

SU(N) magnetism

Interplay between lattice topology and # spin states (see Mila)
SU(2) : 2 atom singlet
SU(4) : 4-atom singlet

NB: correlations arrise at higher entropy!!
(see Takahashi)



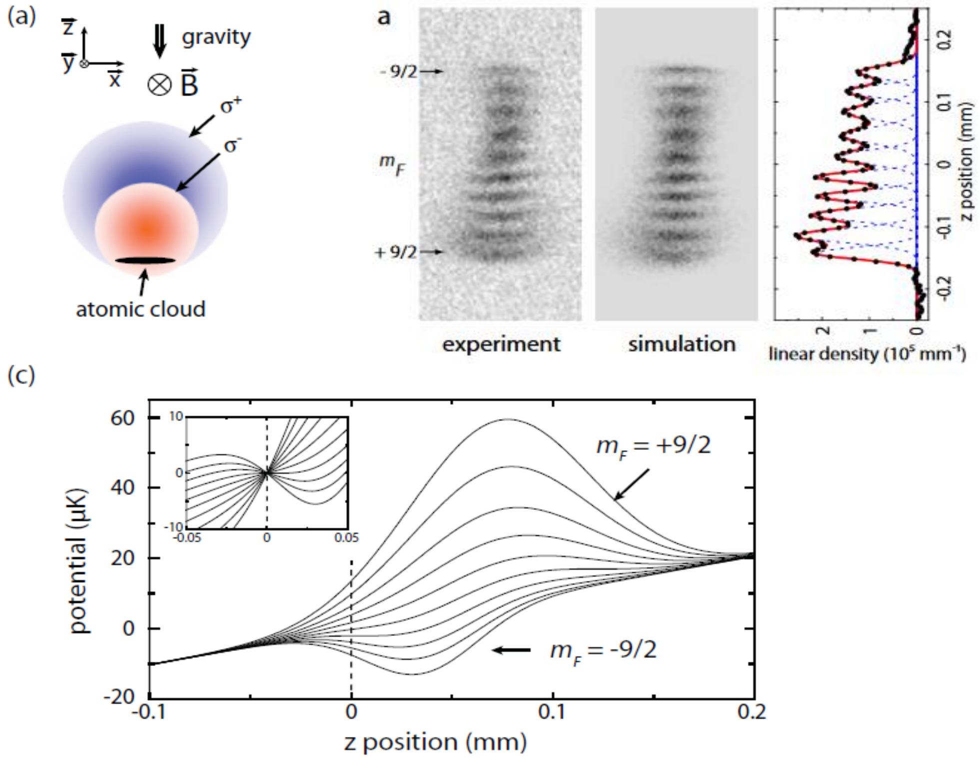
How to manipulate spin?

Purely nuclear spin → magnetic field inconvenient

Create an artificial magnetic field (spin-orbit coupling) (spin-dependent AC-Stark shift)

Use the intercombination line (low scattering rate, large hyperfine structure, large Landé factor)

Example : optical Stern-Gerlach to measure spin



Sr: Stellmer et al, Phys. Rev. A 84, 043611 (2011)

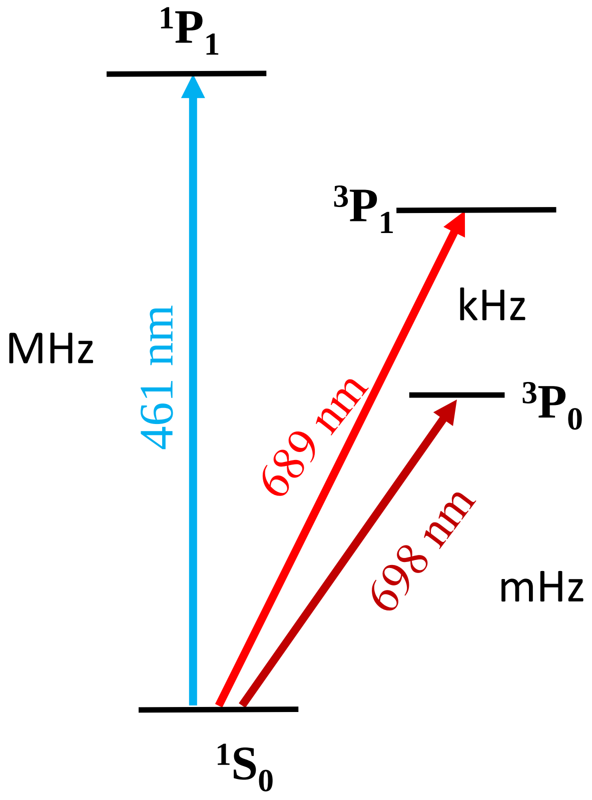
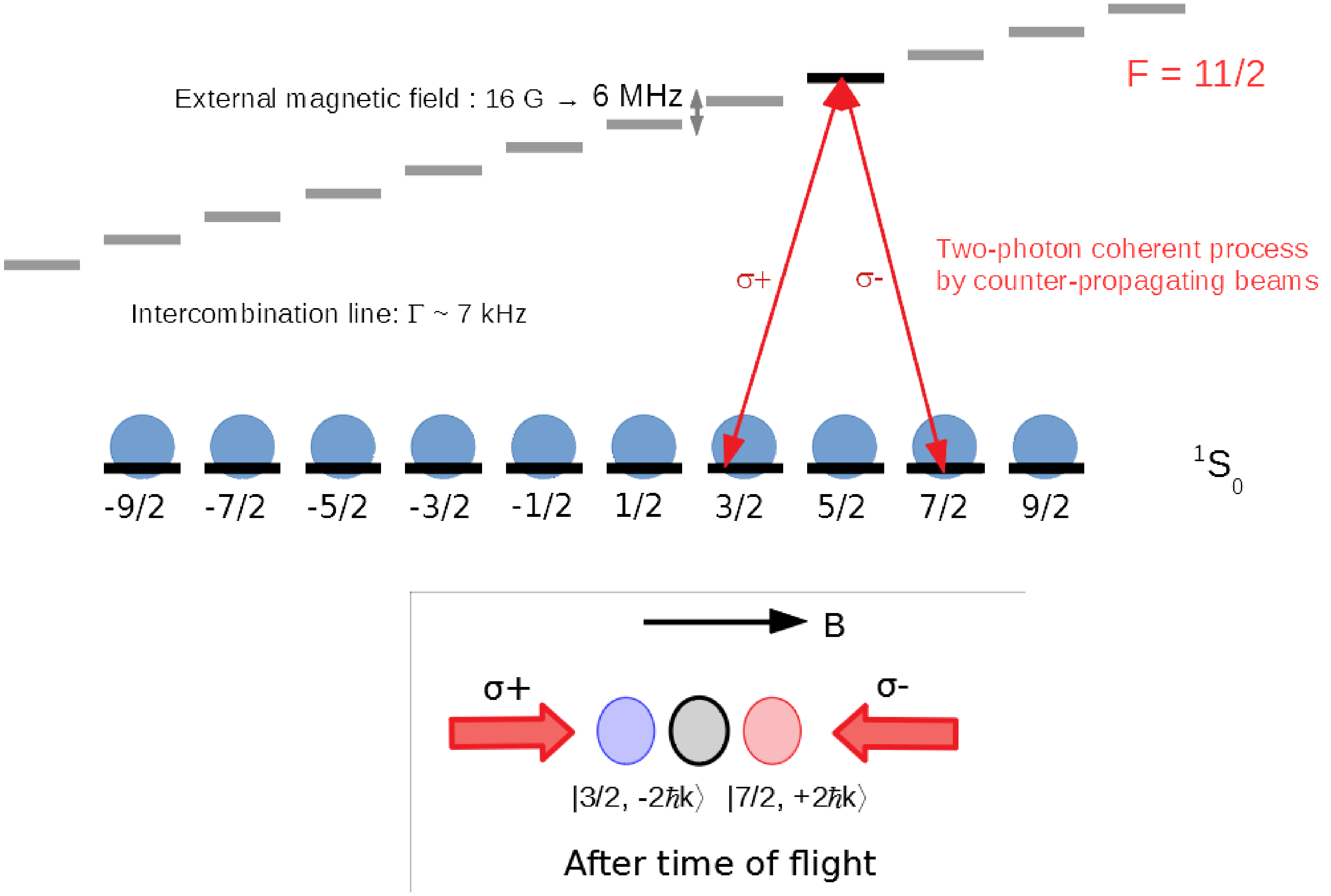


Image on the broad 461 nm transition

One manipulation scheme :
Spin-dependent momentum transfert
using retroreflected light



Recoil extracts atoms from the Fermi sea (then imaging on the broad transition)

Spin and momentum measured simultaneously

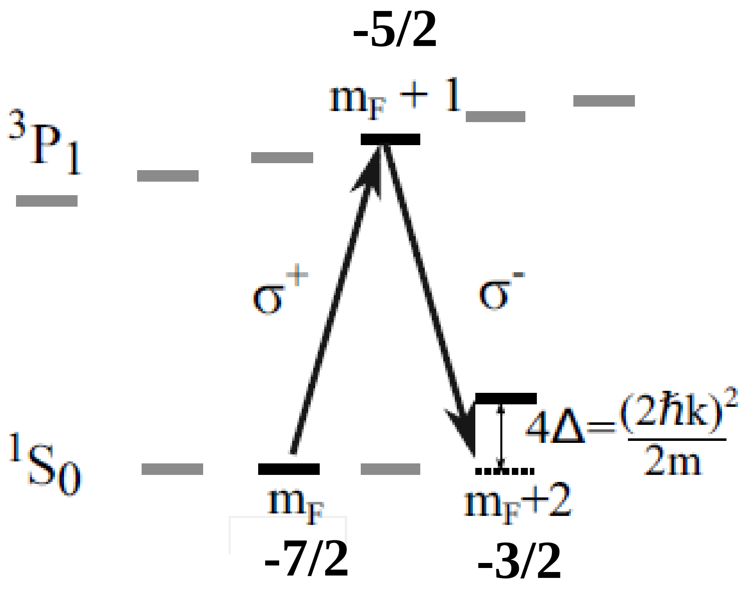
Spin sensitivity insured by Zeeman effect in the excited state ($g\mu_B B >> \Gamma$ easy)

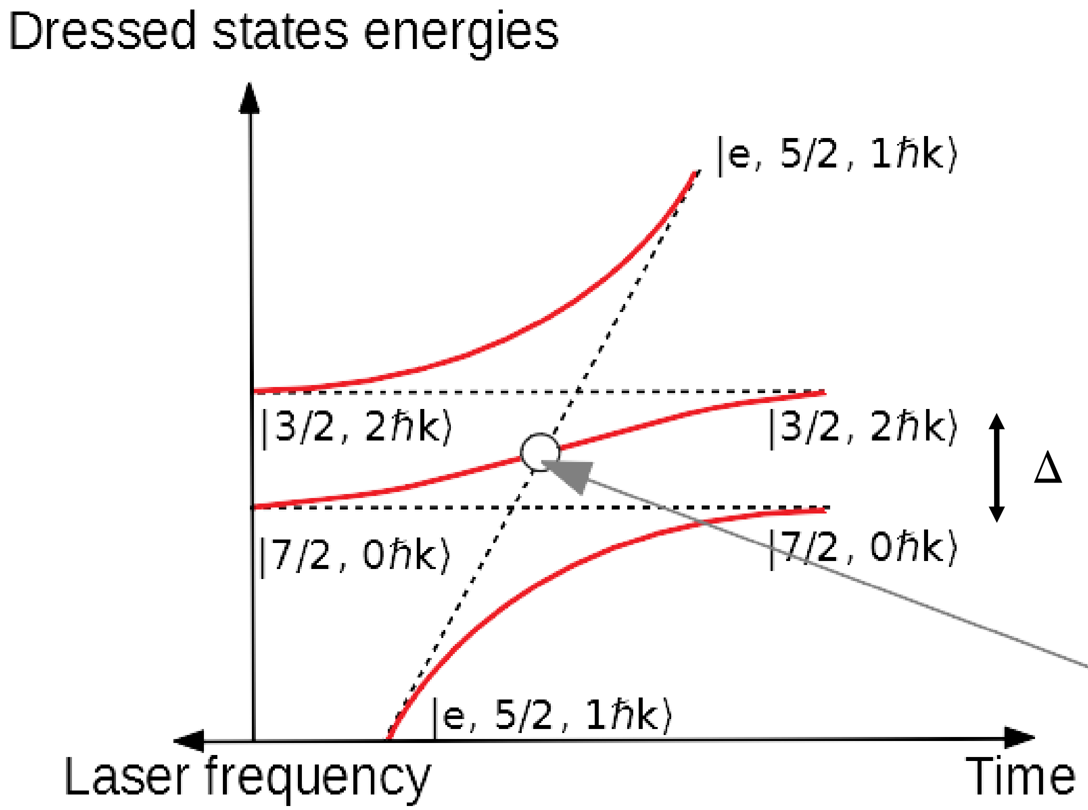
Weak dissipation (use dark states)

Robust (use adiabatic sweep)

Simple (one retrorefected beam)

Spin-dependent momentum transfert with an adiabatic sweep





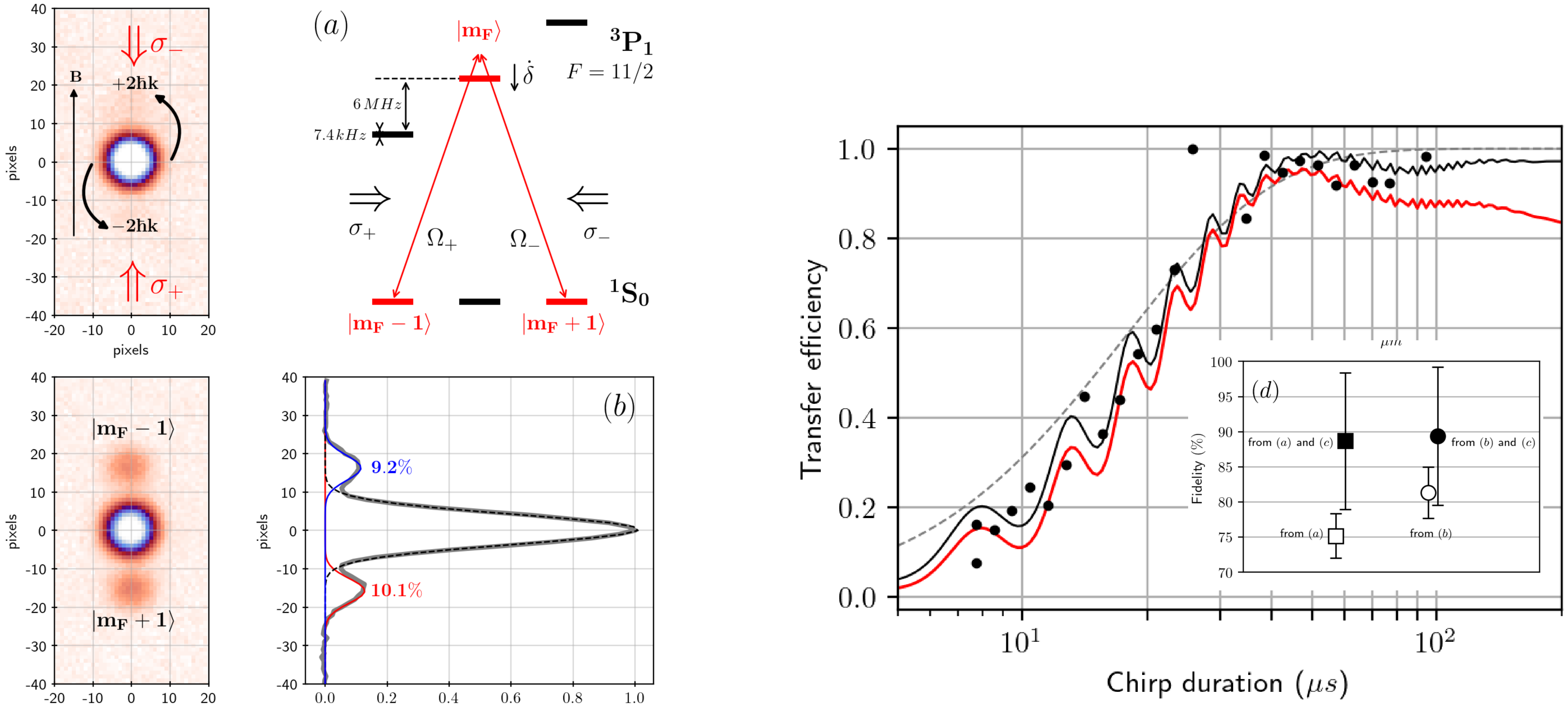
Robust adiabatic transfer
from |7/2, 0ħk⟩ to |3/2, 2ħk⟩

$$|\langle e|\psi\rangle|^2 \propto \frac{8\,Er^2}{\Omega^2}$$

Weak sensitivity to spontaneous emission!!

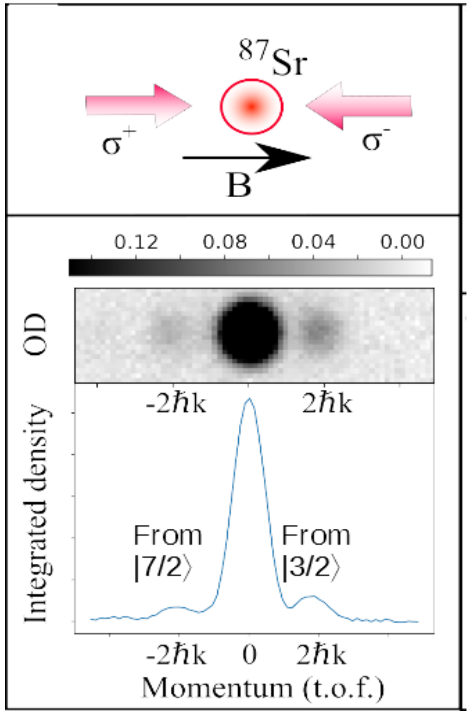
NB ~anti SWAP cooling

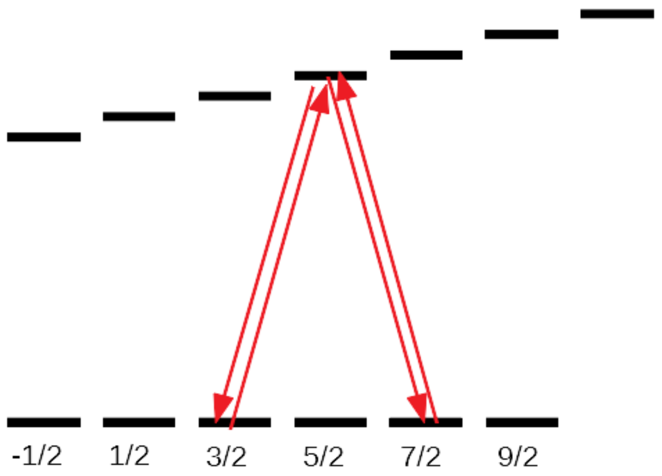
Spin-dependent momentum transfert with an adiabatic sweep



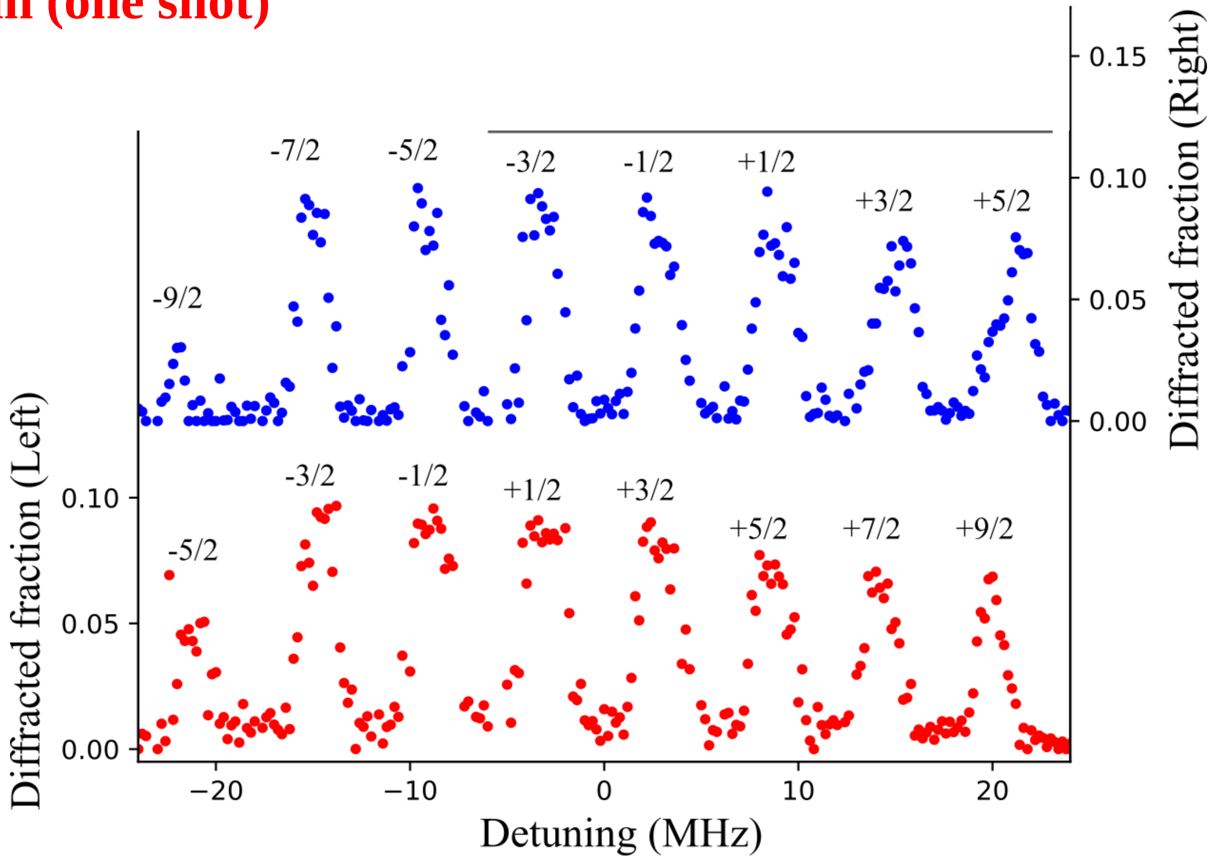
Transfer efficiency > 90%

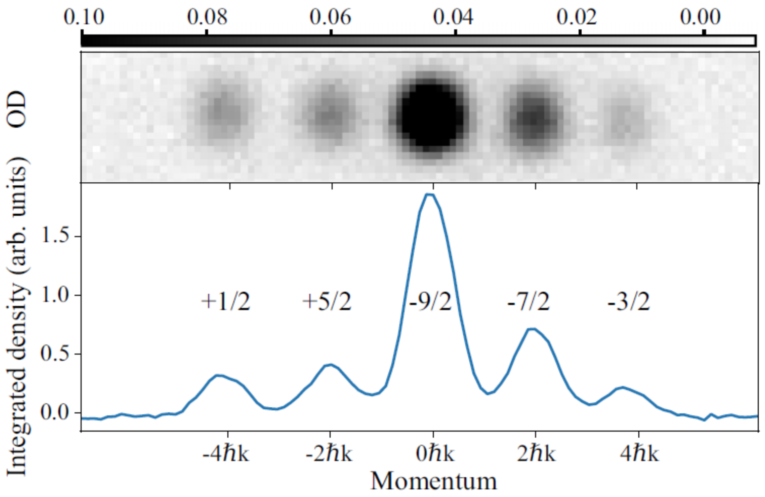
Measuring the spin (one shot)





Two spin state populations measured in one run





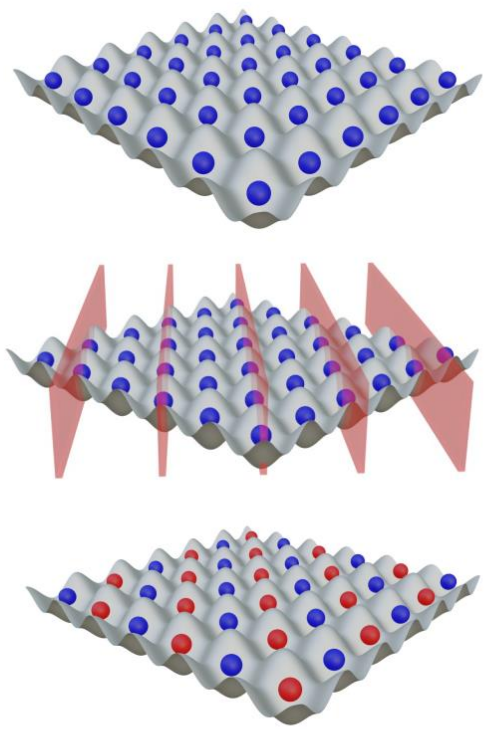
Three pulse sequence

PRA **102**, 013317 (2020)

(one-shot characterization of a SU(5) Fermi gas)

Our project : use the intercombination to manipulate spins / control magnetism with little dissipation

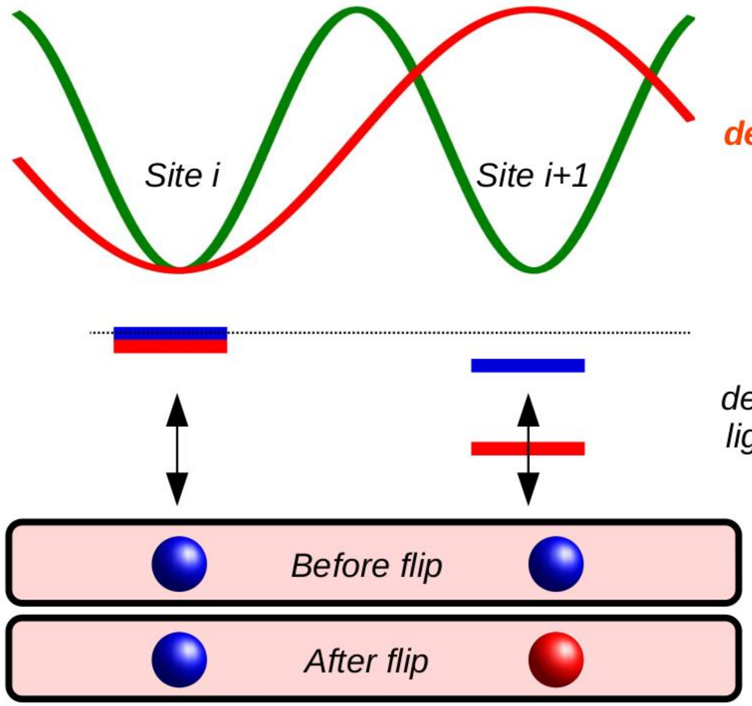
Polarized gas loaded in optical lattice : band insulator

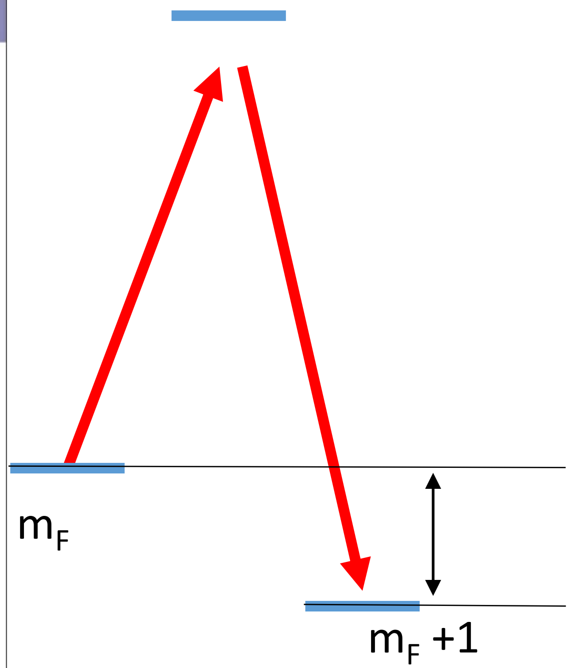


Ground state within spin dependent lattice

Prepare a Néel order by flipping half the spins

- Spectral resolution of the Raman transitions
- Adiabatic passage
- Spatial resolution

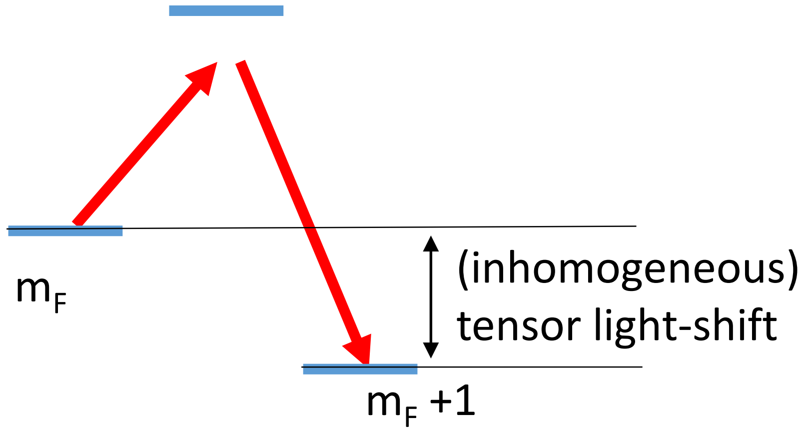


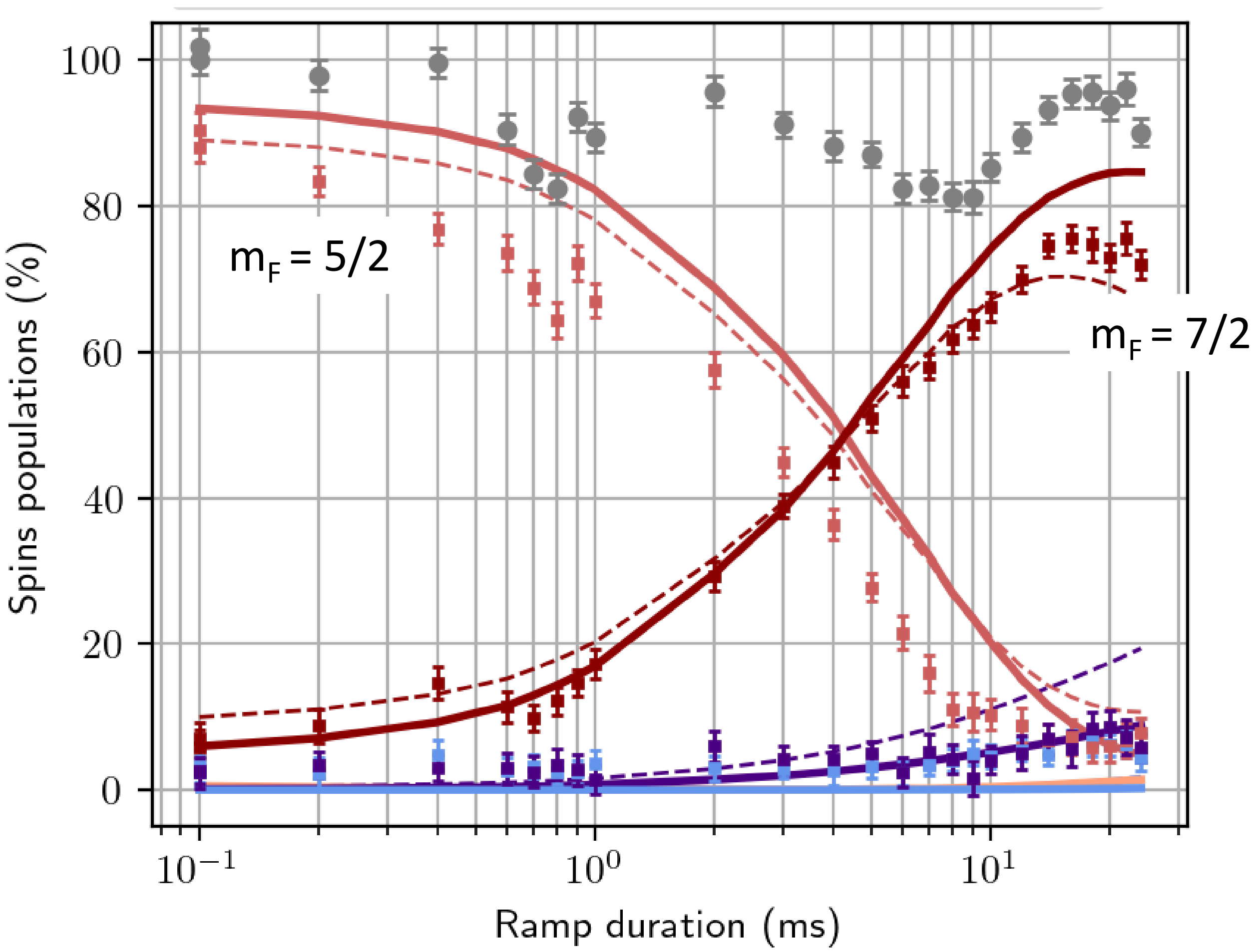


(inhomogeneous) tensor light-shift

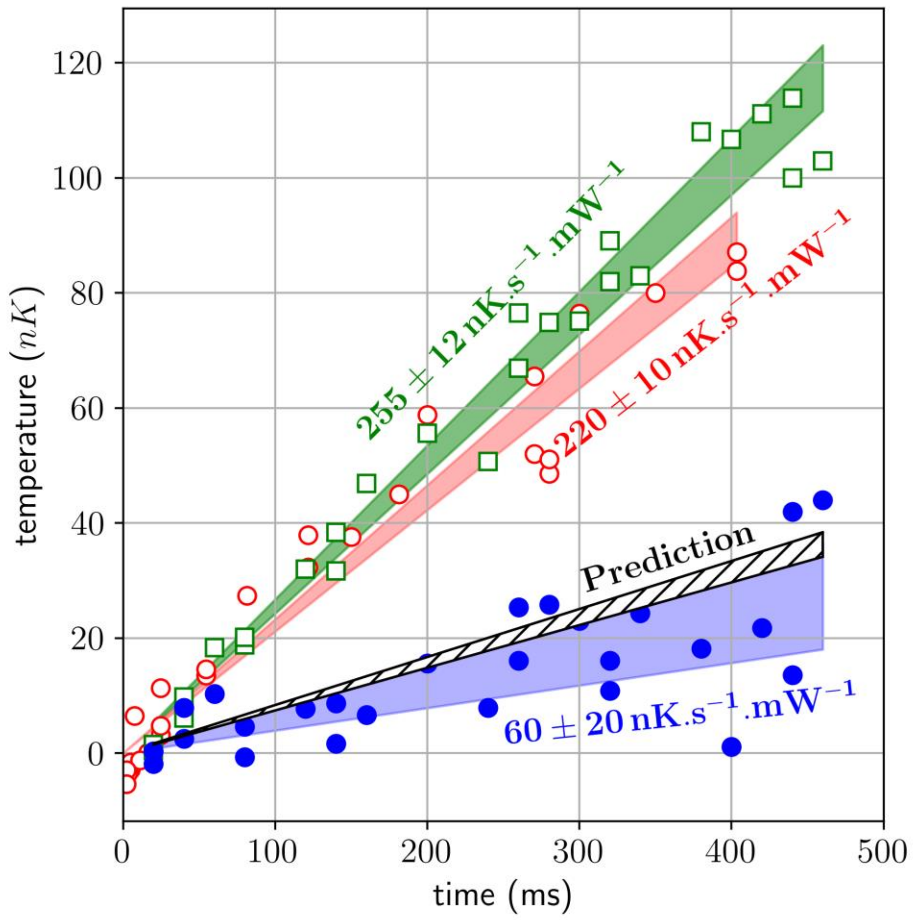
See also Jian-Wei Pan, 2020

Can be used to prepare, or read, spin textures (structure factor measurement)





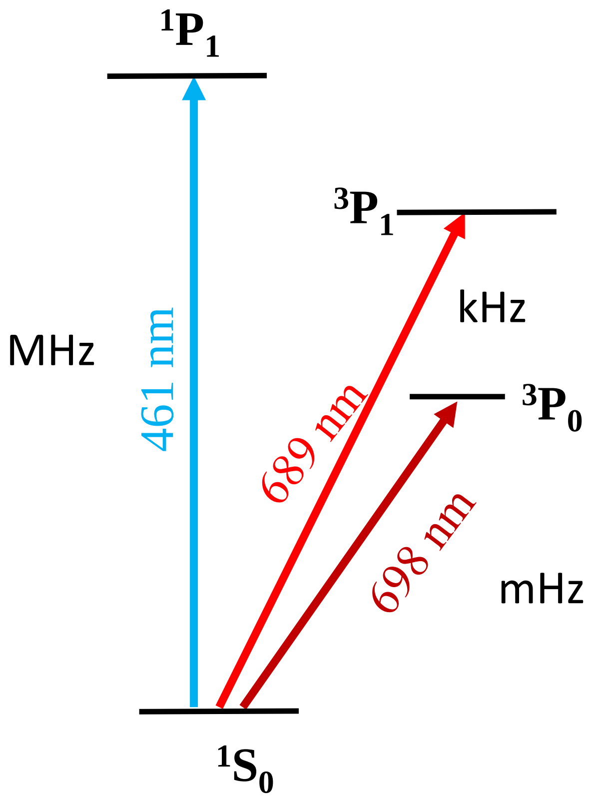
PRELIMINARY

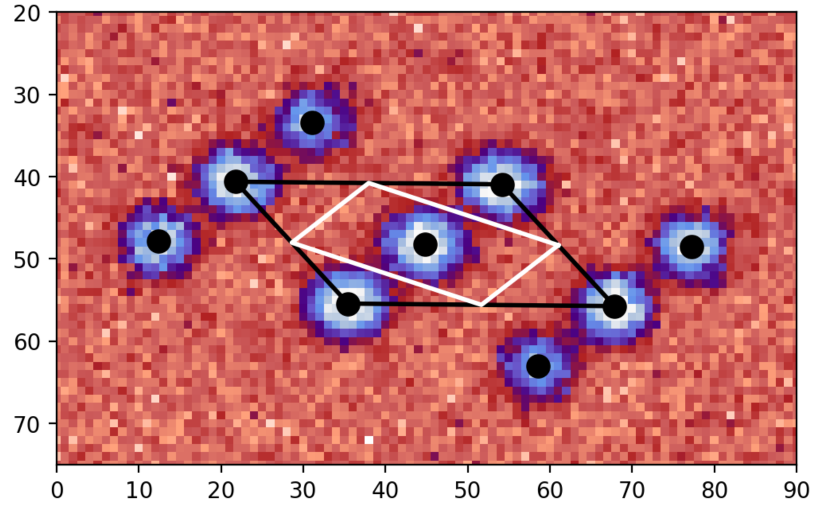


Uses cavity filter

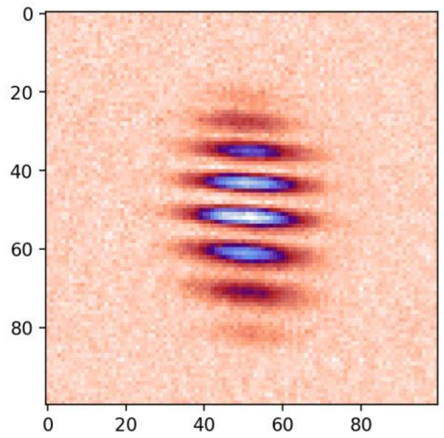
Transfer efficiency ~80%, limited by amplified ASE from laser diode

Strontium ready to go !!



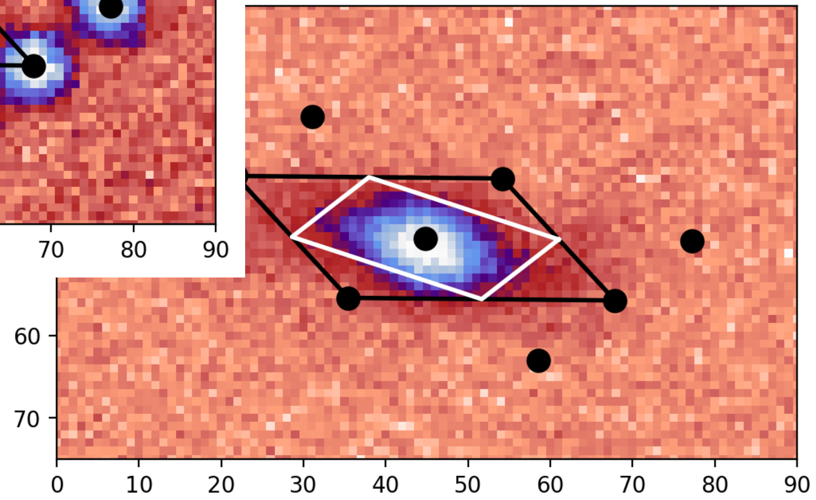


Pulsed 2D lattice



« Quantum magnifier »
~7 2D gases

(see Sengstock)

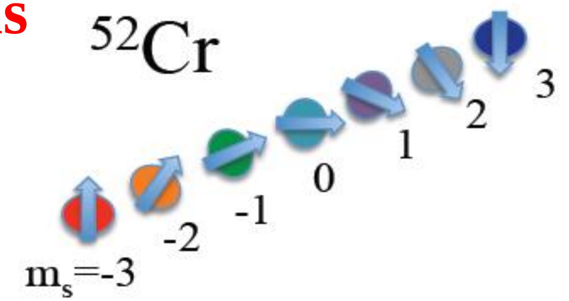


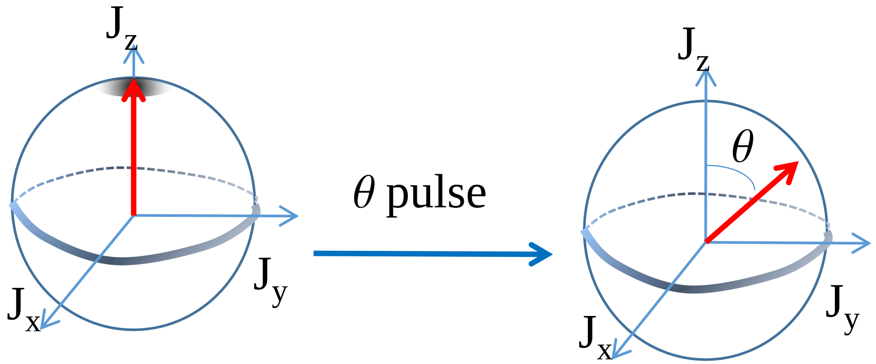
Band mapping

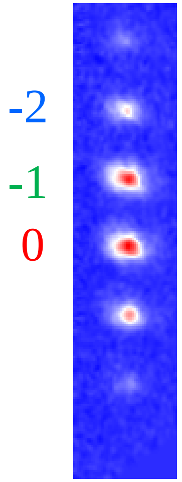
Our project : use the intercombination to manipulate spins / control magnetism with little dissipation
… in a 3D lattice !

PART II : Chromium

Out-of-equilibrium physics with 10^4 magnetic atoms

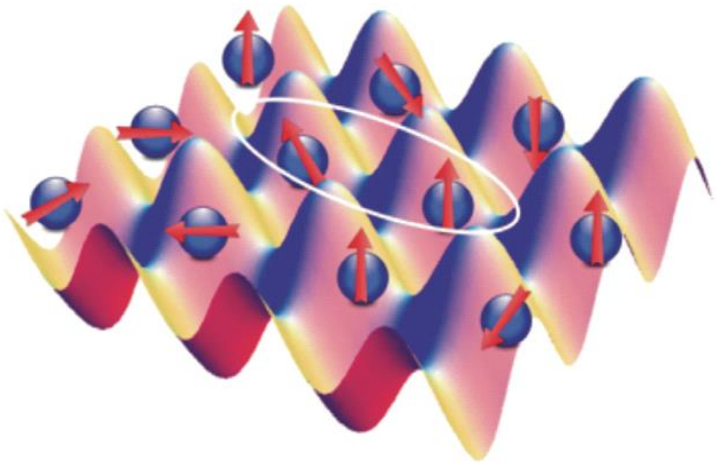






Total populations
in different Zeeman states

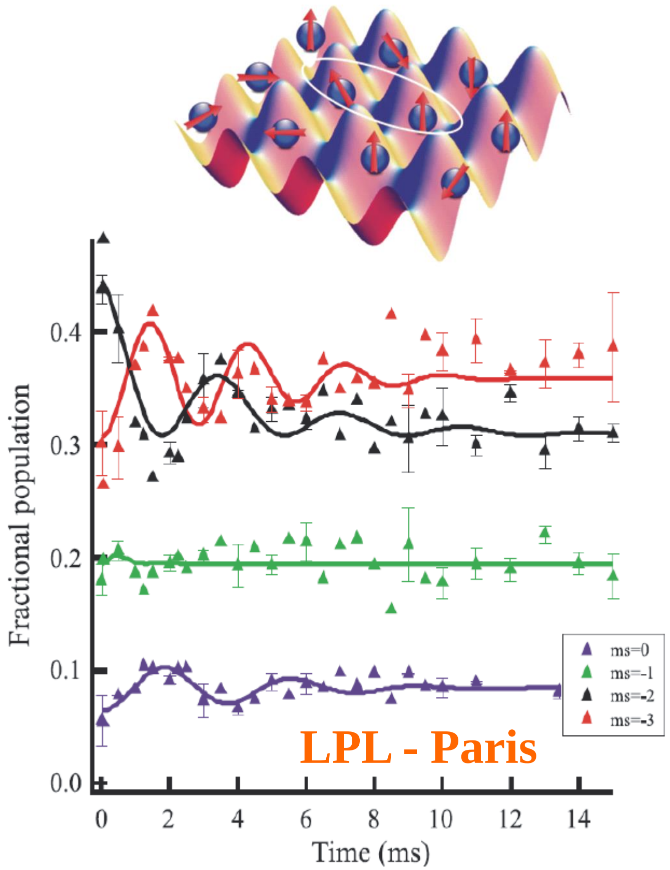
$$\Psi_0 = |3_\theta, 3_\theta, 3_\theta, 3_\theta, \ldots, 3_\theta\rangle$$

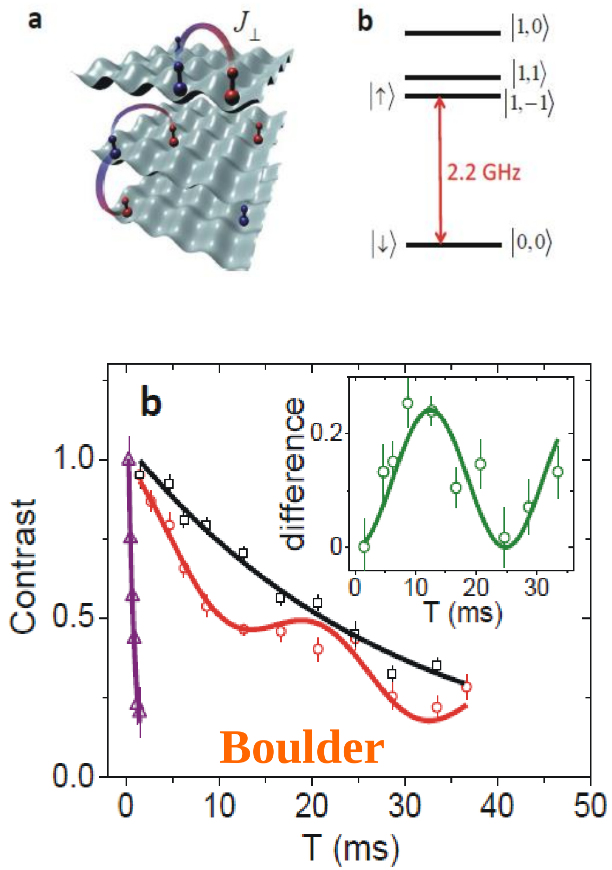


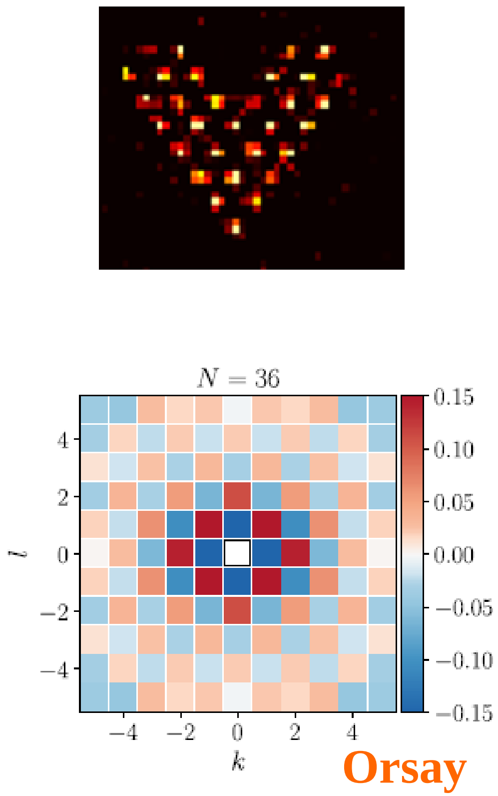
$$H = c_{dd} \sum_{(i,j)} \left[S_i^z . S_j^z - \frac{1}{4} \left(S_i^+ . S_j^- + S_i^- . S_j^+ \right) \right] \frac{(1 - 3\cos^2\theta_{ij})}{r_{ij}^3}$$

Dipolar systems in ultra-cold atoms and molecules

Magnetic atoms vs dipolar molecules vs Rydberg Atoms







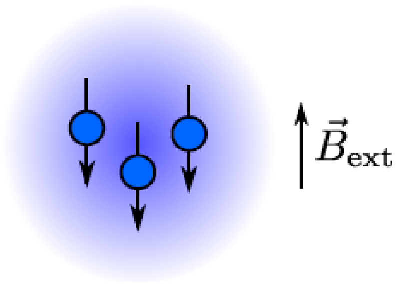
transport possible;
truly macroscopic
Large spin
Individual adressing coming up (Greiner)

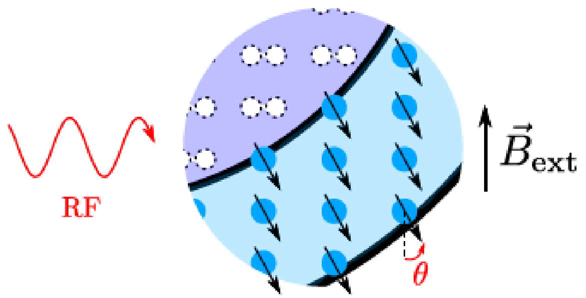
Control of Hamiltonian
Individual adressing
coming up (Bakr)

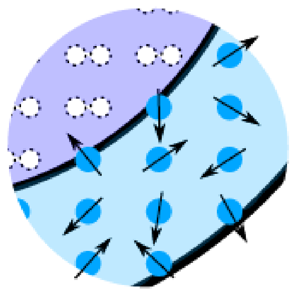
Control of Hamiltonian
And geometry
individual adressing

Quantum Thermalization (Isolated System)

Initial state **Quench** **Evolution to…**







Stationnary state
…
Thermal-like
but very quantum !

Quantum many-body physics of pure states → **Thermodynamics (mixed states)**

Eigenstate Thermalization Hypothesis Deutsch, Srednicki, Olshanii
Growth of entanglement

This talk : use of collective measurements (e.g. $\sum_{i=1}^{N} \hat{s}_{z,i}$ **)**

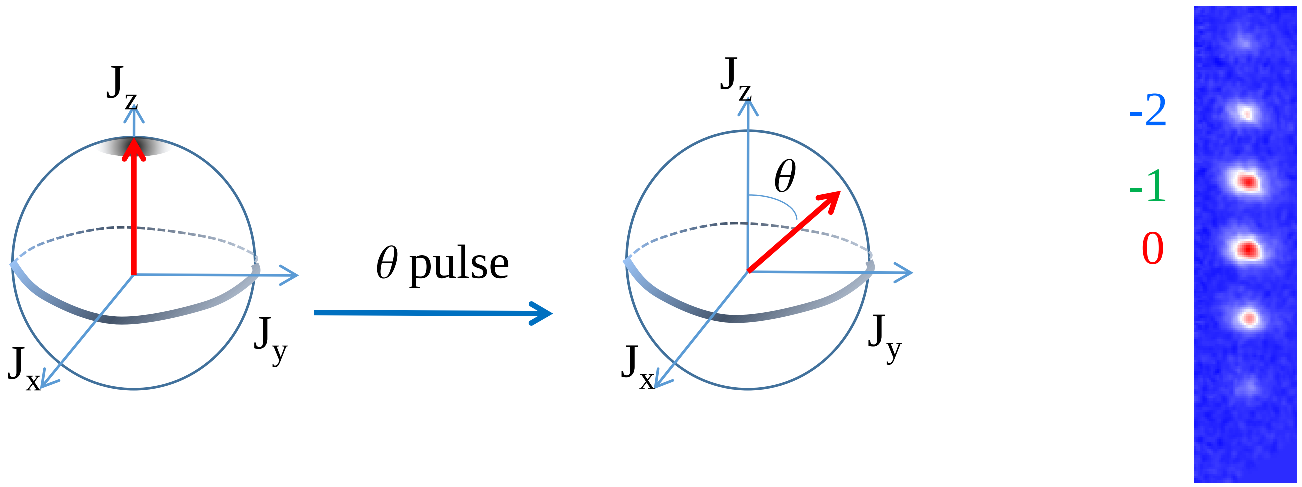
Outline:

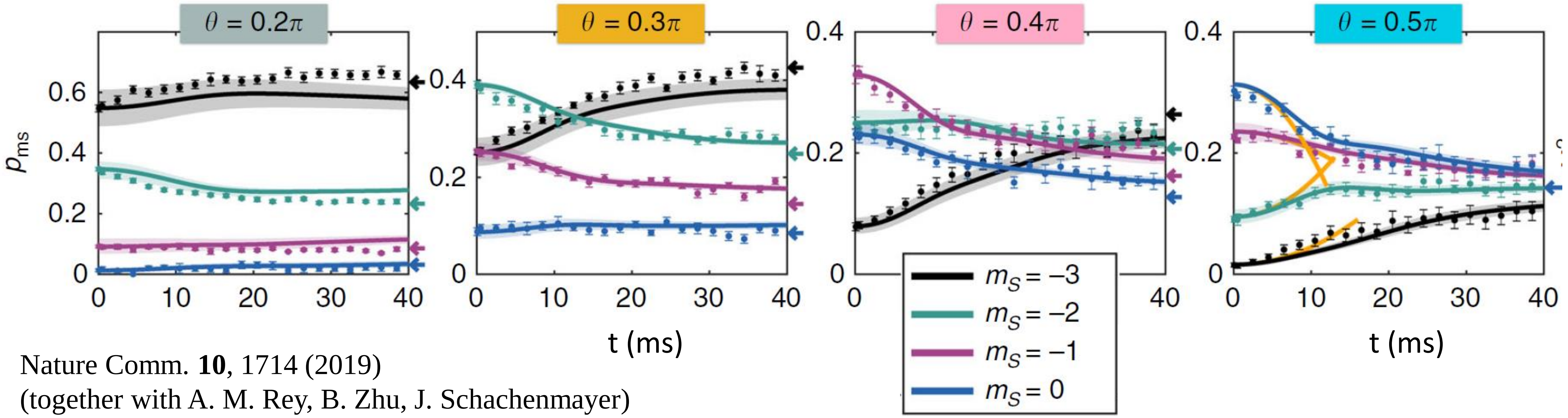
1- Thermalization of the Zeeman populations

2- Thermalization of the collective spin

3- Experimental measurement of correlations using collective measurements

Experimental results

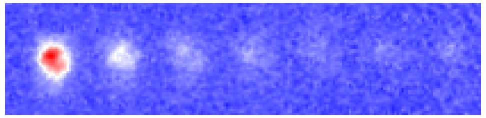


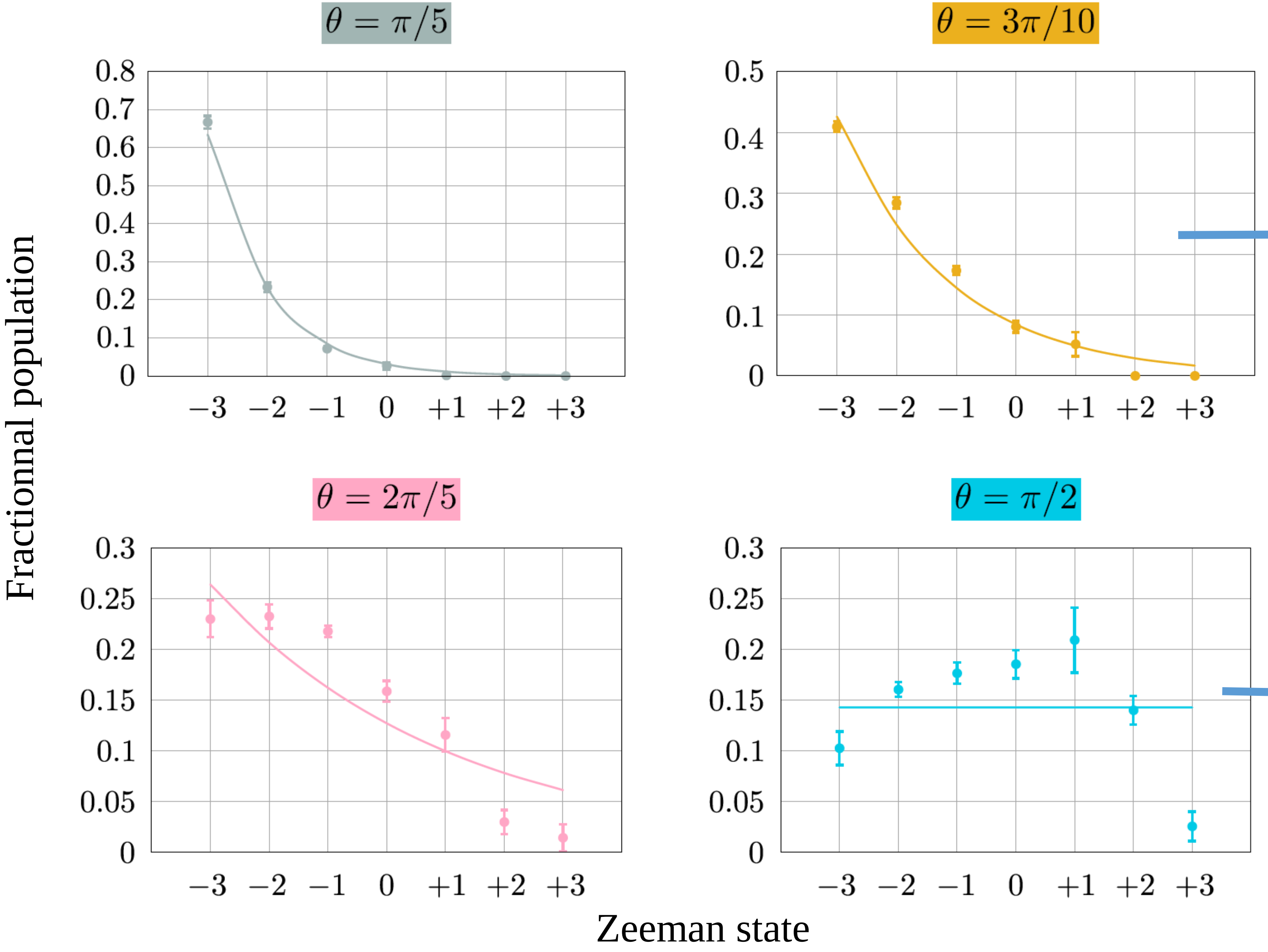


Nature Comm. **10**, 1714 (2019)
(together with A. M. Rey, B. Zhu, J. Schachenmayer)

Agreement with GDTWA indicate the importance of quantum correlations

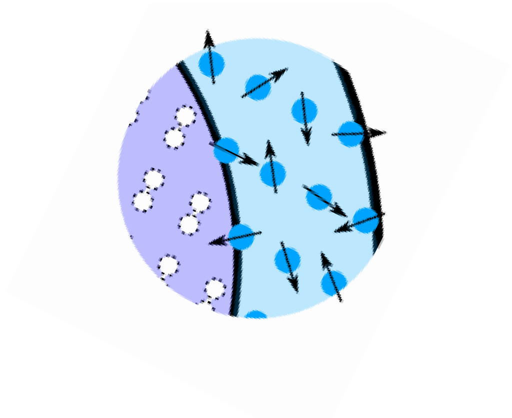
Asymptotic behavior





Small angles:
Exponential,
« Thermal-like behavior »
(maximum of entropy)

$$S_B = -N \sum p_{m_s}(t) \log p_{m_s}(t)$$



Large angles:
Need to go beyond.

Take into account energy constraints.

Two contributions for energy

Dipole-dipole interactions

$$\langle \Psi(t) | V_{dd} | \Psi(t) \rangle$$

Difficult to calculate
except at t=0

Tensor light-shift leads to
an effective quadratic Zeeman effect

$$E(m_s) = B_Q m_s^2$$

Simple to evaluate
using experimental data

$$\sum m_s^2 p_{m_s}$$

Analytic model for quantum thermalization

Look at the thermal state that corresponds to the initial energy

High-temperature expansion (A.M. Rey)

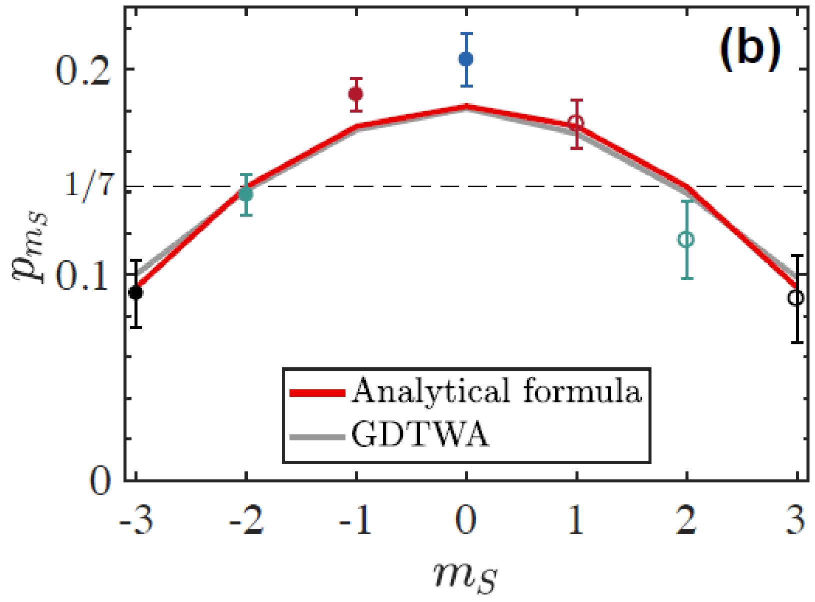
$$\hat{\rho} = \exp[-\beta\hat{H}] \approx Id - \beta\hat{H} \qquad \langle A \rangle = \frac{Tr[\rho.A]}{Tr[\rho]}$$

$$\frac{1}{k_B T} = \frac{5B_Q + 9\overline{V}}{24B_Q^2 + 24V_{eff}^2} \qquad P_{m_s} = \frac{1}{7}\left(1 + \beta B_Q(4 - m_s{}^2)\right)$$

where: $V_{eff}^2 = \sum V_{(i,j)}^2 \qquad \overline{V} = \sum V_{(i,j)}$

Consistent with the eigenstate thermalization hypothesis

(~the thermal character is built in the eigenstates themselves)



An effective temperature (a few nK) for an isolated (somewhat pure) system

Outline:

1- Thermalization of the Zeeman populations

2- Thermalization of the collective spin

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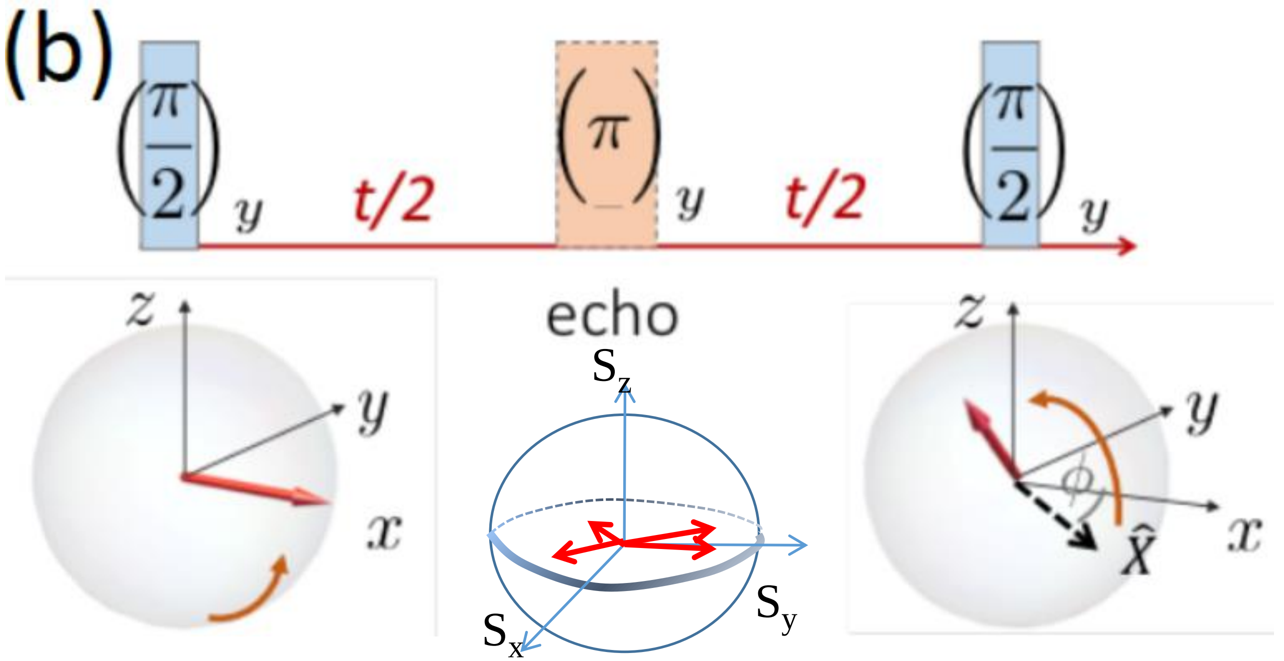
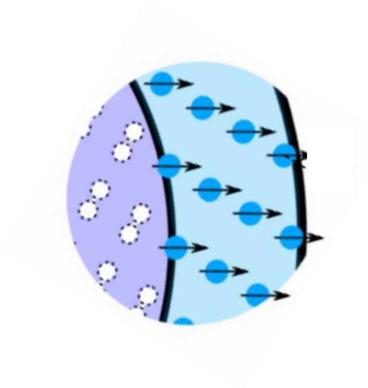
Measuring the collective spin through Ramsey interferometry

$$\ell = \sqrt{\langle S_x \rangle^2 + \langle S_y \rangle^2 + \langle S_z \rangle^2}$$

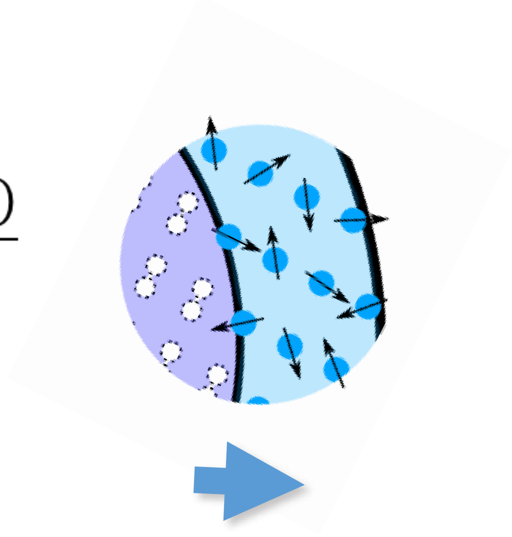


Random direction
shot-to-shot

Fluctuations of Sz
provide a
measurement of
contrast, i.e.
spin length



$$H = c_{dd} \sum_{(i,j)} \left[S_i^z . S_j^z - \frac{1}{4} (S_i^+ . S_j^- + S_i^- . S_j^+) \right] \frac{(1 - 3\cos^2\theta_{ij})}{r_{ij}^3}$$



Measuring the collective spin through Ramsey interferometry

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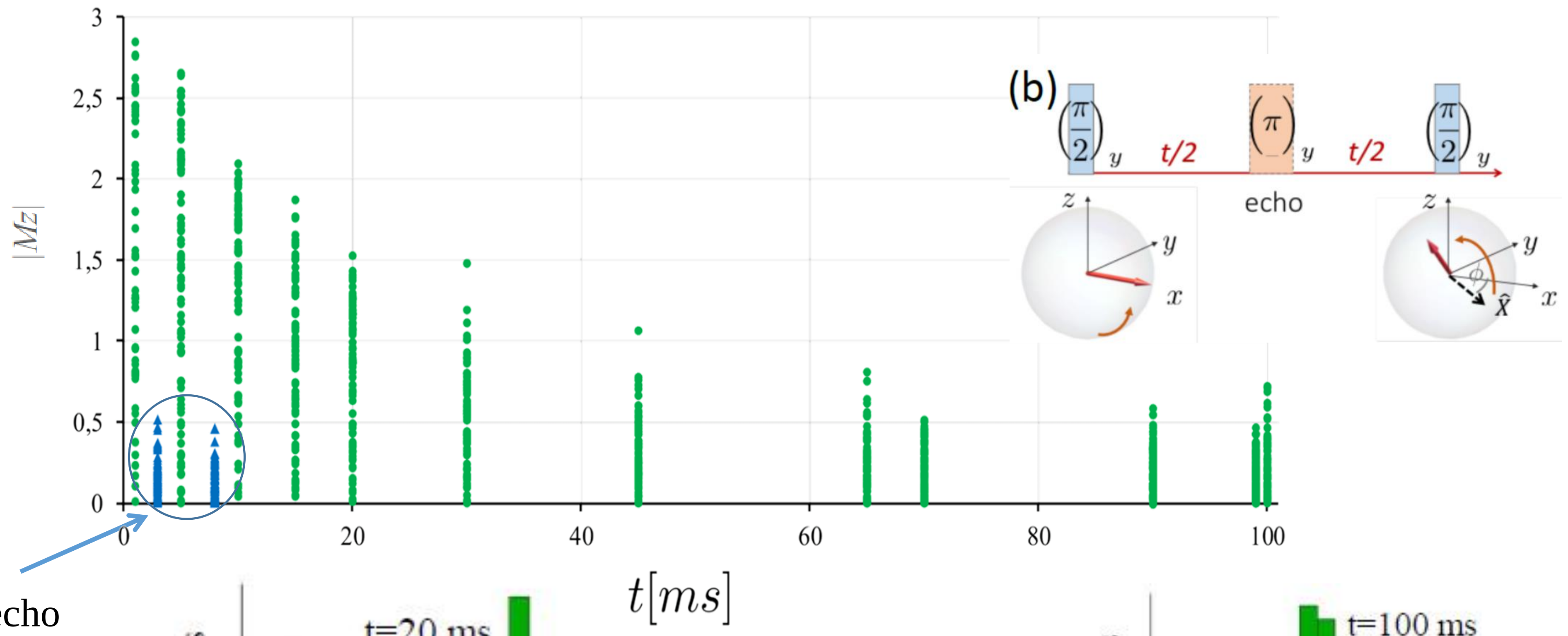
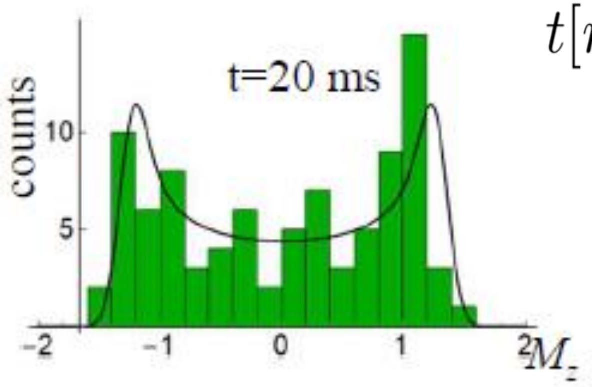
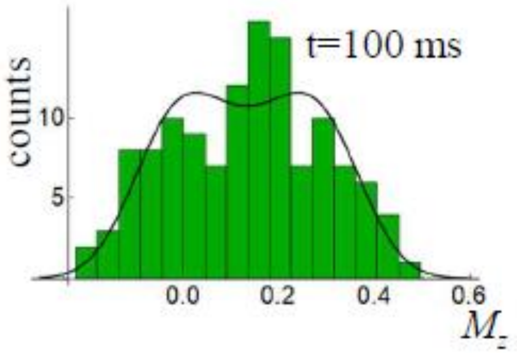
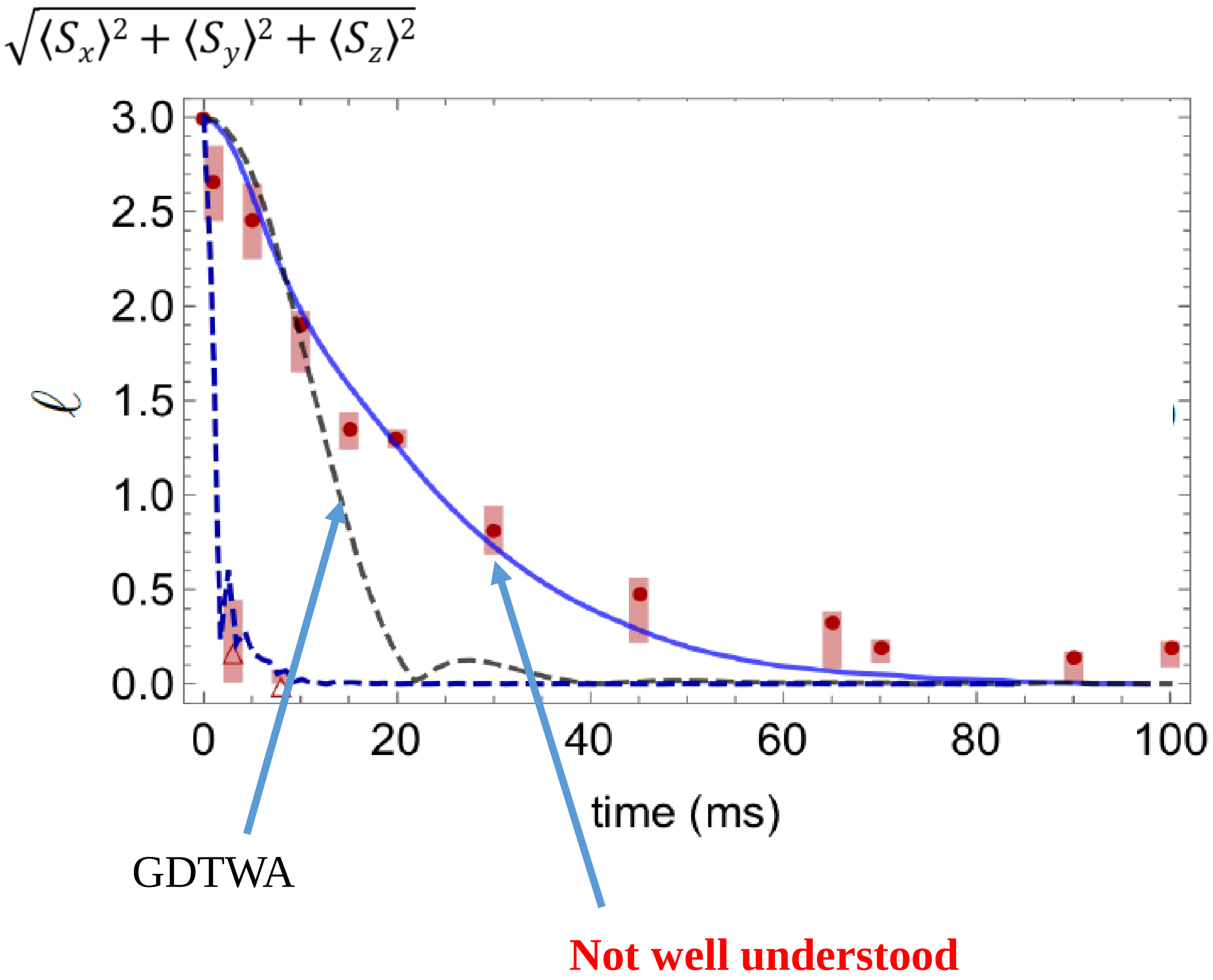


Without spin echo





Damping of the collective spin due to dipolar interactions



$$|S_\perp|(t) \underset{t\to 0}{\approx} |S_\perp|\left(1 - t^2\left[\Delta B^2 + \frac{1}{N}\sum V_{i,j}^2\right]\right)$$

Classical inhomogeneous precession (↔variance of mean-field)

Beyond mean-field

Hazzard et al., PRL **110**, 075301 (2013)

Note that the damping of the spin is a purely dipolar beyond mean-field effect for a homogeneous system, associated with the growth of entanglement

Good agreement at short times
Good agreement with second-order perturbation theory too

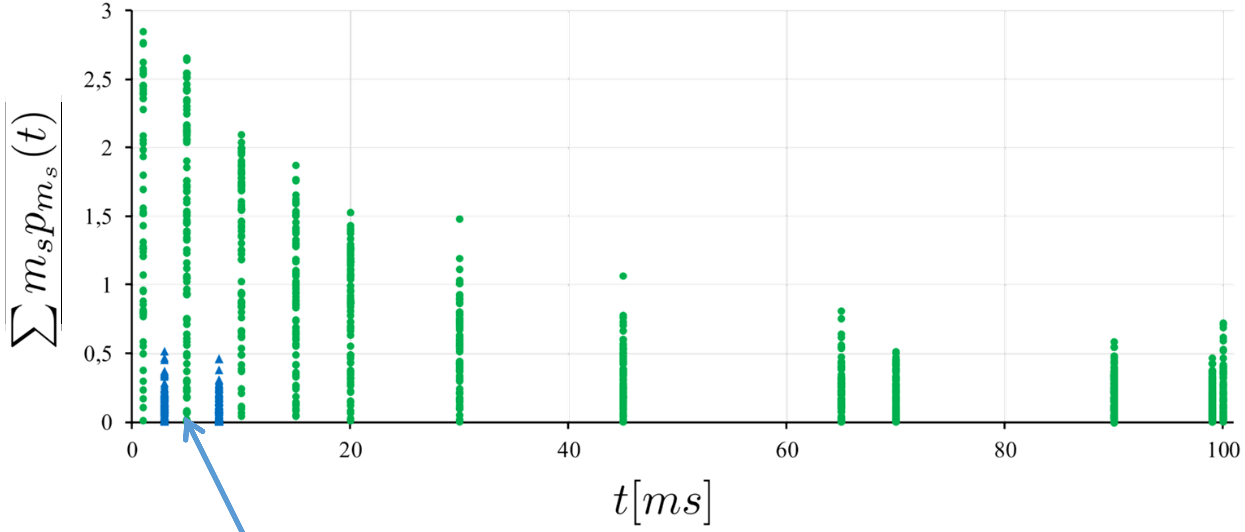
See also J. Ye (KRb molecules)
Weidemüller (Rydberg atoms)

Spin-length data with and without lattice

Collective spin length $\sqrt{\langle S_x\rangle^2 + \langle S_y\rangle^2 + \langle S_z\rangle^2}$ from Ramsey interferometer

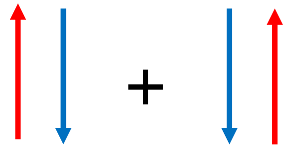
quantum thermalization

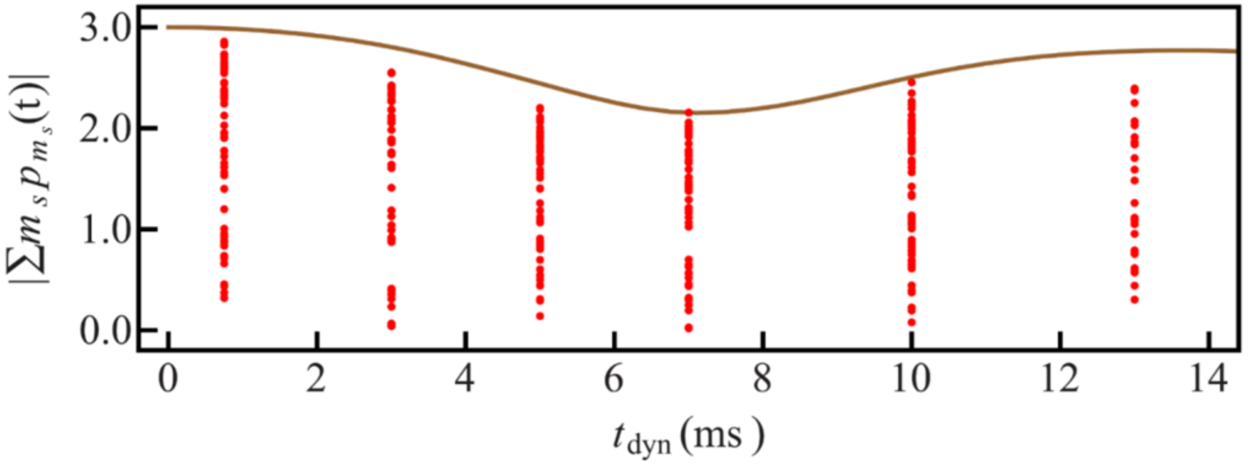
PRL 125, 143401 (2020)



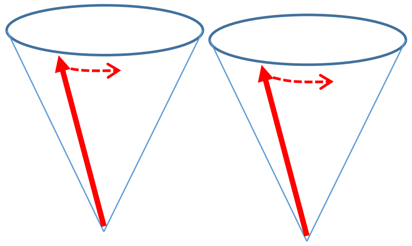
Without spin echo

Lattice case : decrease of spin length due to dipolar interactions





Bulk case :
Spins remain almost locked despite magnetic field gradient



preservation of ferromagnetism

Classical ferrofluid

Spin gap $\propto \dfrac{(a_6 - a_4)\hbar^2 n}{m}$

PRL 121, 013201 (2018)

Partial conclusions on the collective spin measurements

Strong decay of collective spin, associated with dipole-dipole interactions

The decays is « too » slow.

→ **heating** in the lattice ?
→ Are there more **holes** than we thought ?
→ effect of **losses** ?
→ more subtle effect associated with possibly disorder ?
(see glassy dynamics observed with Rydberg atoms Phys. Rev. X **11**, 011011 (2021))

The measurement of coherences (the contrast of the interferometer) gives access to information we could not reach by simply measuring populations.

NB: $\sqrt{\langle S_x \rangle^2 + \langle S_y \rangle^2 + \langle S_z \rangle^2}\Big|_{t\to\infty} \approx 0$ can be related to $P_{ms} = \frac{1}{7}\left(1 + \beta B_Q(4 - m_s{}^2)\right) \approx exp[-\beta B_Q m_s^2]$

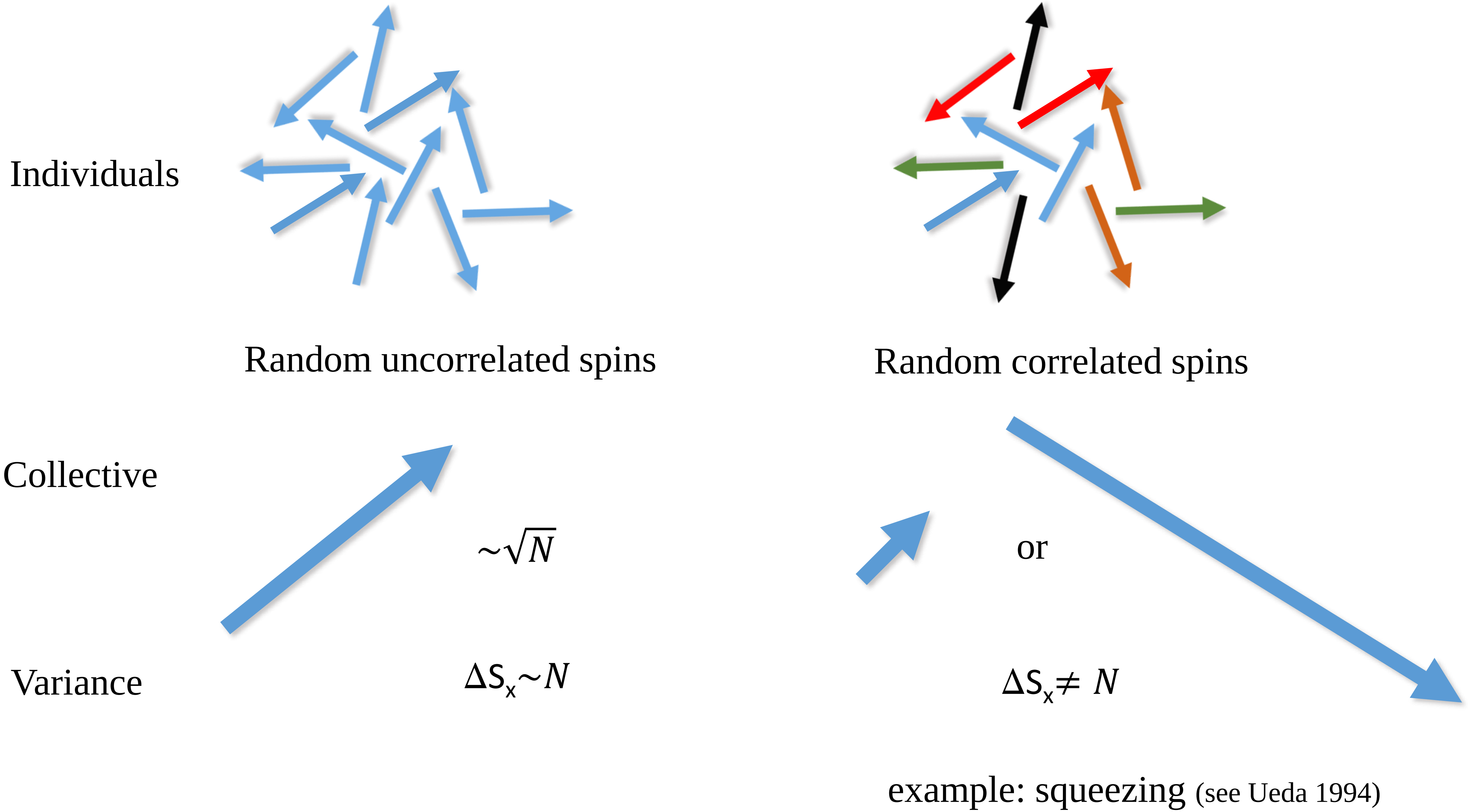
At equilibrium, the strongly interacting many-body system looks like a non-interacting one !

Outline:

1- Thermalization of the Zeeman populations

2- Thermalization of the collective spin

3- Experimental measurement of correlations using collective measurements


Individuals
Random uncorrelated spins
Random correlated spins
Collective
$\sim\sqrt{N}$
or
Variance
$\Delta S_x \sim N$
$\Delta S_x \neq N$
example: squeezing (see Ueda 1994)

What we learn from variances (ONLY for s>1/2)

$$\left\langle \hat{S}_z^2 \right\rangle = \left\langle \left(\sum_{i=1}^{N} \hat{s}_{z,i} \right)^2 \right\rangle = \left\langle \sum_{i=1}^{N} \hat{s}_{z,i}^2 \right\rangle + \left\langle \sum_{i\neq j} \hat{s}_{z,i}.\hat{s}_{z,j} \right\rangle$$

$$\textsc{Var}\left(\hat{S}_z \right) + \sum_i \langle \hat{s}_i^z \rangle^2 = \sum_{m_s} m_s^2 P_{m_s} + \left\langle \sum_{i\neq j} \hat{s}_{z,i}.\hat{s}_{z,j} \right\rangle - \sum_{i\neq j} \langle \hat{s}_{z,i} \rangle . \langle \hat{s}_{z,j} \rangle$$

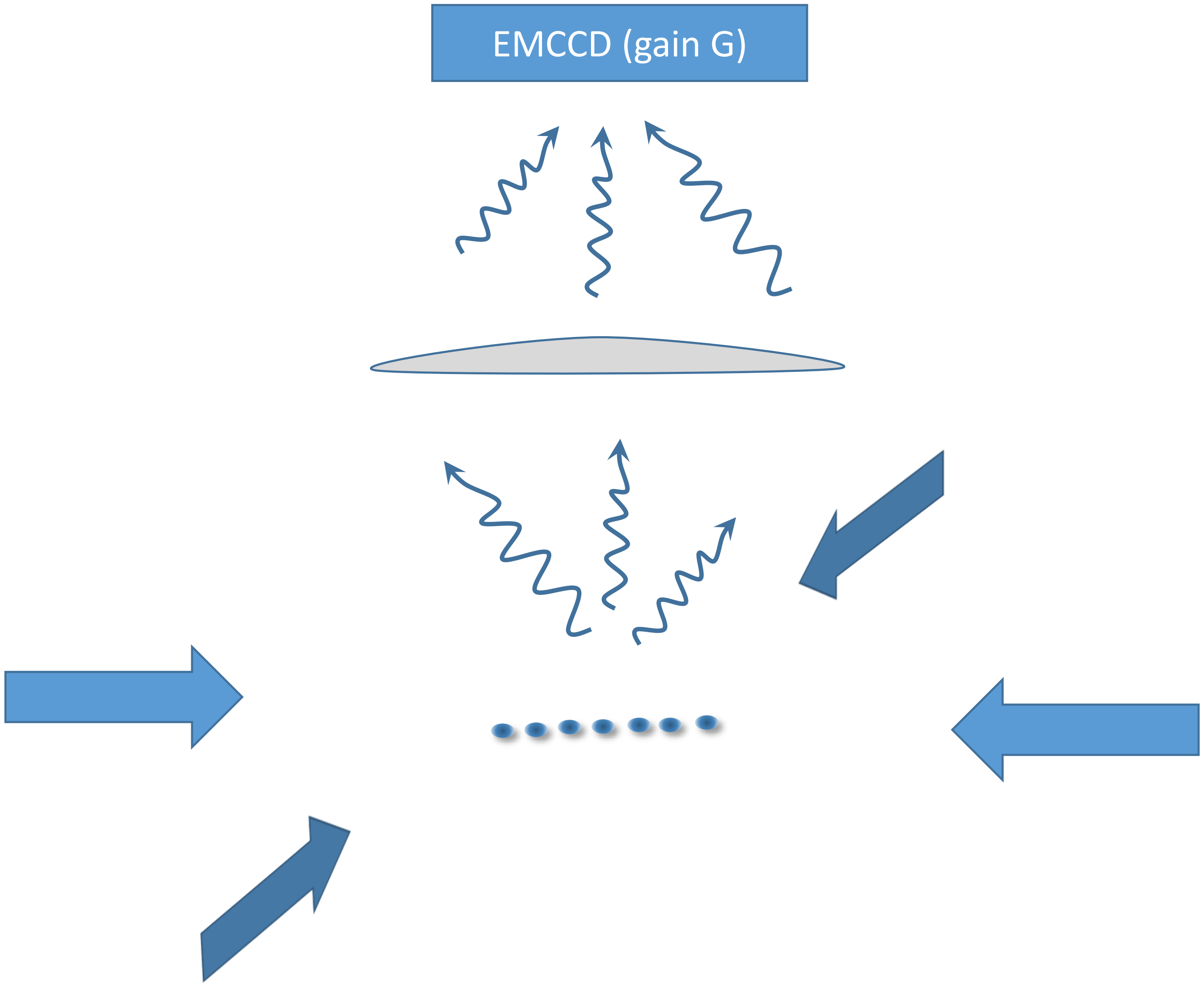
Measured by the fluctuation of the collective spin (many images)

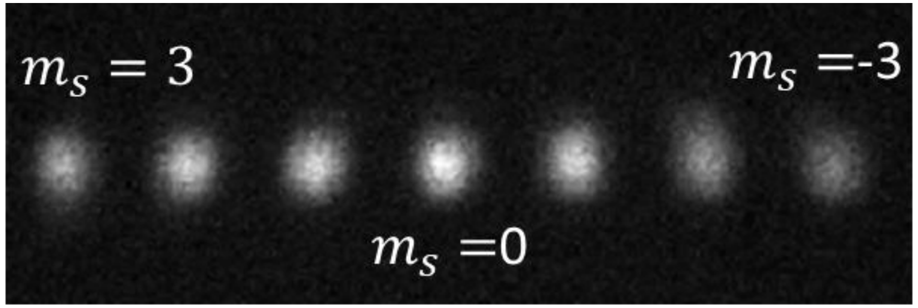
Nm_z^2 for a homogeneous system

Measured by a single image

$$\textsc{CoVar}(N_{m_1} N_{m_2}) = -NP_{m_1}P_{m_2} + \left\langle \sum_{i\neq j} \hat{n}_{m_1,i}.\hat{n}_{m_2,j} \right\rangle - \sum_{i\neq j} \langle \hat{n}_{m_1,i} \rangle . \langle \hat{n}_{m_2,j} \rangle$$

Measuring quantum variance from fluorescence imaging



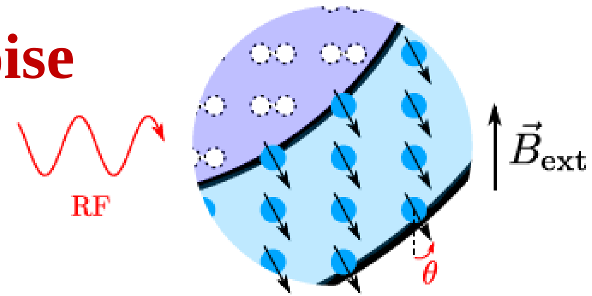


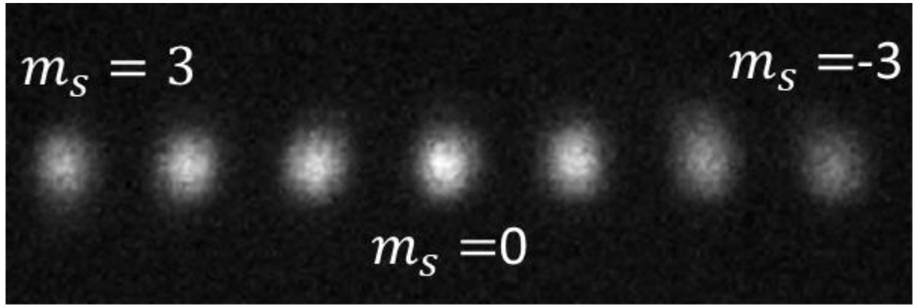
(New imaging system at LPL)

Measuring quantum variance from fluorescence imaging

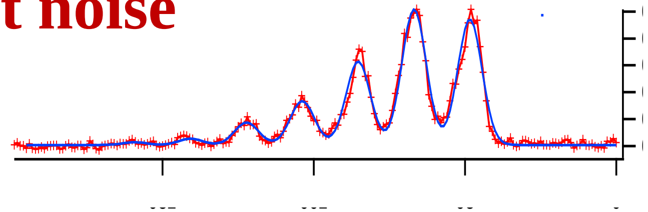
Preparation noise

$Var(Sz) \propto N^2$





Fit noise



$Var(Sz) \propto 1$

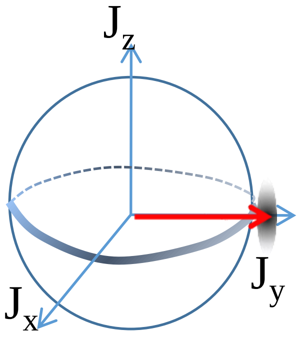
Photon shot noise

$$\frac{Noise}{Signal} = \sqrt{\frac{1}{\beta N}} \qquad Var(Sz) \propto \frac{1}{\beta} N$$

Photon collection efficiency

Quantum projection noise

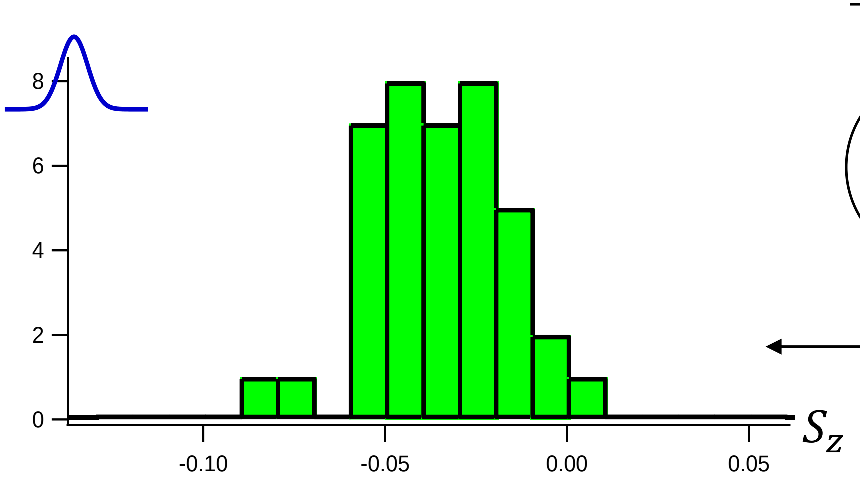
$$Var(S_z) = \frac{3}{2} N$$

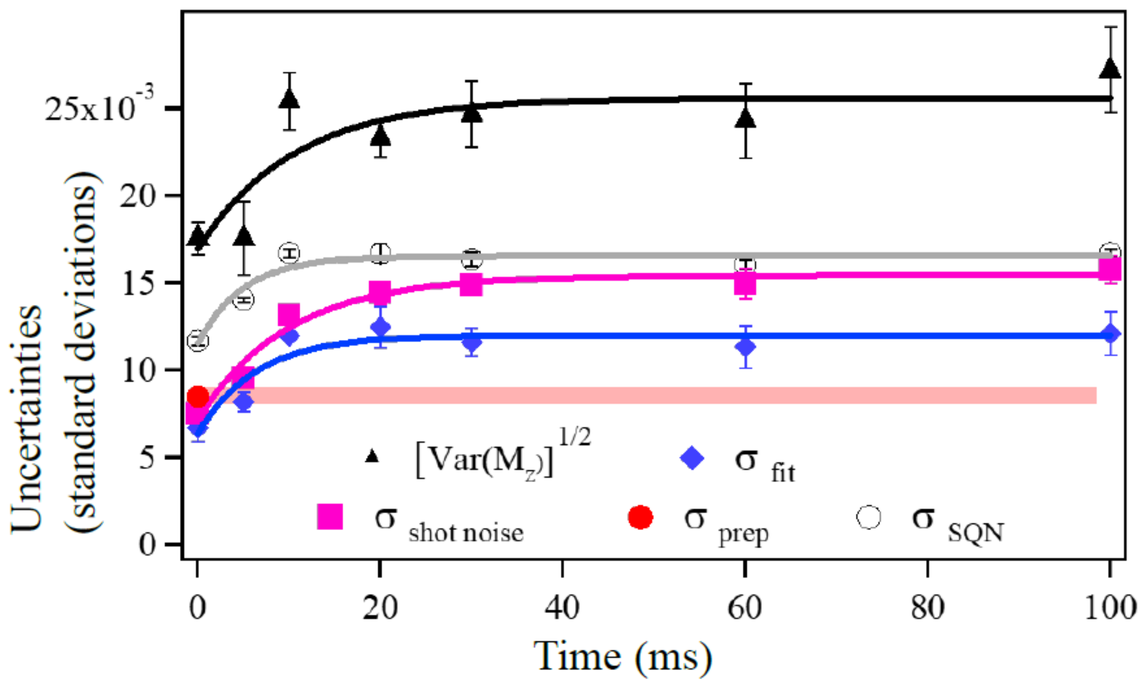


Currently: Technical Noise $(\alpha N^2) \sim .5\ SQN(\alpha N)$ (@10000 $Atoms$)

Measuring quantum variance from fluorescence imaging

(Magnetization) $S_z = \sum m_s p_{m_s}$





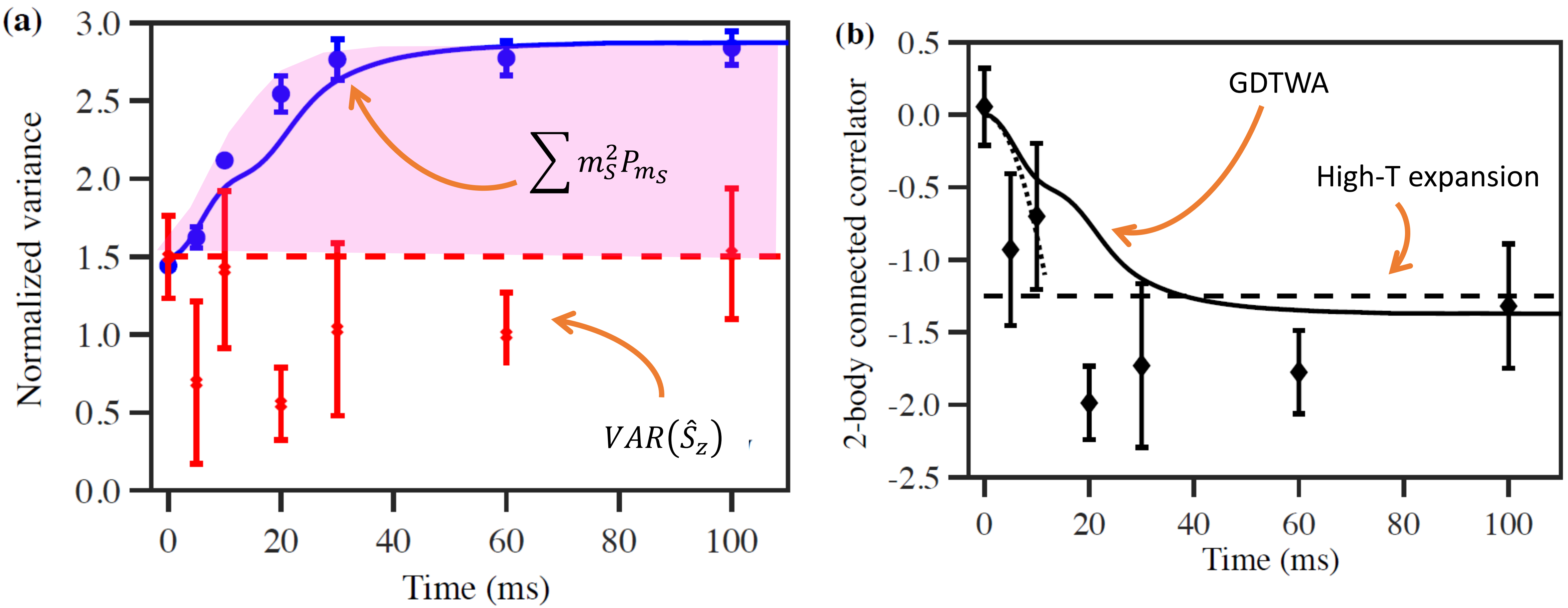
Estimated from first principles

Estimate the quantum projection noise by:

$$Var(Mz) = Var_{exp} - Var_{rf} - Var_{fit} - Var_{shotnoise}$$

Fitted to data at short time

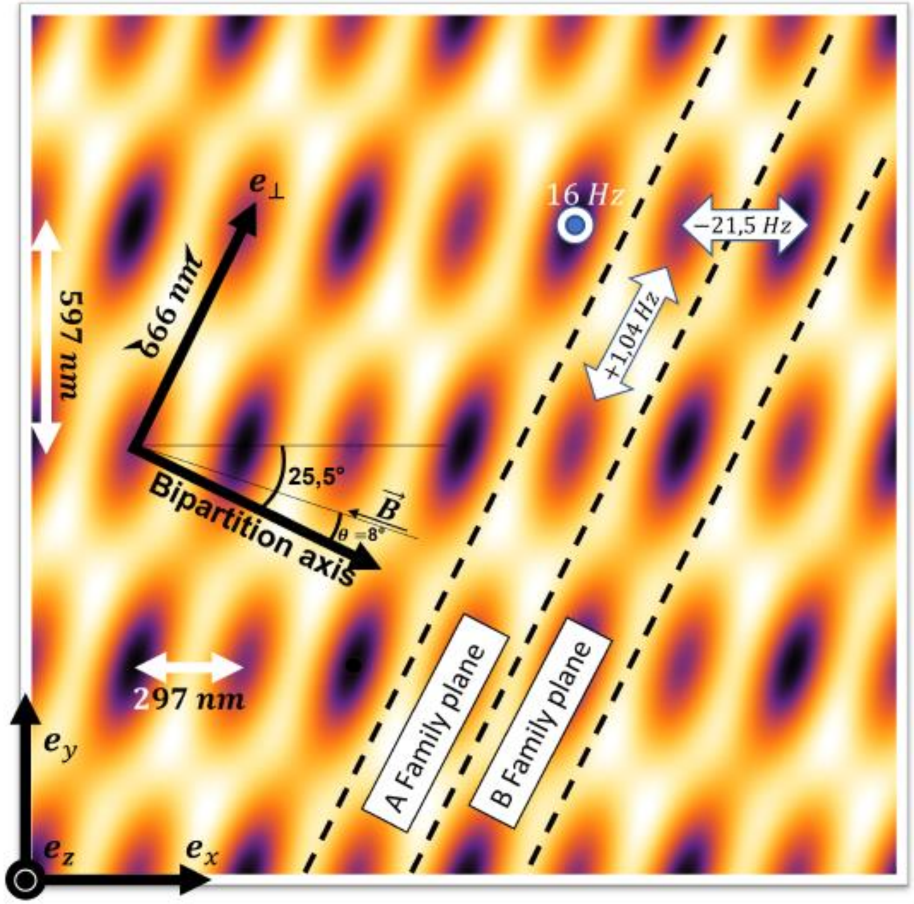
Measurement of connected two-body correlations

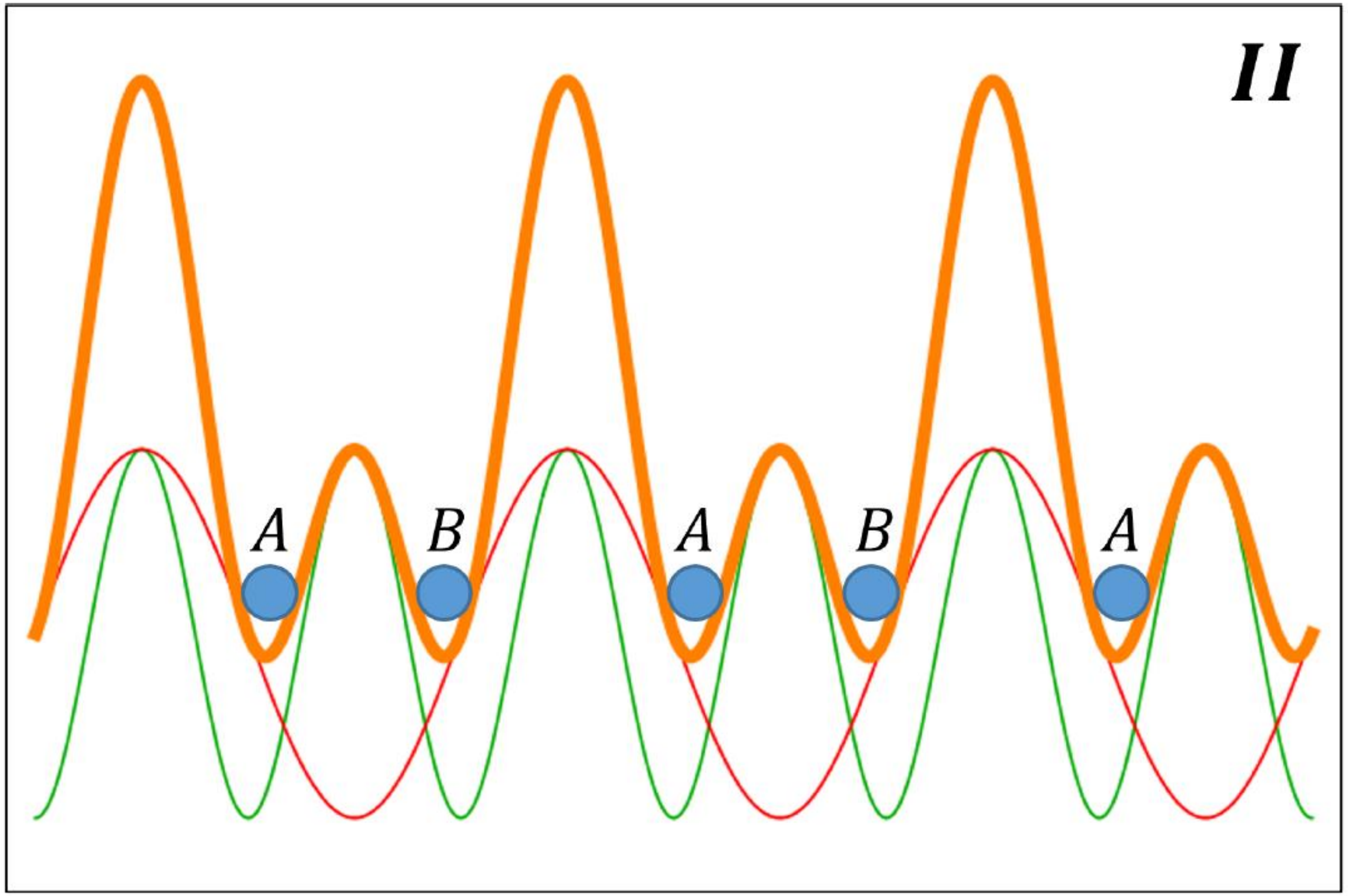


$$\mathrm{VAR}\left(\hat{S}_z\right) + \sum_i \langle \hat{s}_i^z \rangle^2 = \sum_{m_s} m_s^2 P_{m_s} + \left\langle \sum_{i \neq j} \hat{s}_{z,i} . \hat{s}_{z,j} \right\rangle - \sum_{i \neq j} \langle \hat{s}_{z,i} \rangle . \langle \hat{s}_{z,j} \rangle$$

~0

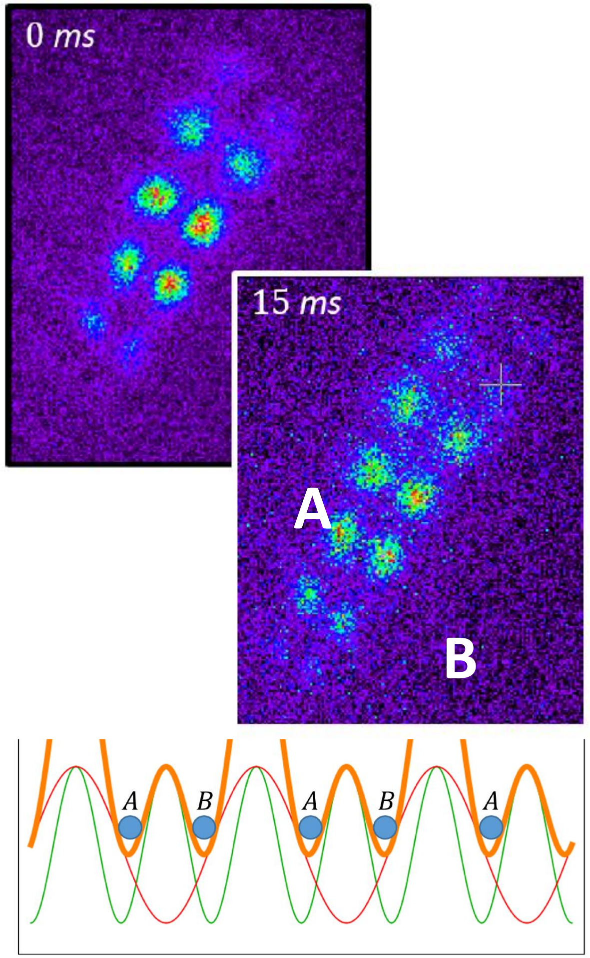
Bi-partite measurements

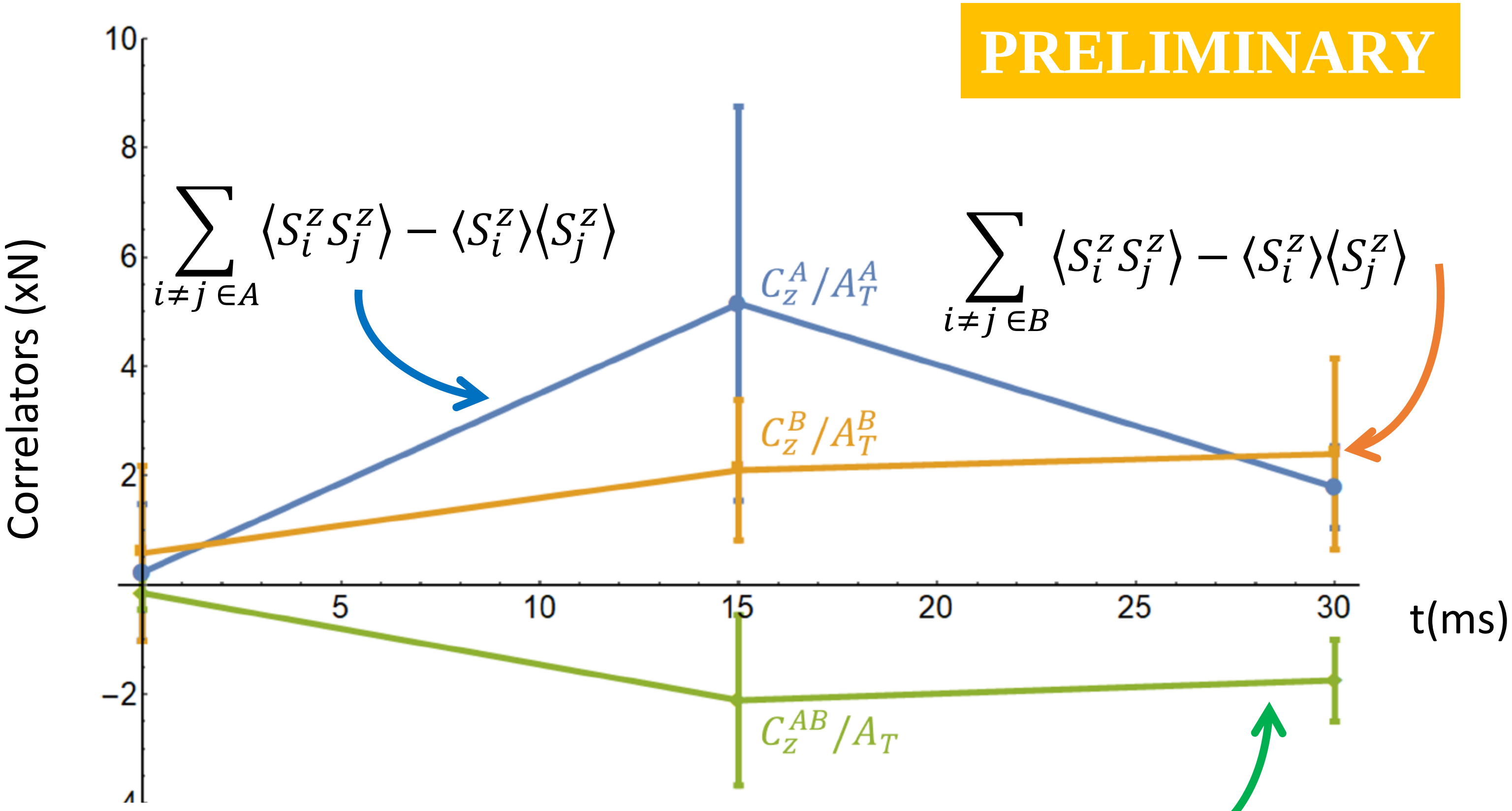




PRELIMINARY

Observation of anisotropic correlations due to anisotropy of dipolar interactions





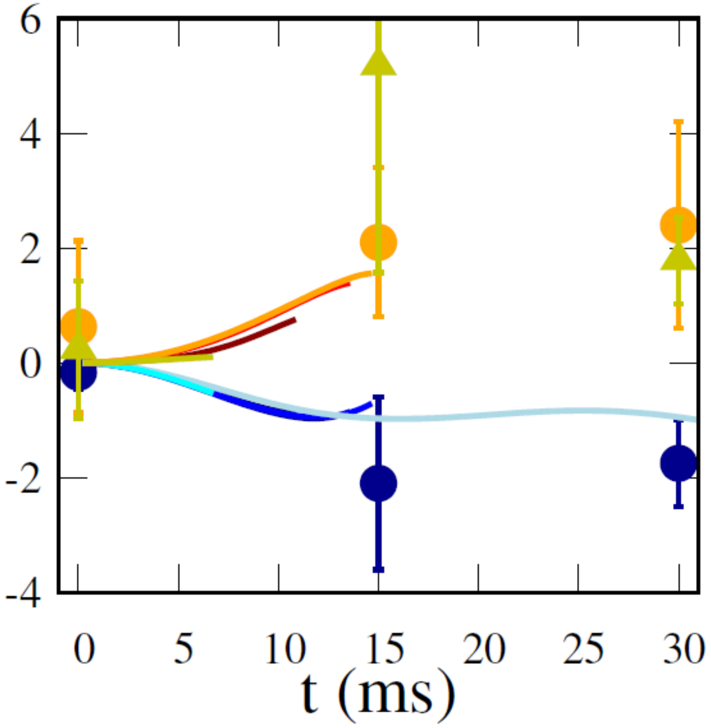
$$\sum_{i\in A,j\in B}\langle S_i^z S_j^z\rangle-\langle S_i^z\rangle\langle S_j^z\rangle = CoVar(S_A^z,S_B^z)$$

See also W. Bakr (NaRb molecules under microscope)

$2\,CoVar(S_A^z,S_B^z)$ = Var(S^z)-Var(S_A^z)-Var(S_B^z)<0 for « classical enough system » where Var(S^z)=0

Comparaisons to simulations

PRELIMINARY



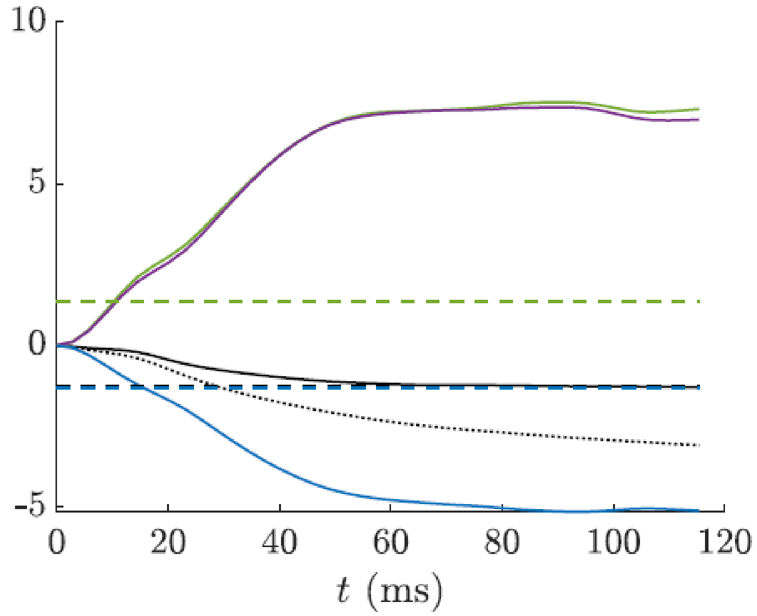
Second order cumulant approach

Gaussian Ansatz for S=3 Holstein-Primakokk Bosons

(Tommaso Roscilde)

GDTWA

(Ana Maria Rey, Sean Muleady)



Some comments:

- These measurements are well-suited to macroscopic systems (but beware of the preparation noise)
- We have found new ways to experimentally characterize various two-body correlations for s>1/2
- Some of those are rather immune to technical/detection noise.
- But require lots of data
- We could maybe go to higher-order correlations using higher order moments (more difficult)

A list of over simplifications I've made:

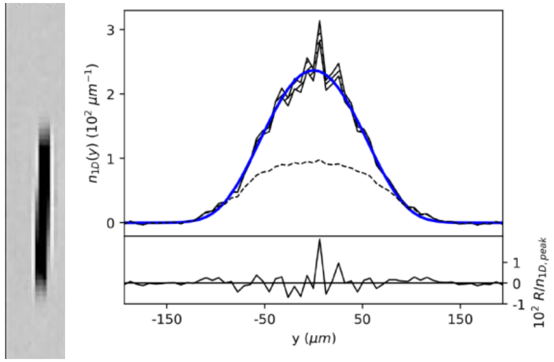
- neglecting (dipolar relaxation) losses (can modify correlations)
- physics beyond singly-occupied sites rather unexplored.
- assuming inhomogeneity can be bad (here it looks ok)

and some questions:

- could we turn the measurements based on co-variances into entanglement witnesses?

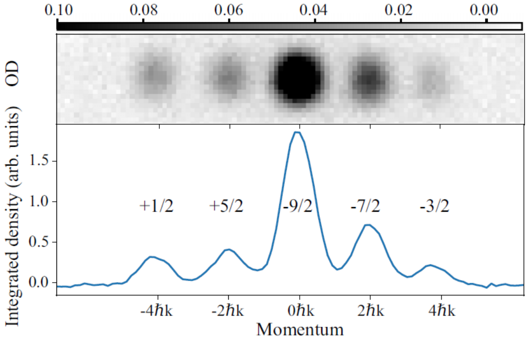
See Irénée Frérot et al., arXiv:2203.13547 (2022)

Measuring objects smaller than resolution limit



PRA **104**, 033309 (2021)

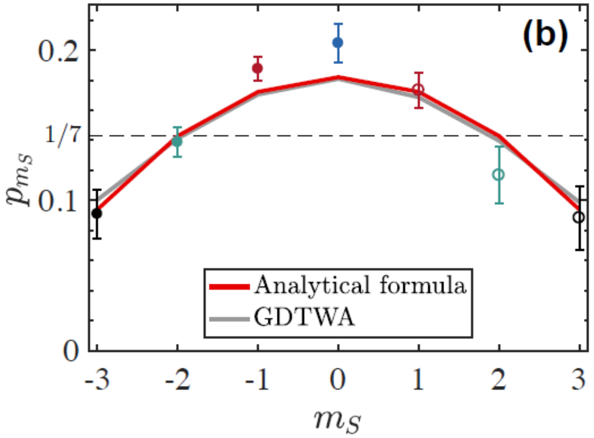
A new spin-orbit scheme to characterize SU(N) Fermi gases



PRA **102**, 013317 (2020)

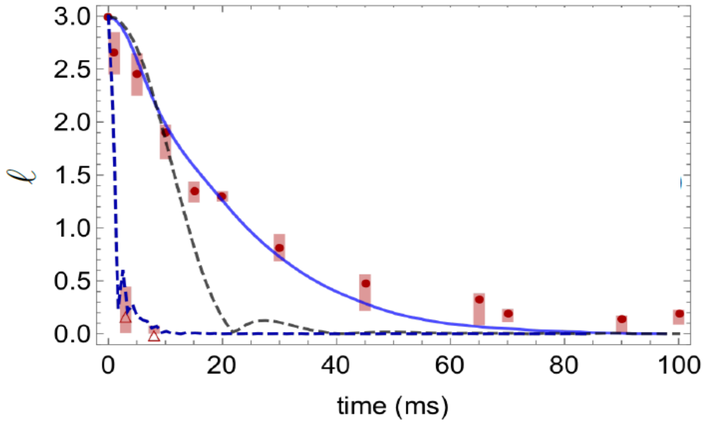
CONCLUSION

Quantum thermalization of populations



Nature Comm. **10**, 1714 (2019)

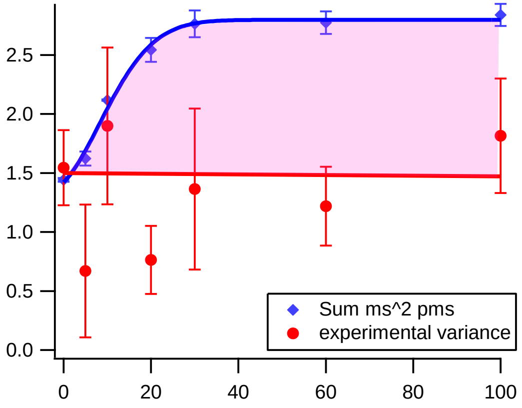
Quantum thermalization of collective spin



PRL **125**, 143401 (2020)

PRL **121**, 013201 (2018)

Measuring correlations from (co-)variances



PRL **129**, 023401 (2022)

Dipolar physics: A review of experiments with magnetic quantum gases [arXiv:2201.02672]

Youssef Aziz Alaoui, Thomas Lauprêtre,
William Dubosclard, S. Lepoutre, L. Gabardos
Ziyad Amodjee, Andrea Litvinov, Pierre Bataille, Husain Ahmed
B. Laburthe-Tolra, O. Gorceix, E. Maréchal, L. Vernac,
M. Robert-de-St-Vincent, Benjamin Pasquiou
K. Kechadi, P. Pedri

A. M. Rey, J. Schachenmayer, B. Zhu, S. Muleady
B. Blakie, Petra Fersterer, Arghavan Safavi-Naini,
T. Roscilde

LPL Laboratoire de physique des lasers

UNIVERSITÉ PARIS 13 NORD

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CEFIPRA

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AGENCE NATIONALE DE LA RECHERCHE ANR

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MINISTÈRE DE L'ENSEIGNEMENT SUPÉRIEUR ET DE LA RECHERCHE

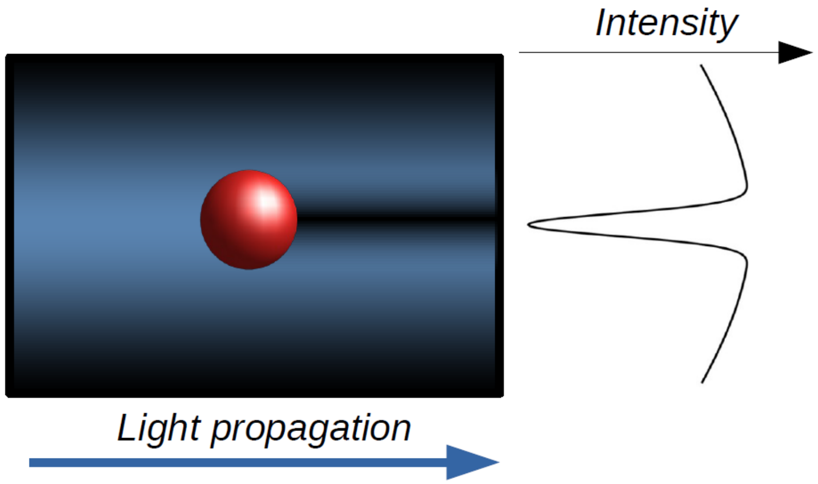
cnrs dépasser les frontières

Warm-up….

Measuring objects smaller than the imaging resolution

Measuring objects smaller than the imaging resolution

Ideal Case

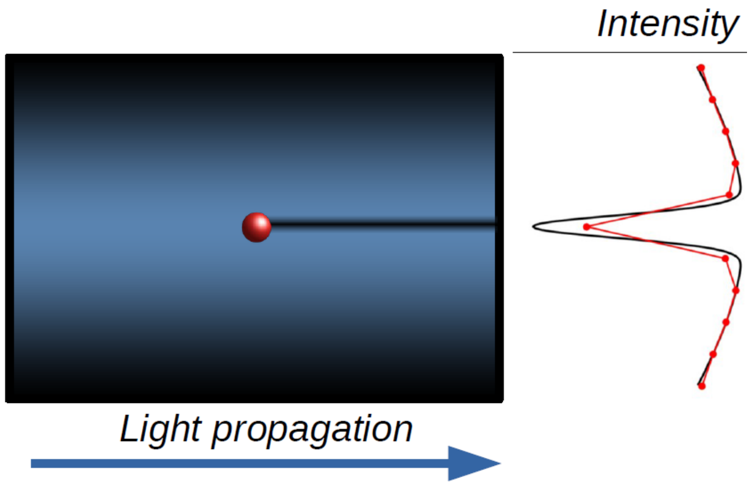


Beer Lambert Law

$$\frac{dI(x,y,z)}{I(x,y,z)} = -n(x,y,z)\sigma_0 dz$$

$$Log\left(\frac{I}{I_0}\right) = -\int dz\, n(x,y,z)\sigma := -OD(x,y)$$

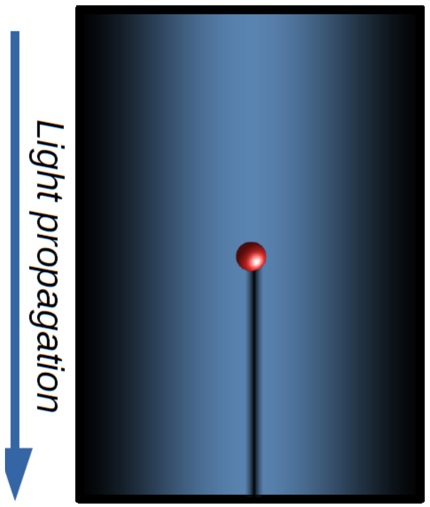
Small objects



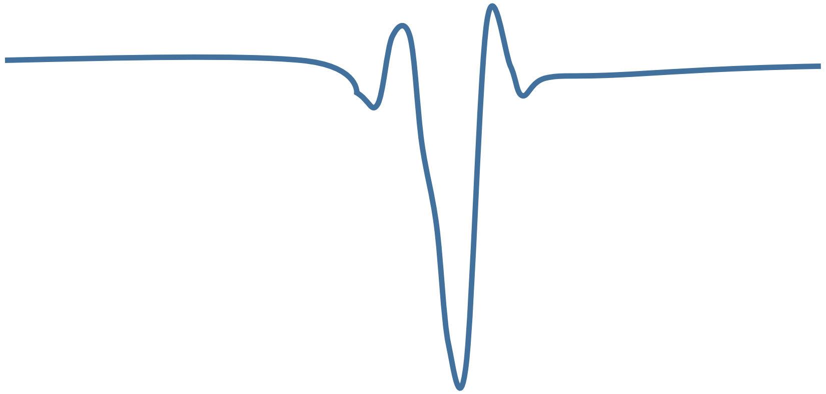
Pixelation

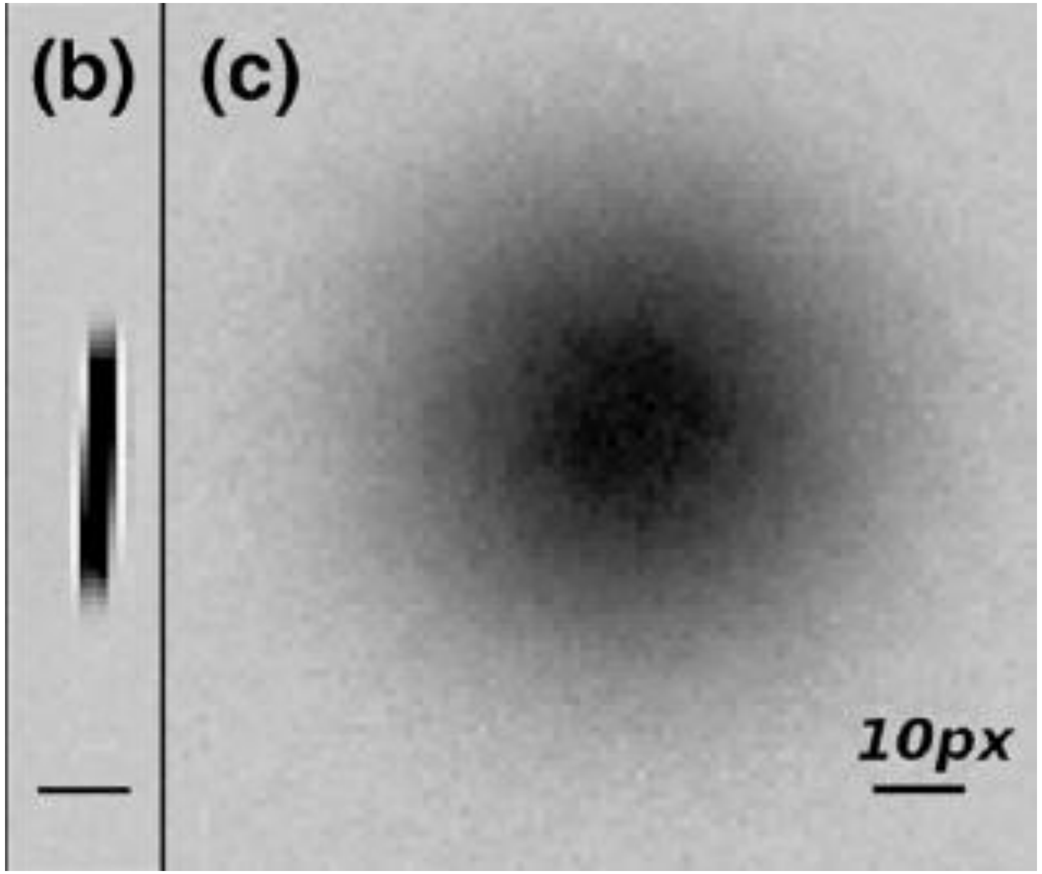
$$P(i,j) = \iint_{D_{i,j}} I(x,y)dx\,dy$$

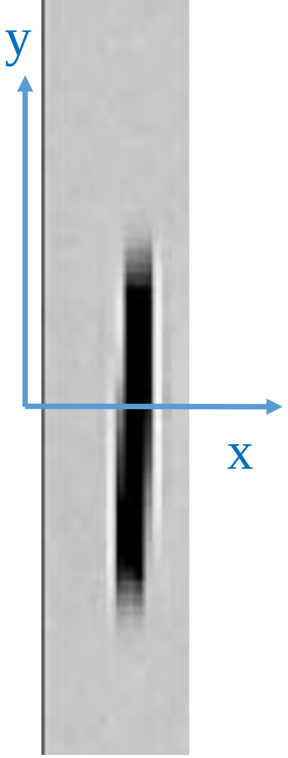
$$\ln\left(\frac{P(i,j)}{P_0(i,j)}\right) \neq \frac{1}{a^2}\iint_{D_{i,j}} \ln\left(\frac{I(x,y)}{I_0(x,y)}\right)dx\,dy$$



+ add diffraction







Idea: counting missing photons

$$Log\left(\frac{I}{I_0}\right) = -\int dz\, n(x,y,z)\sigma \coloneqq -OD(x,y)$$

$$OD(x,y) = OD_m(y) e^{-x^2/w^2}$$ **Ansatz**

$$\frac{P(y)-P_0}{P_0} = \int dx \left(1 - e^{-OD_m(y) e^{-x^2/w^2}}\right)$$

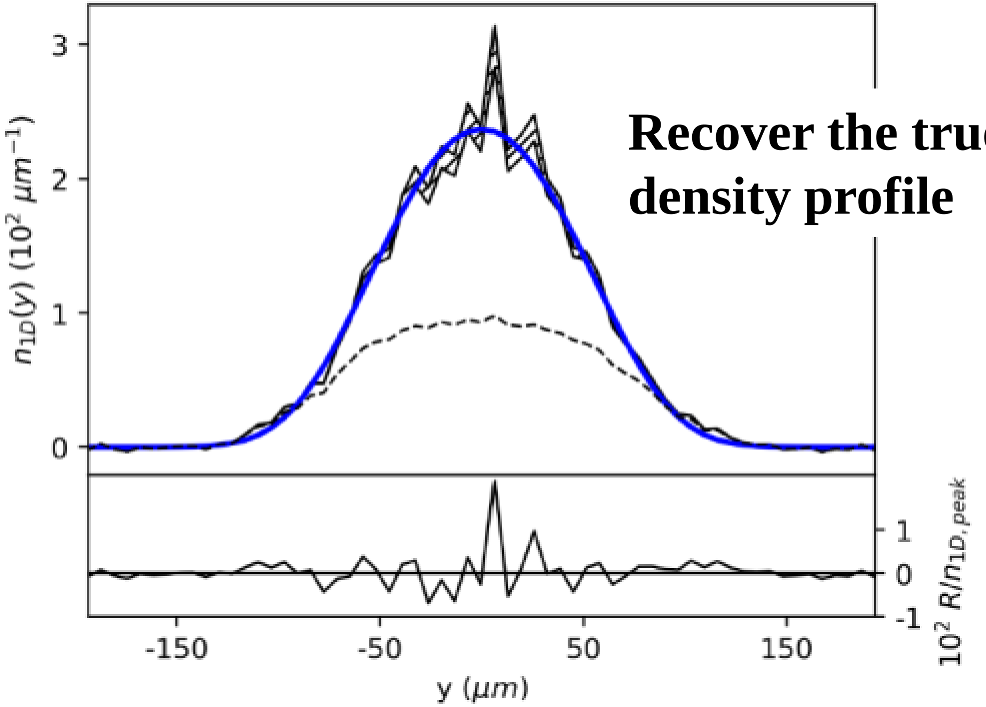
$$= w \int du \left(1 - e^{-OD_m(y) e^{-u^2}}\right)$$

Deduce $OD_m(y)$ from experimentally measured missing photon number.

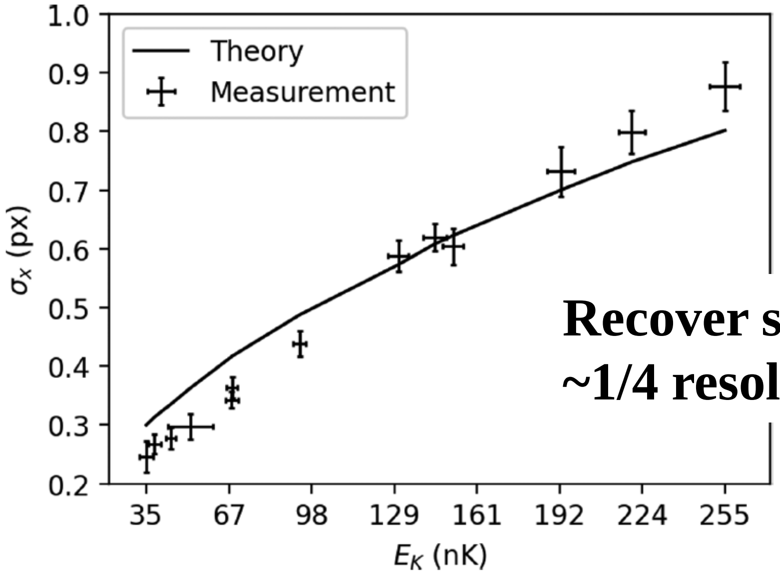
Need additional information

Add the requirement that $\int dx\, dy\, OD(x,y) = N\,\sigma$

Find w, and OD(y)



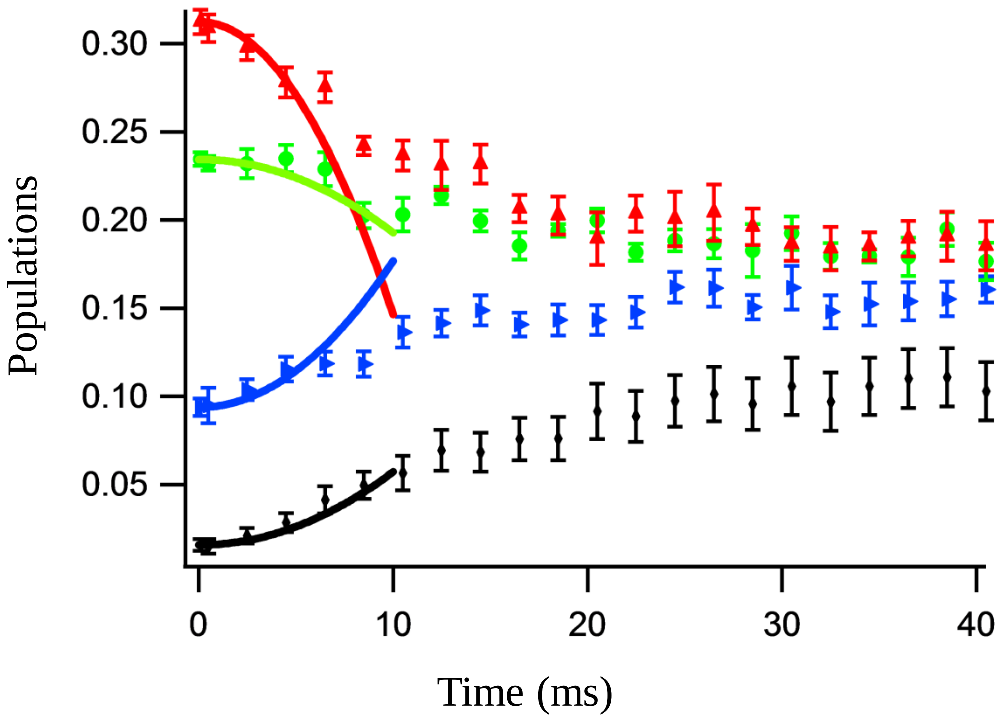
Recover the true density profile

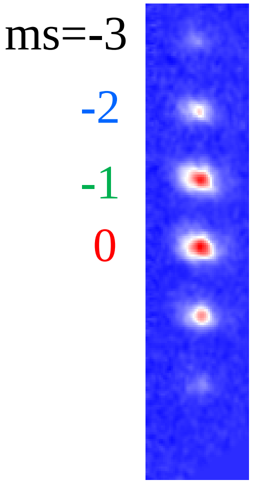


Recover size ~1/4 resolution

PRA **104**, 033309 (2021)

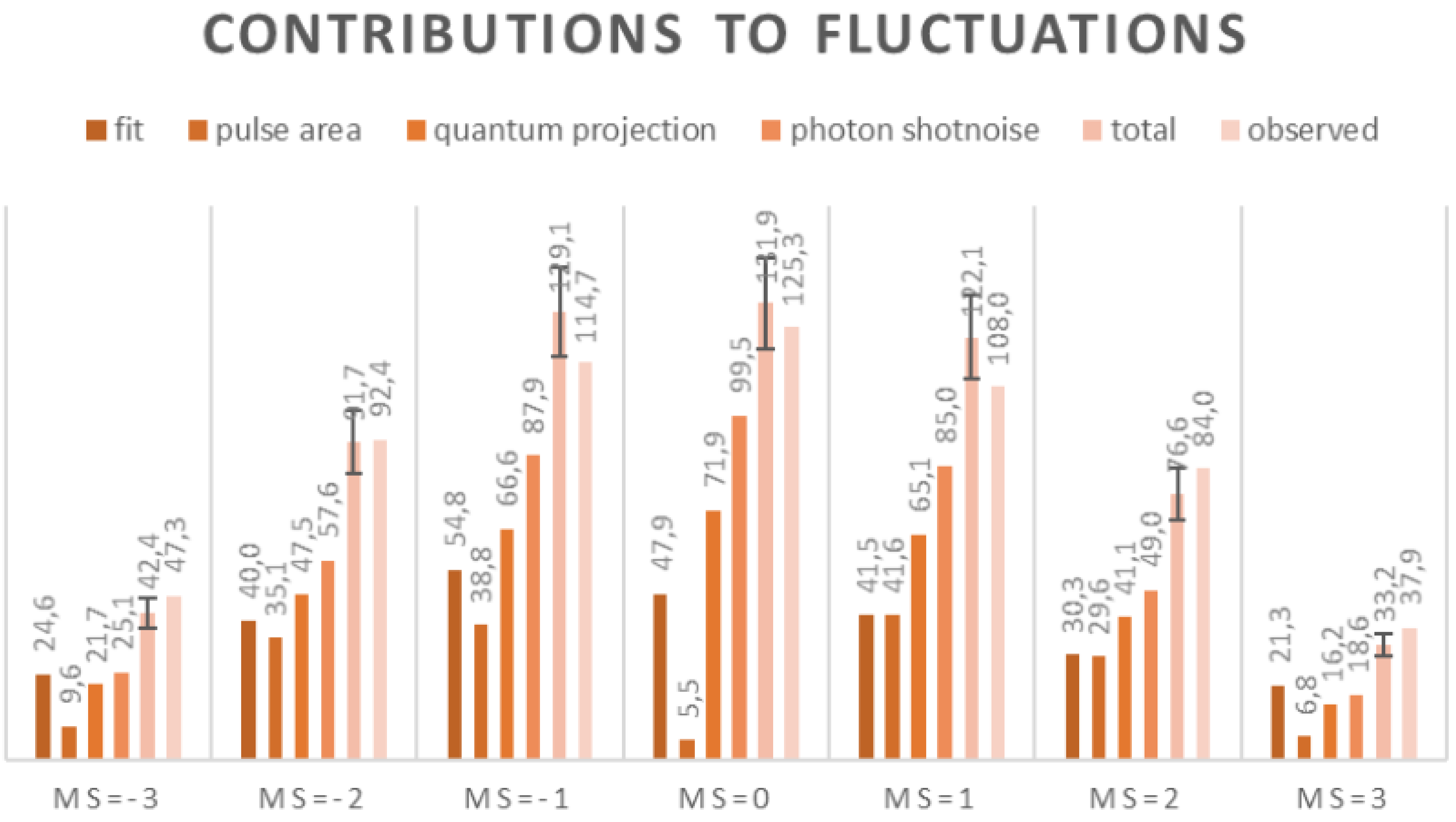
Short term dynamics of the many-body system





Perturbation theory

$$\langle A(t)\rangle = \langle A\rangle - it\langle [A,H]\rangle - \frac{t^2}{2}\langle [[A,H],H]\rangle + \ldots \longrightarrow \quad \Gamma = \sqrt{\sum_{(i,j)} \left(V_{(i,j)}{}^2\right)}$$

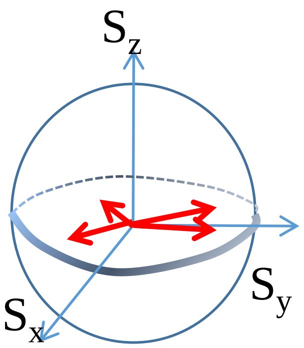


CONTRIBUTIONS TO FLUCTUATIONS
fit
pulse area
quantum projection
photon shotnoise
total
observed
24,6
9,6
21,7
25,1
42,4
47,3
40,0
35,1
47,5
57,6
92,4
54,8
38,8
66,6
87,9
114,7
47,9
5,5
71,9
99,5
125,3
41,5
41,6
65,1
85,0
108,0
30,3
29,6
41,1
49,0
84,0
21,3
6,8
16,2
18,6
33,2
37,9
MS=-3
MS=-2
MS=-1
MS=0
MS=1
MS=2
MS=3

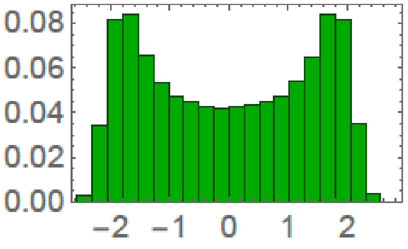
Data analysis: measure collective spins from probability distributions

$$\ell = \sqrt{\langle S_x\rangle^2 + \langle S_y\rangle^2 + \langle S_z\rangle^2}$$

Assume Classical Spin:

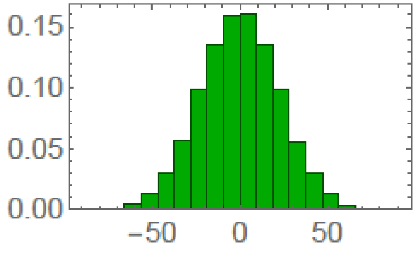


$$\frac{dN}{dM_z} = \frac{1}{\pi\ell}\frac{1}{\sqrt{1-\frac{M_z^2}{\ell^2}}}$$

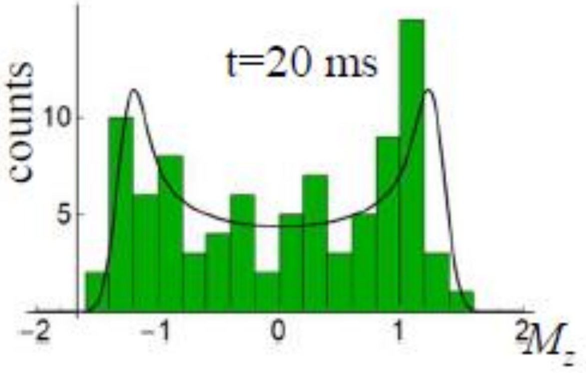


Assume ℓ=0, and Gaussian noise:

$$\frac{dN}{dM_z} = \frac{1}{\sqrt{\pi}\sigma}\exp\left(-\frac{M_z^2}{\sigma^2}\right)$$

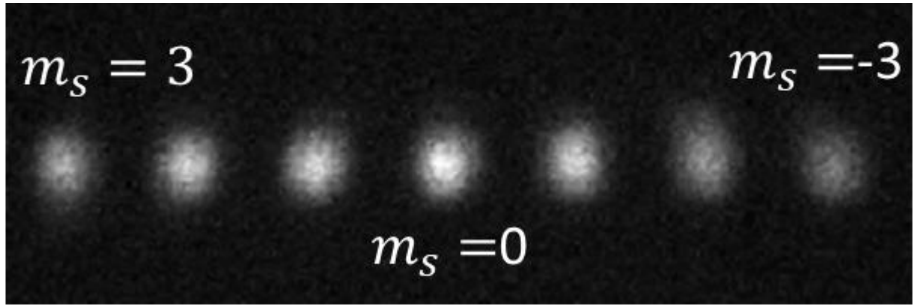


Method to derive ℓ:
fit probability distributions with a convolution of the two distributions

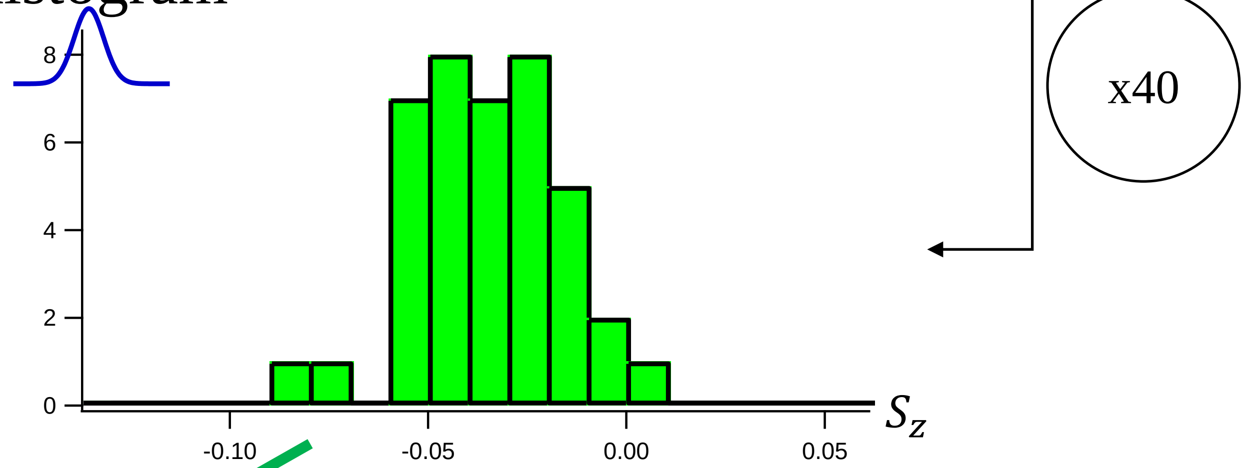


Measuring the variance of magnetization

(Magnetization) $S_z = \sum m_s p_{m_s}$



S_z histogram



Estimated from first principles

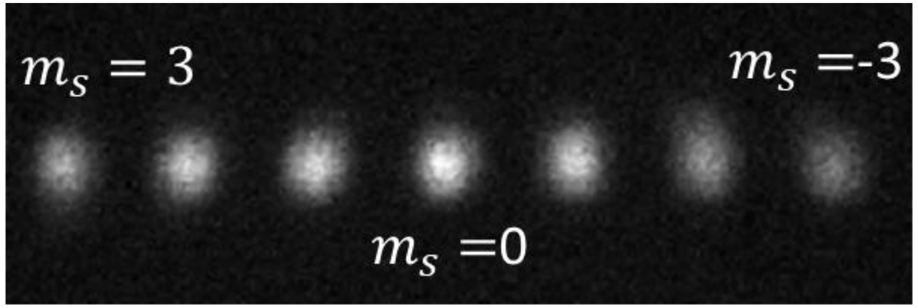
Estimate the quantum projection noise by:

$$Var(Mz) = Var_{exp} - Var_{rf} - Var_{fit} - Var_{shotnoise}$$

Fitted to data at short time

How many images?

One image



N atoms provide **N** measurements for individual populations

N atoms provide **1** measurement for the collective spin S_z

$$P(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-x_0)^2}{2\sigma^2}\right)$$

$$< x^2 > - < x > 2 = \ ^2$$

$$< x^4 > - < x^2 > ^2 = 2(\sigma^4 + 2\sigma^2 x_0^2))$$

Standard deviation on the measurement of x^2 :

$$\sqrt{2(\sigma^4+2\sigma^2 x_0^2))}$$

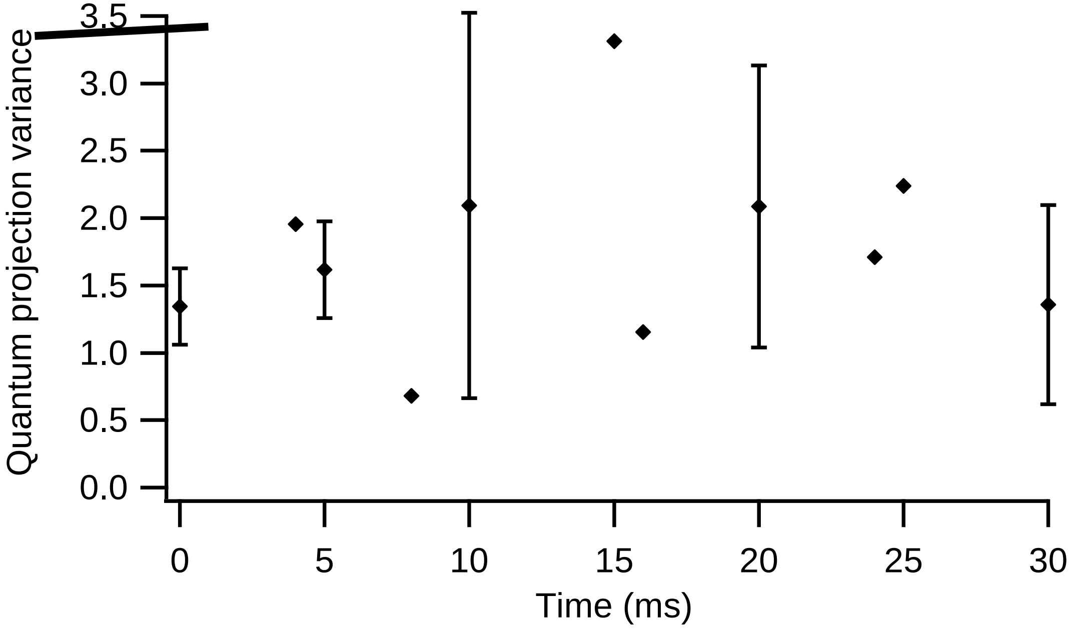
$x_0 \ll \sigma$ $\quad \sqrt{\frac{2}{M}} \ll 1$

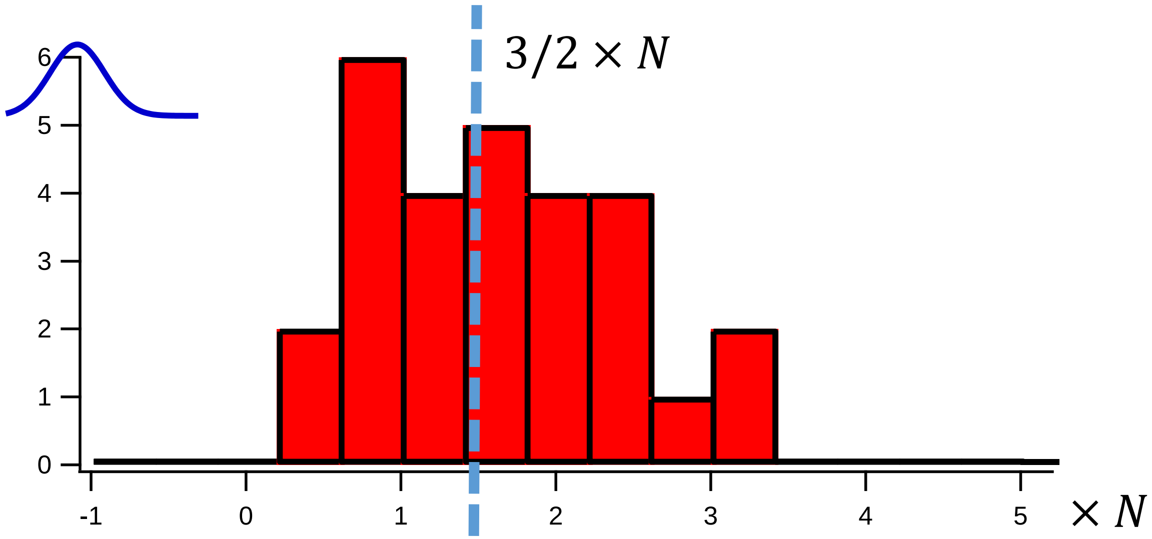
$x_0 \gg \sigma$ $\quad \frac{2x_0}{\sigma}\sqrt{\frac{1}{M}} \ll 1$

(M images)

Variance of the variance

$$Var(Mz) = Var_{exp} - Var_{rf} - Var_{fit} - Var_{shotnoise}$$





Variance is constant $[H, Sz] = 0$

$$H = c_{dd} \sum_{(i,j)} \left[S_i^z . S_j^z - \frac{1}{4}(S_i^+ . S_j^- + S_i^- . S_j^+) \right] \frac{(1 - 3\cos^2\theta_{ij})}{r_{ij}^3}$$

Histogram of the estimated variances

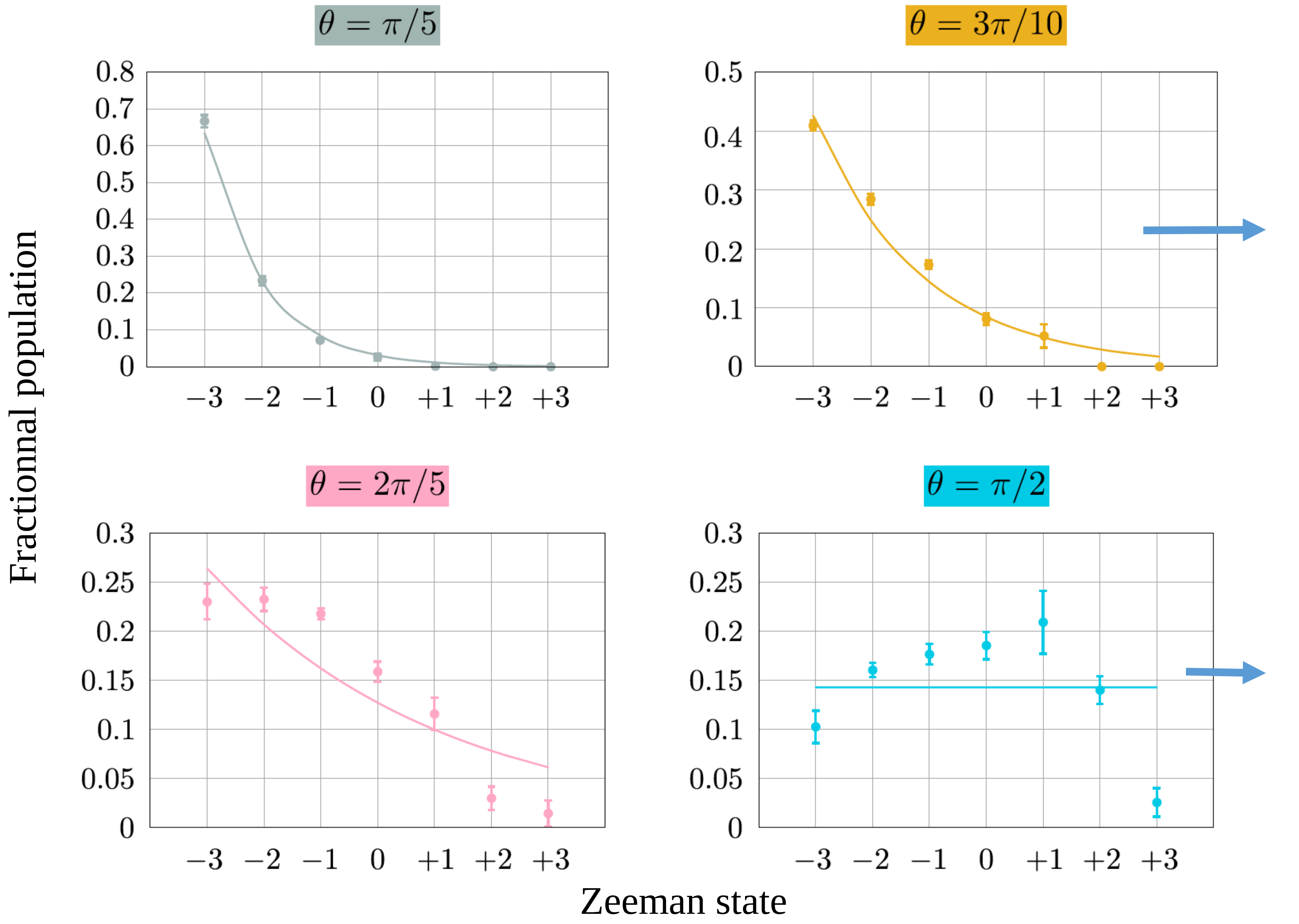
Mean = 1.3

sdv of measurement = .86

Number of measurement = 28

→ error of estimate $.86/\sqrt{28} = .16$

Asymptotic behavior



Small angles:
« Thermal-like behavior »
(maximum of entropy)

Large angles:
Need to go beyond.

Take into account energy constraints.

Two contributions for energy

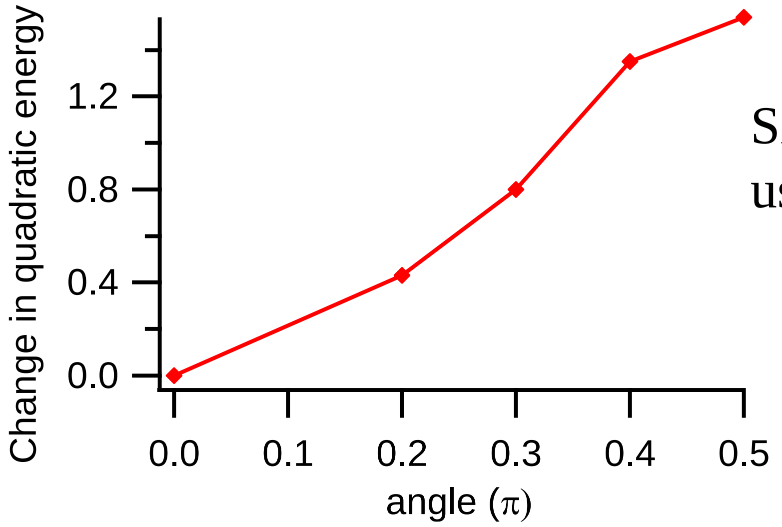
Dipole-dipole interactions

$$\langle \Psi(t) | V_{dd} | \Psi(t) \rangle$$

Difficult to calculate
except at t=0

Tensor light-shift leads to
an effective quadratic Zeeman effect

$$E(m_s) = B_Q m_s{}^2$$



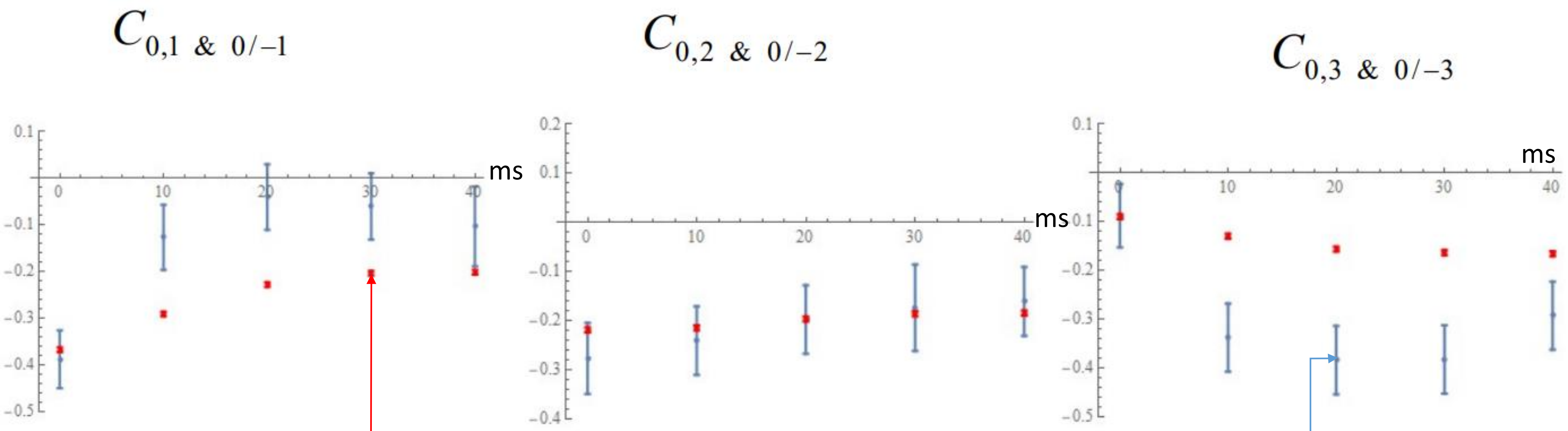
Simple to evaluate
using experimental data

$$\sum m_s^2 p_{m_s}$$

→ **This explains why simply maximizing entropy is sufficient at small angles**

❑ Some results:

- Comparison between measured spin correlations and expected correlations for independent spins (red dots)



$$C(P_{m1},P_{m2})=\frac{-p_{m1}p_{m2}}{\sqrt{p_{m1}-p_{m1}^{2}}\sqrt{p_{m2}-p_{m2}^{2}}}$$

$$C(P_{m_1},P_{m_2})=\frac{Cov(P_{m_1},P_{m_2})}{\sqrt{var(P_{m_1})var(P_{m_2})}}$$

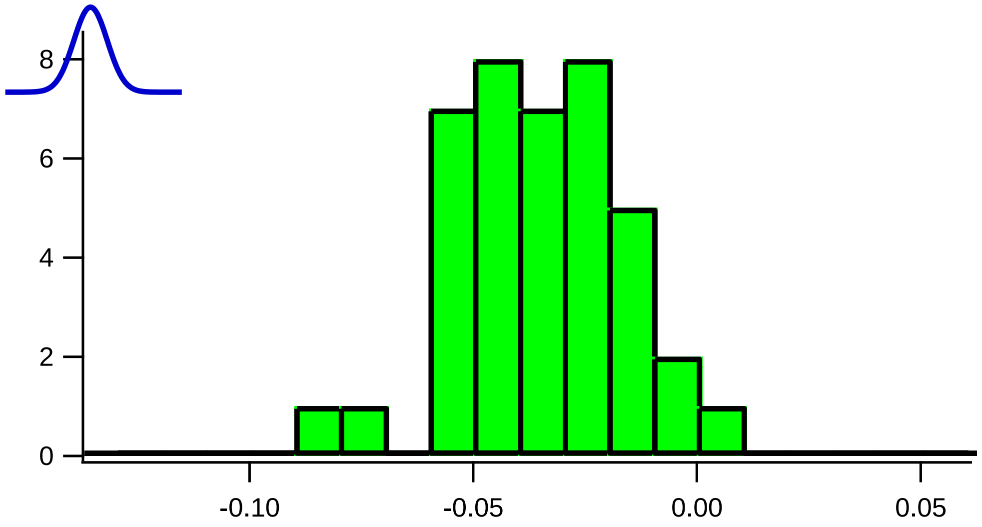
Idea 1: compare variance of Mz and Sum ms^2 Pms

Interest*: *provides a correlation witness and a measurement of the correlations

Weakness: sensitive to detection noise and rf noise

One measurement = Repeat 40 times the same experiment and measure Mz each time

Histogram of results

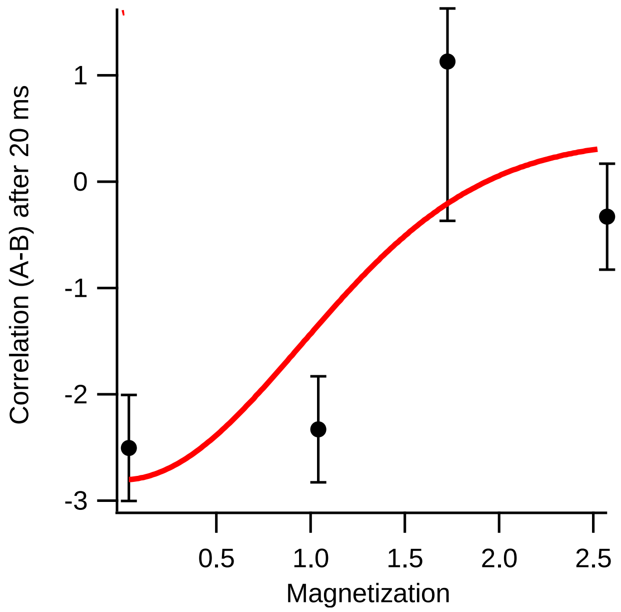


Estimated from first principles

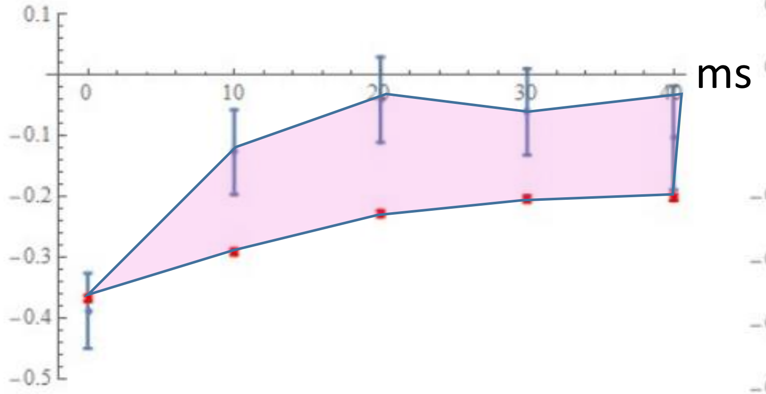
Estimate the quantum projection noise by:

$$Var(Mz) = Var_{exp} - Var_{rf} - Var_{fit} - Var_{shotnoise}$$

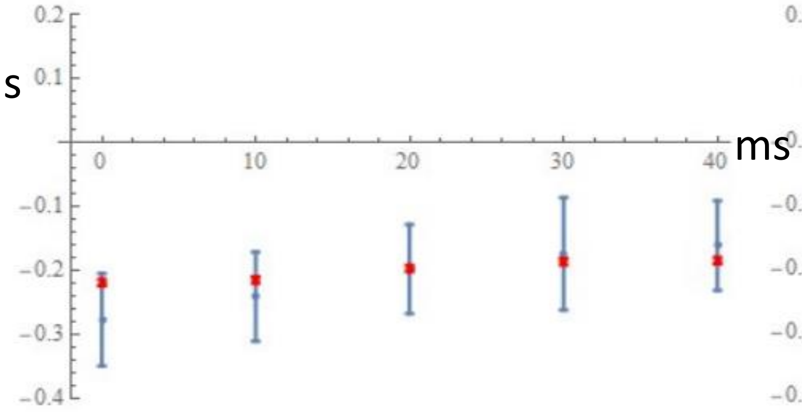
Fitted to data at short time



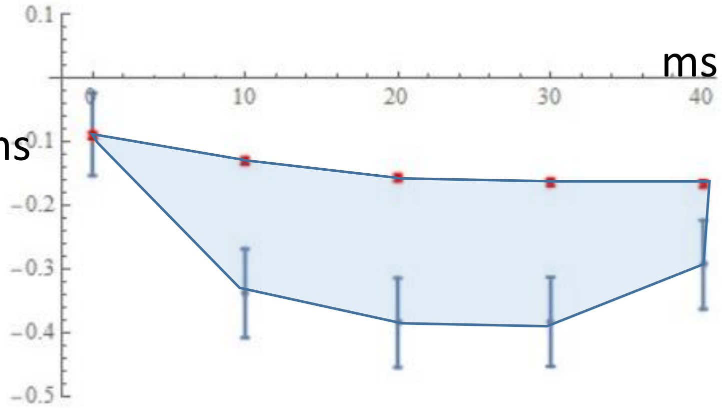
$C_{0,1}$ & 0/−1



$C_{0,2}$ & 0/−2



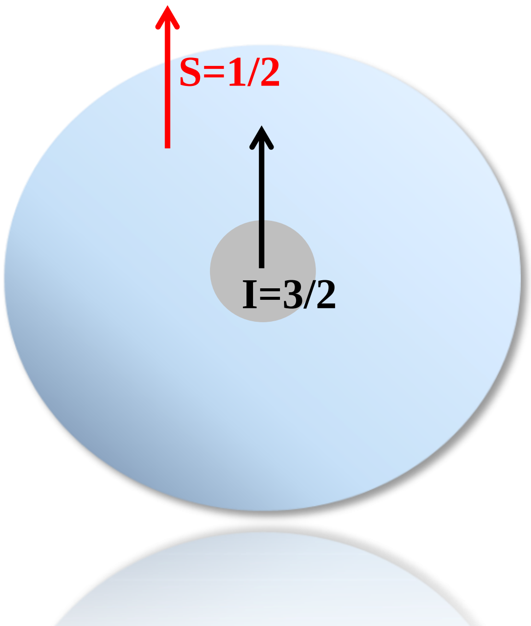
$C_{0,3}$ & 0/−3



Atoms are composite objects, whose spin can be larger than 1/2

$$\vec{F} = \vec{S} + \vec{I}$$

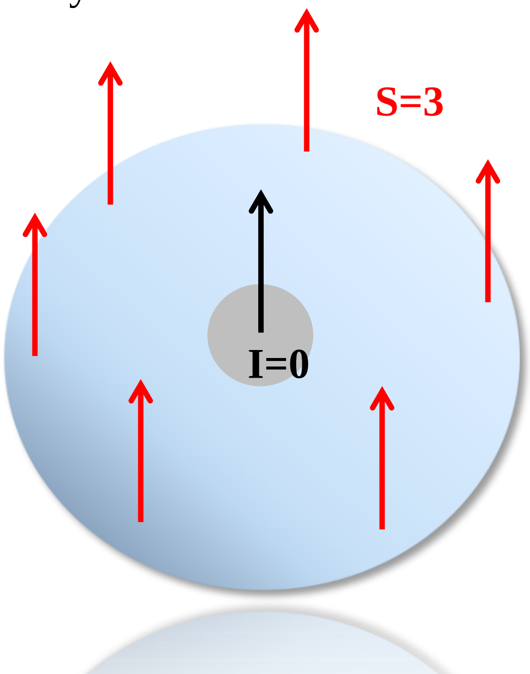
Alkali: spin arrises both from nuclear and electronic spins



e.g. Na, Rb

Spin-dependent
contact interactions

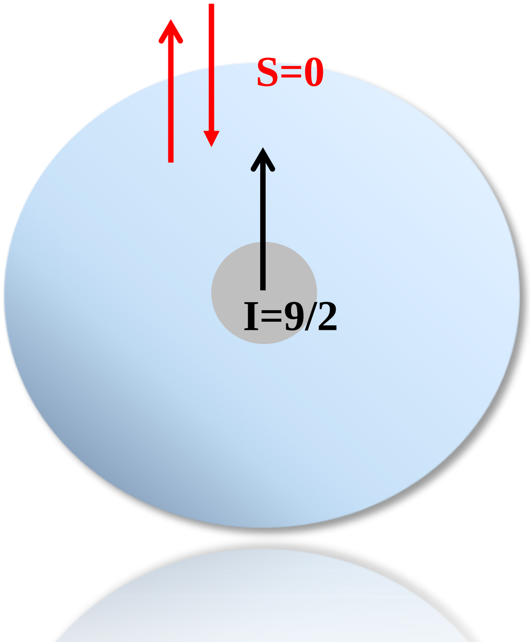
« magnetic atoms »: spin is purely electronic



e.g. Cr, Er, Dy

Strong dipole-dipole
long-range interaction

Alkaline-earth: spin is purely nuclear



e.g. Sr, Yb

Spin-independent
contact interactions

From 2 to N atoms

$$S \times S = i\hbar S$$

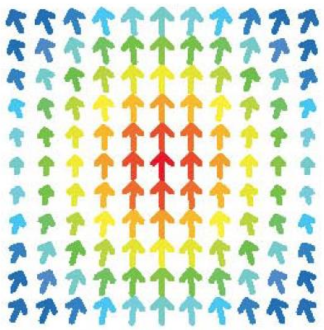
$$\frac{dS}{dt} = \frac{i}{\hbar}[H, S] \longrightarrow \frac{dS}{dt} = \gamma S \times B$$

$$\frac{dS}{dt} = \gamma S \times (B + \sum_j B_j)$$

$\vec{B}_j$ **Field created by atom j**

$$\frac{d\langle S\rangle}{dt} = \gamma \left\langle S \times (B + \sum_j B_j) \right\rangle$$
$$= \gamma \langle S\rangle \times (B + \sum_j \langle B_j\rangle)$$

Neglect correlations



$$\Gamma(r) = \int dr' V_{dd}(r - r')n(r')$$

Mean-field interaction ↔ Gross-Pitaevskii equation

In the mean-field approximation, atoms undergo (classical) precession
Mean-field may be inhomogeneous, and total spin may not be conserved

With and without lattice: the main difference (Lanczos approach)

Lattice case :

$$H_{dd} = -\frac{1}{2}S'_x.S'_x - \frac{3}{8}(S'^+S'^+ + S'^-S'^-) + \frac{1}{8}[\,(S'^+S'^- + S'^-S'^+)]$$

$$\Psi_0 = |3_x, 3_x, 3_x, 3_x, \dots, 3_x\rangle \xrightarrow{\ T\ } \Psi_1 = \sum_{(i,j)} |3_x, \underset{i}{2_x}, 3_x, 3_x, \dots 3_x, \underset{j}{2_x}, 3_x\rangle \xrightarrow{\ T\ } (\dots)$$

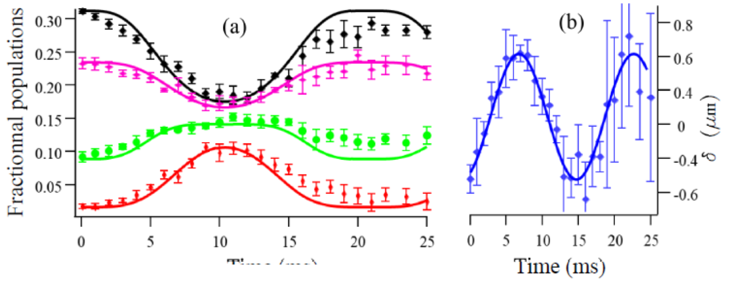
BEC case :

$$\Psi_0 = |3_x, 3_x, 3_x, 3_x, \dots, 3_x\rangle \xrightarrow{\ T\ } \Psi_1 = |2_x, 2_x, 3_x, 3_x, \dots 3_x, 3_x, 3_x\rangle \xrightarrow{\ T\ } (\dots)$$

$$\text{Spin gap} \propto \frac{(a_6 - a_4)\hbar^2 n}{m}$$

In the BEC case, protection of ferromagnetism after the quench due to a spin gap

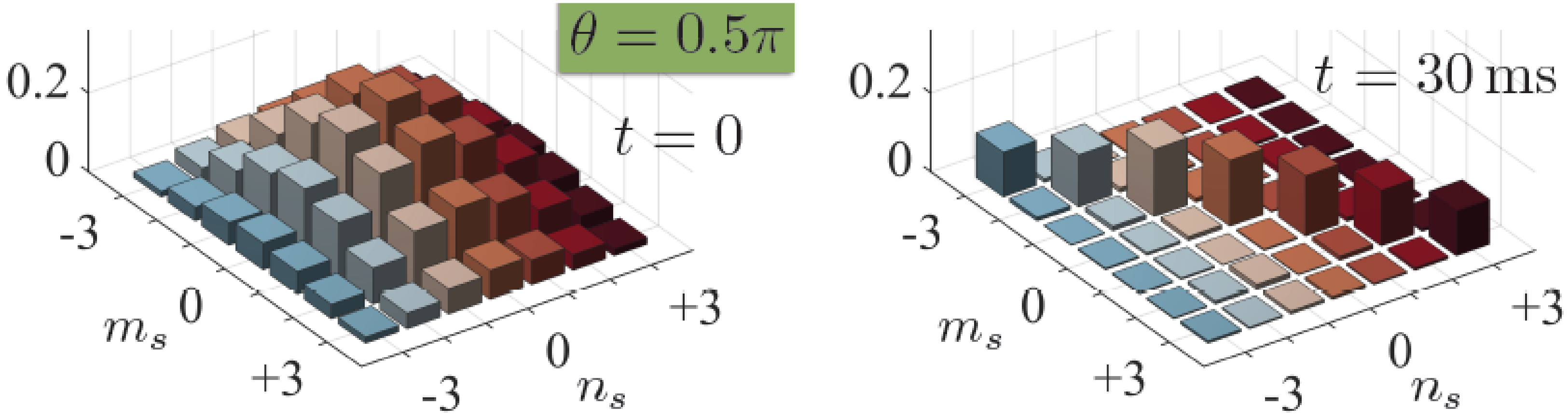
Quench results in the excitation of trapped of magnon mode
(and the retardation of thermalization)



PRL 121, 013201 (2018)

Outlook : Quantum thermalization, entanglement

Reduced density matrix (isolate one spin and trace over the rest of the system)



« coherent »

« mixed »

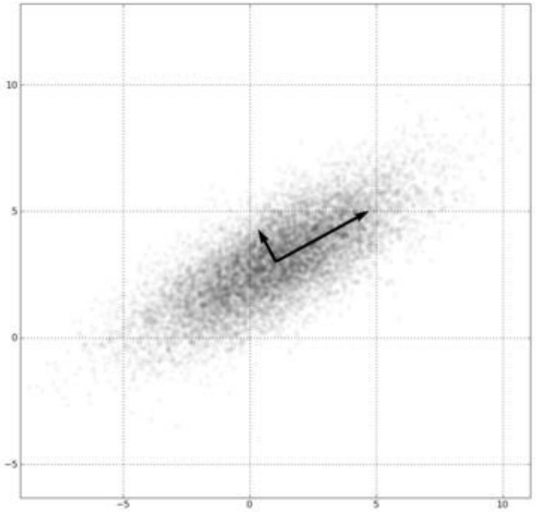
!! However still a pure state !!

(calculations by J. Schachenmayer)

$\mathrm{Tr}(\rho^2)=1$

$\mathrm{Tr}((\rho|_A)^2)\neq 1$

What we learn from co-variances (S>1/2 only)



$$\textsc{CoVar}(N_{m_1} N_{m_2}) = \left\langle \hat{N}_{m_1} \hat{N}_{m_2} \right\rangle - \left\langle \hat{N}_{m_1} \right\rangle \left\langle \hat{N}_{m_2} \right\rangle$$

$$\left\langle \hat{N}_{m_1} \hat{N}_{m_2} \right\rangle = \left\langle \sum_{i=1}^{N} \hat{n}_{m_1,i} . \hat{n}_{m_2,i} \right\rangle + \left\langle \sum_{i \neq j} \hat{n}_{m_1,i} . \hat{n}_{m_2,j} \right\rangle$$

0 for single sites

$$\textsc{CoVar}(N_{m_1} N_{m_2}) = -N P_{m_1} P_{m_2} + \left\langle \sum_{i \neq j} \hat{n}_{m_1,i} . \hat{n}_{m_2,j} \right\rangle - \sum_{i \neq j} \langle \hat{n}_{m_1,i} \rangle . \langle \hat{n}_{m_2,j} \rangle$$

Measured by fluctuations (many images)

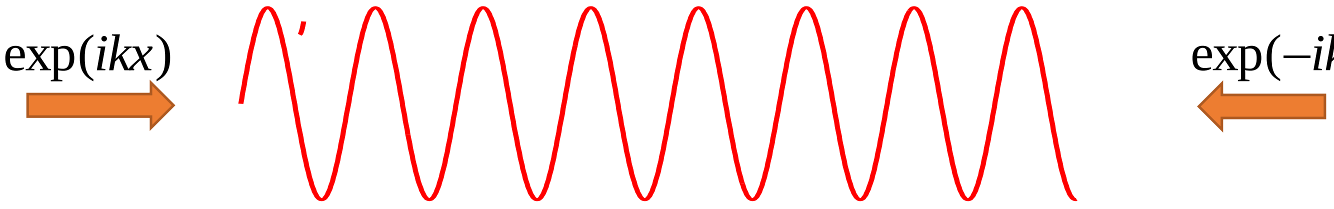
Assumption: homogeneous populations

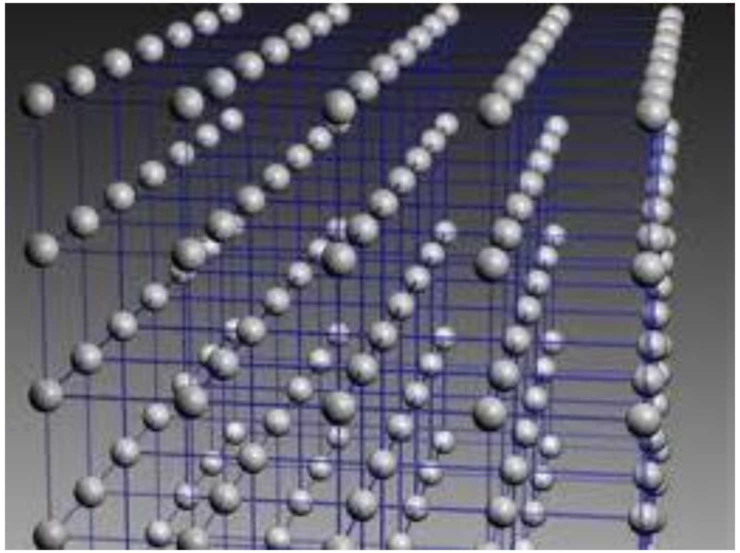
$$\sum_{i} \langle \hat{n}_{m_1,i} \rangle . \langle \hat{n}_{m_2,i} \rangle = N P_{m_1} P_{m_2}$$

Cold atoms revisit condensed matter physics

Optical lattices

Perriodic potential introduced by stationnary wave

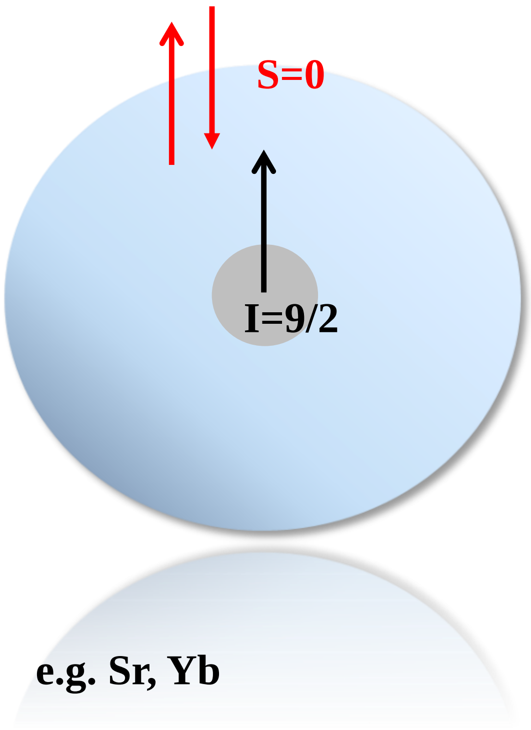
$\exp(ikx)$  $\exp(-ikx)$



Atoms in optical lattices can mimick electrons in solids

Quantum Magnetism

Fermionic isotope in the ground state: SU(N) symmetry



Spin entirely due to nucleus

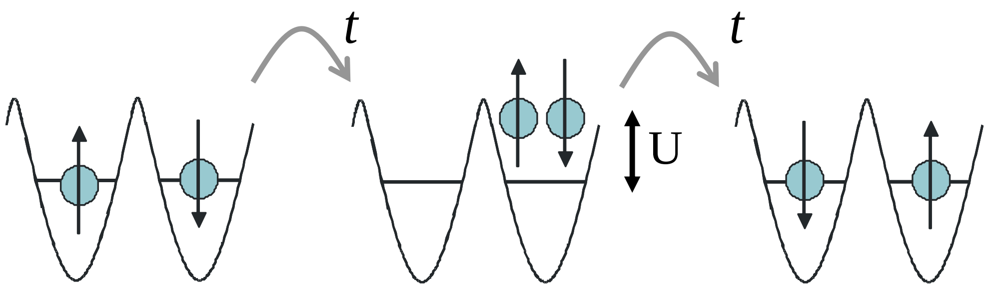
Spin-independent interactions

One consequence : no spin-exchange dynamics

- Can prepare arbitrary number of (fixed) « colours »

Proposal : interplay between SU(N) magnetism and lattice topology

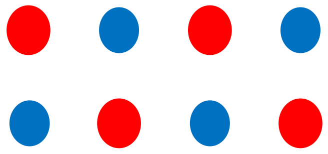
Rule of filling : Two atoms in different states can reduce their energy by tunneling

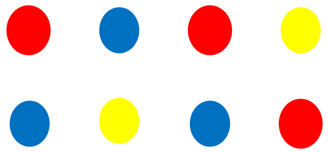


$$\Gamma = -\frac{t^2}{U}$$

Examples:

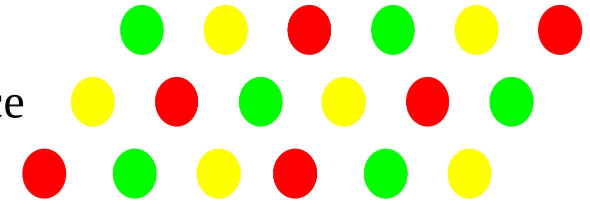
2 colors
Square
Ordered





3 colors
Square
Dis-ordered

3 colors
Triangular lattice
Ordered



Frederic Mila

For a square lattice:
SU(2) ordered
SU(3 and 4) disordered
SU(5) ordered (very low T’s)
SU(6) disordered...
Honeycomb and Kagomé lattice very interesting for SU(N=3,4).

SU(2) : 2 atom singlet
SU(4) : 4-atom singlet
(see Takahashi)

NB: correlations arrise at higher entropy!!

Partial conclusions on the collective spin measurements

Strong decay of collective spin, associated with dipole-dipole interactions

The decays is « too » slow.

→ **heating** in the lattice ?

→ Are there more **holes** than we thought ?

→ effect of **losses** ?

→ more subtle effect associated with possibly disorder ?

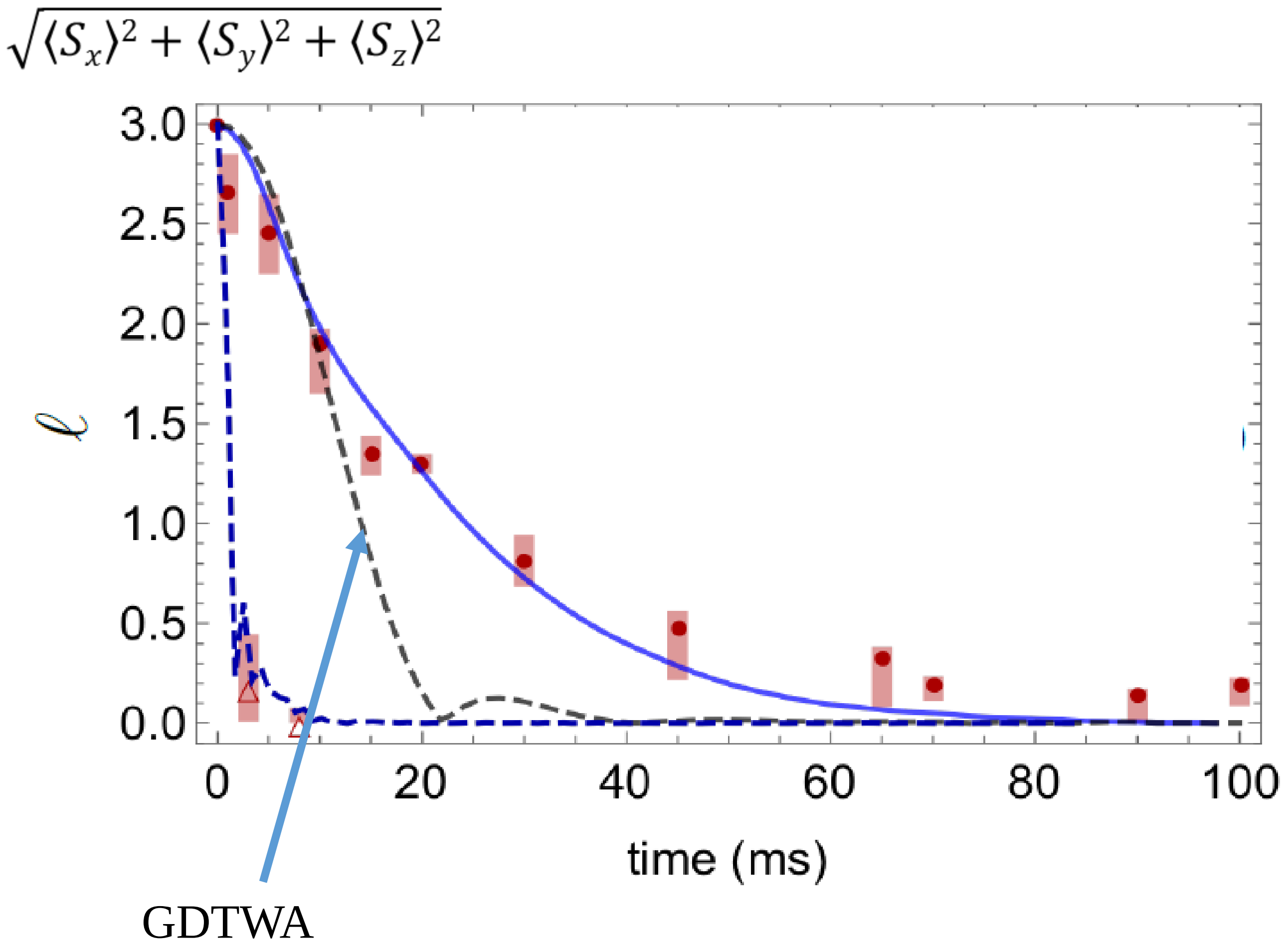
(see glassy dynamics observed with Rydberg atoms Phys. Rev. X **11**, 011011 (2021))

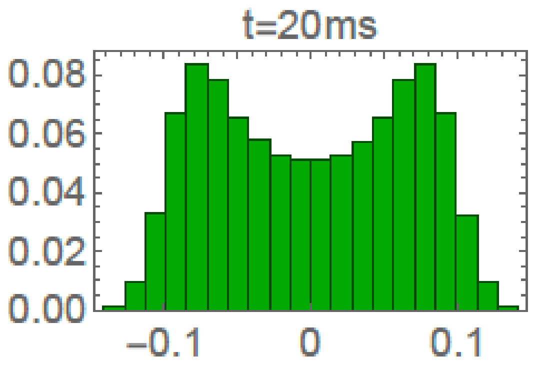
The measurement of coherences (the contrast of the interferometer) gives access to information we could not reach by simply measuring populations.

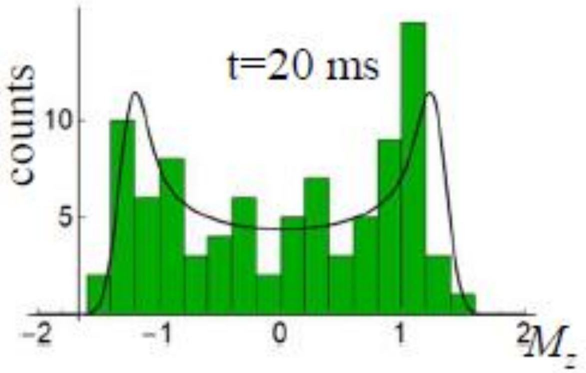
NB: $\sqrt{\langle S_x\rangle^2 + \langle S_y\rangle^2 + \langle S_z\rangle^2}\Big|_{t\to\infty} \approx 0$ can be related to $P_{ms} = \frac{1}{7}\left(1 + \beta B_Q(4 - m_s{}^2)\right) \approx exp[-\beta B_Q m_s^2]$

At equilibrium, the strongly interacting many-body system looks like a non-interacting one !

Damping of the collective spin due to dipolar interactions







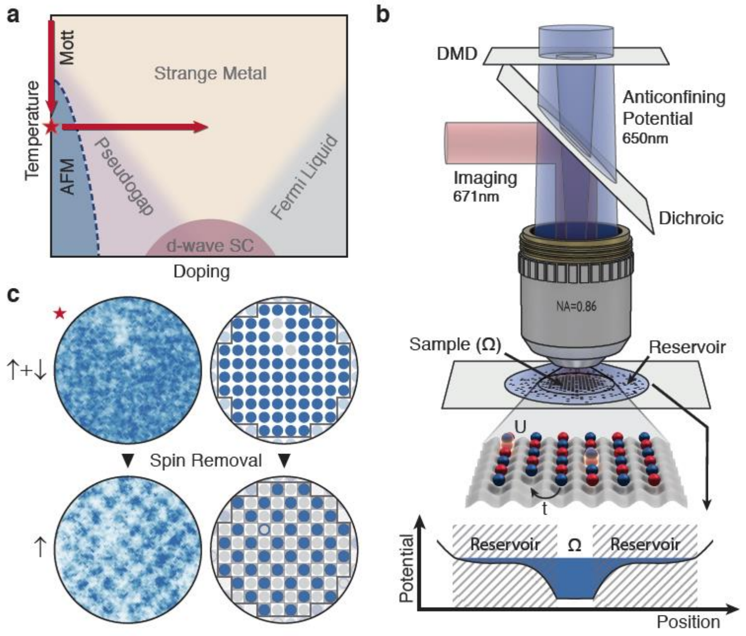
Good agreement at short times
Good agreement with second-order perturbation theory too

At long times, the collective spin decays **SLOWER** than it should !!

Two realizations with cold atoms

Ground state atoms in optical lattices and super-exchange interactions

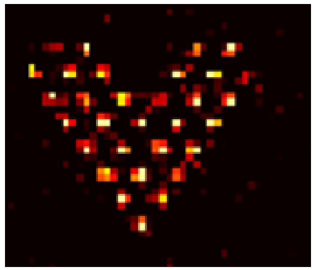
$$S_{1z}S_{2z}+\frac{1}{2}\left(S_{1+}S_{2-}+S_{1-}S_{2+}\right)$$

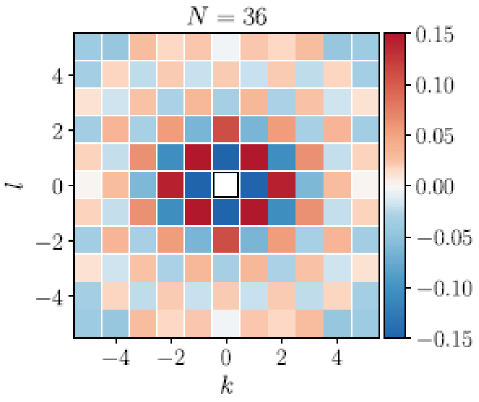


M. Greiner's experiment
(see also Hulet, Kohl, Zwierlein, Bloch…)

Rydberg atoms in tweezers and dipole-dipole interactions

$$\alpha S_{1z}S_{2z}+\beta\frac{1}{2}\left(S_{1+}S_{2-}+S_{1-}S_{2+}\right)$$





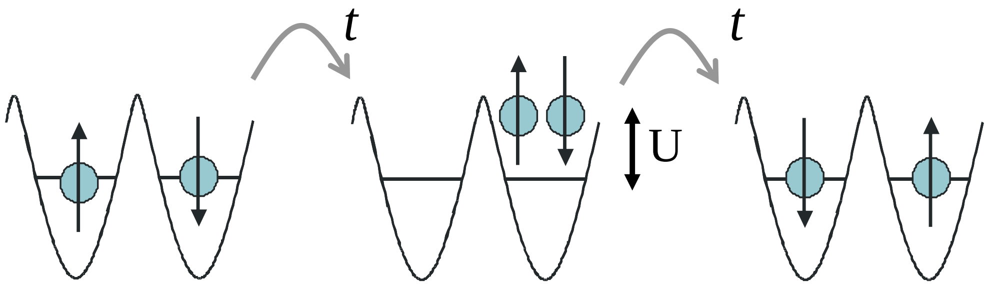
(Browaeys, …)

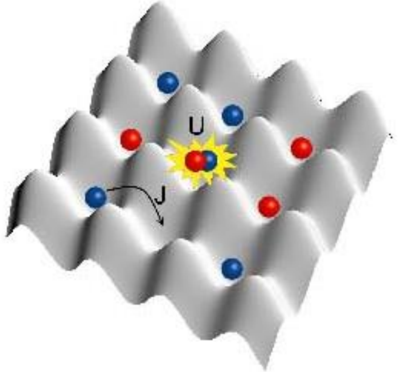
Two realizations with cold atoms

Ground state atoms in optical lattices and super-exchange interactions

Van-der-Waals interactions

$$V(R) = -\frac{C_6}{R^6} \longrightarrow V(R) = \frac{4\pi\hbar^2}{m} a_S \delta(R)$$





$$\Gamma = -\frac{t^2}{U}$$

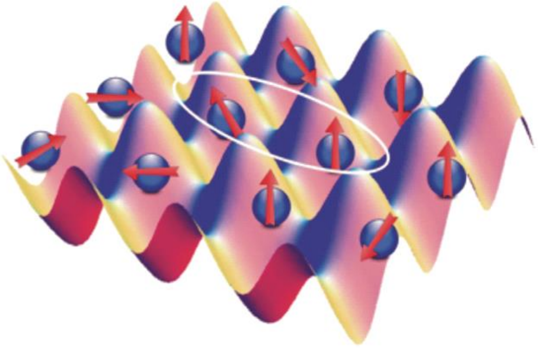
$$S_{1z}S_{2z} + \frac{1}{2}\left(S_{1+}S_{2-} + S_{1-}S_{2+}\right)$$

Ising Exchange

Atoms or molecules with dipole-dipole interactions

Dipole-dipole interactions

$$V_{dd} = \frac{\mu_0}{4\pi} S^2 \left(g_J \mu_B\right)^2 \left(1 - 3\cos^2(\theta)\right) \frac{1}{R^3}$$



$$S_{1z}S_{2z} - \frac{1}{4}\left(S_{1+}S_{2-} + S_{1-}S_{2+}\right)$$

Ising Exchange

How to characterize a quantum many-body system

Partial information : trace over subsystem

Example for a pure state:

$$\rho = |\Psi\rangle\langle\Psi|$$

$$\mathrm{Tr}(\rho^2)=1$$

« pure »

$$\mathrm{Tr}((\rho|_A)^2)\neq 1$$

« locally mixed »

Measures the **entropy associated to entanglement**

See Greiner 2016
Klempt; Treutlein

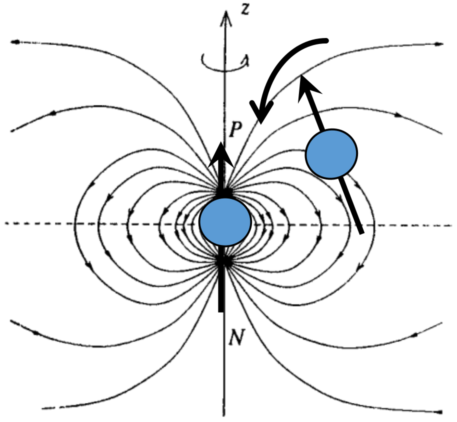
$$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

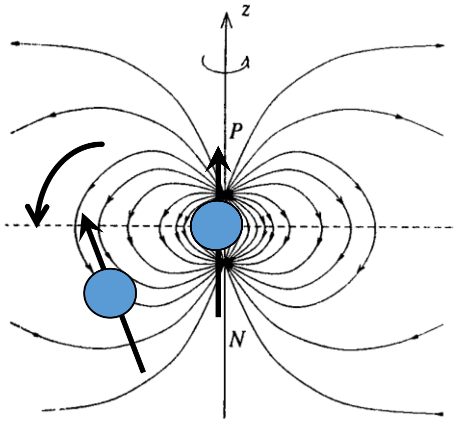
$$\left(\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)\right)^2$$

Basic idea $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$

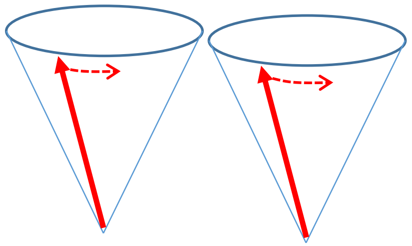
But measurement performed in just one lattice site will show random fluctuations $(|m_s = \pm 1/2\rangle)$ → **associated entropy**

Length of spin vs correlations – Two Atoms

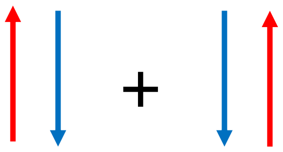




Two initially polarized atoms (A and B):



$$\vec{B}_A(B) = \vec{B}_B(A)$$



Classically:

These two atoms undergo identical precession

Total spin is conserved in time

Quantum-mechanically:

Possibility for entanglement

Total spin is NOT conserved

Measuring interatomic correlations through covariances

$$\textsc{CoVar}(N_{m_1}N_{m_2}) = -NP_{m_1}P_{m_2} + \left\langle \sum_{i\neq j} \hat{n}_{m_1,i}.\hat{n}_{m_2,j} \right\rangle - \sum_{i\neq j} \langle \hat{n}_{m_1,i} \rangle . \langle \hat{n}_{m_2,j} \rangle$$

Measured by fluctuations (many images)

Assumption: homogeneous populations

$$\sum_{i} \langle \hat{n}_{m_1,i} \rangle . \langle \hat{n}_{m_2,i} \rangle = NP_{m_1}P_{m_2}$$

A few general properties – correlations in the steady « thermalized » state

NB: $$P_{m_s} = \frac{1}{7}\left(1+\beta B_Q(4-m_s{}^2)\right)$$ for simplicity we focus on $B_Q=0 \rightarrow P_{m_s}=1/7$ $$\sum m_s^2 P_{m_s} = \frac{9}{2}N$$

$$[S_z, H] = 0$$

$$\Psi_{ini} = (|ms=3\rangle_x)^N \qquad VAR(S_z) = \frac{3}{2}N \qquad \sum_{i\neq j}\langle S_i^z S_j^z\rangle - \langle S_i^z\rangle\langle S_j^z\rangle = -\frac{5}{2}N$$

$$\Psi_{ini} = \left(1/\sqrt{2}(|ms=3\rangle + |ms=-3\rangle)\right)^N \qquad VAR(S_z) = 9N \qquad \sum_{i\neq j}\langle S_i^z S_j^z\rangle - \langle S_i^z\rangle\langle S_j^z\rangle = \frac{9}{2}N$$

S=1/2 or mixtures : $\sum_{i\neq j}\langle S_i^z S_j^z\rangle - \langle S_i^z\rangle\langle S_j^z\rangle = 0$