Detecting entanglement in large ensemble of large spin atoms

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Magnetic Quantum Gases group

Laboratoire de Physique des Lasers, Villetaneuse – Paris 13

Chromium Experiment
Quantum dipolar gases

BEC (2007)
Fermi Sea (2014)

Strontium Experiment (in progress)
SU N Quantum Magnetism

Fermi sea (2019)
No dipolar interaction
No spin dependent interactions

Theory team

Theoretical collaborations
Luis Santos (Hanovre)
Perola Milman (MPQ)
Tommaso Roscilde (Ens Lyon)
Marius Gadja (Varsovie)
Anna Maria Rey (Boulder)

Magnetic Quantum Gases group

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Chromium

Laurent Vernac
Youssef El Alahoui (PhD)
Lucas Gabardos (PhD)

we are looking for a postdoctorate!

Strontium

Martin Robert de Saint Vincent
Olivier Gorceix
Pierre Bataille (PhD)
Andrea Litvinov (PhD)

Theory

Paolo Pedri

Detecting entanglement in large ensemble of large spin atoms

This has been done with BECs interacting with spin dependent Van der Waals interactions spin ½, spin 1

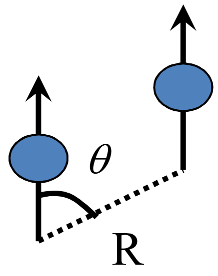
We are studying growth of entanglement
between spin 3 chromium atoms
interacting with dipolar interactions
in optical lattices

Two interactions at play for spin dynamics in chromium quantum gases

Spin =3 for chromium

Dipole-dipole interactions

$$V_{dd} = \frac{\mu_0}{4\pi} S^2 \left(g_J \mu_B\right)^2 \left(1 - 3\cos^2(\theta)\right) \frac{1}{R^3}$$



Long range   Anisotropic

Van-der-Waals (contact) interactions

$$V(R) = \frac{4\pi\hbar^2}{m} a_S \delta(R)$$

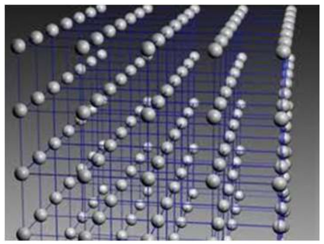
Short range   Isotropic

Both interactions create **Spin Exchange** processes

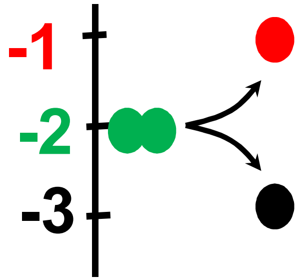
$$\hat{H}_{dd} \propto S_{1z}S_{2z} - \frac{1}{4}\left(S_{1+}S_{2-} + S_{1-}S_{2+}\right)$$

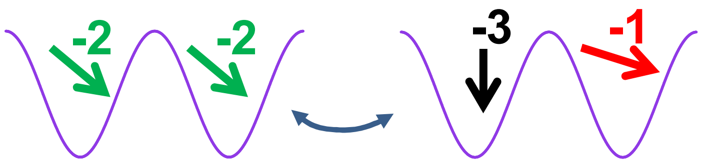
Ising   Exchange

In a BEC spin dynamics is mostly driven by spin dependent contact interactions



In lattice with one atom per site spin dynamics is purely dipolar





Entanglement generation!

$$\Gamma = \frac{4\pi\hbar^2}{m} n\left(a_6 - a_4\right)$$

dipolar atomic systems: Stuttgart (Dy), Innsbruck (Er), Stanford (Dy), Paris (Dy), …

Ingredients for spin dynamics in chromium quantum gases

Effective dipolar Hamiltonian

$$\hat{H}_{dd} = \sum_{i>j} V_{i,j} \left(\hat{S}_i^z \hat{S}_j^z - \frac{1}{2} \left(\hat{S}_i^x \hat{S}_j^x + \hat{S}_i^y \hat{S}_j^y \right) \right)$$

$$V_{i,j} = \frac{\mu_0}{4\pi} (g\mu_B)^2 \frac{1 - 3\cos^2 \theta_{i,j}}{R_{i,j}{}^3} \qquad \frac{\mu_0}{4\pi} (g\mu_B)^2 \frac{1}{(\lambda/2)^3} \approx 2.8 \ \text{Hz}$$

Quadratic effect

$$\hat{H}_Q = B_Q \sum_i \hat{S}_i^{z\,2}$$

Due to the tensorial light shift created by lattice lasers

$$B_Q \approx -5, +5 \quad \text{Hz}$$

Zeeman term

$$\hat{H}_Z = g\mu_B \sum_i B(i) \hat{S}_i^z$$

$$\vec{B}(\vec{r}) = \left(B_0 + \vec{b}.\ \vec{r} \right) \vec{z} + \ldots$$

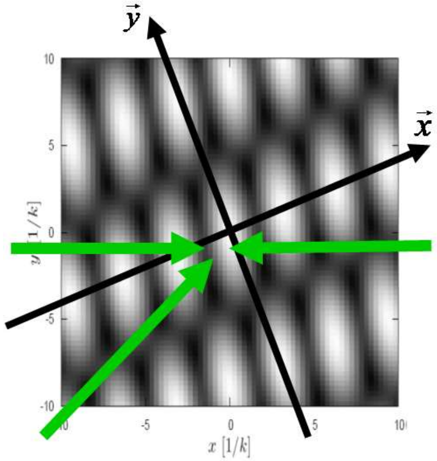
$\vec{b}$: Magnetic gradients

↓ (B_0) Eliminated in the rotating frame

Spin dynamics in deep 3D lattices: preparation

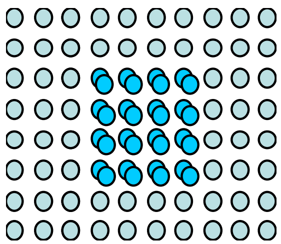
Our lattice architecture:
(Horizontal 3-beam lattice) x (Vertical retro-reflected lattice)

Anisotropic lattice
$f_x \sim 170$ kHz
$f_y \sim 50$ kHz
$f_z \sim 100$ kHz



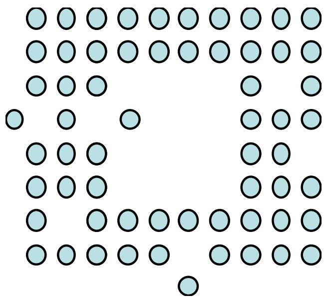
Rectangular lattice of anisotropic sites

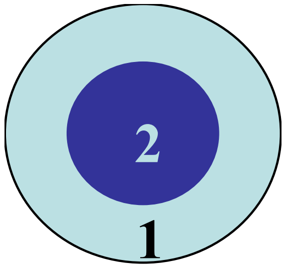
deep 3D lattice → strong correlations, Mott transition



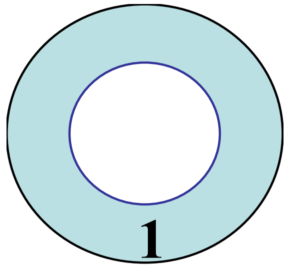
Dipolar relaxation

$$g\mu_B B = \Delta E_c >> U_{lattice}$$





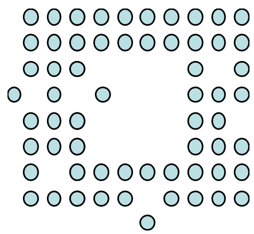
Purification (empty doublons)



One atom per site ensures a pure dipolar spin dynamics

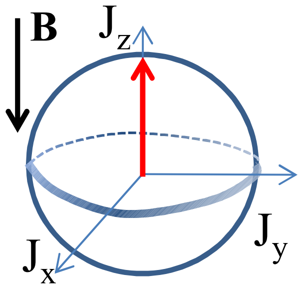
Principle of out of equilibrium spin dynamics experiments

Initial state:



Spin =3 for chromium

$$\Psi_{initial} = \left| -3_z, -3_z, \ldots, -3_z, -3_z \right\rangle$$



1- Excite the spins

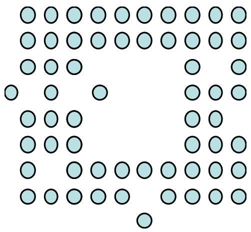
2- Free evolution under the effect of interactions

3- Measurement of Spin populations

4- Prove entanglement

Principle of out of equilibrium spin dynamics experiments

Initial state:



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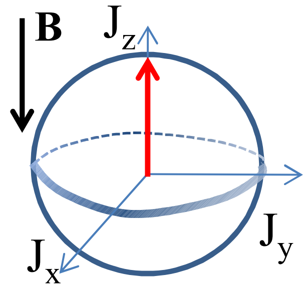


1- Excite the spins

Use of light: large quadratic light shift to beat the linear Zeeman shift

$$\Psi_{(t=0)} = \left| -2_z, -2_z, \ldots, -2_z, -2_z \right\rangle$$ dePaz et al, PRL 2013

Use of Radio Frequency: induce spin rotations

$$\Psi_{(t=0)} = \left| -3_\theta, -3_\theta, \ldots, -3_\theta, -3_\theta \right\rangle$$ Lepoutre et al, NatCom 2019

2- Free evolution under the effect of interactions

Minimize all source of noise: magnetic noise

3- Measurement of Spin populations

Absorption imaging; to be improved

4- Prove entanglement

Open question…

Prove entanglement in large ensemble of large spins interacting at a distance through dipolar interactions

I- Comparison of spin populations dynamics with quantum simulations

Quantum thermalization: an isolated system thermalizes due to growth of entanglement

II- Measurement of the norm of the collective spin

Is it an entanglement witness?

III- Finding well adapted entanglement witness

Spin squeezing like inequalities?
Bipartite measurement?

An experimental talk

Out of equilibrium spin dynamics after rotation of the spins

Prediction (Ana Maria Rey):
PRL **110**, 075301 (2013)

θ small → classical precession
θ large → entanglement grows

Short time exact results:

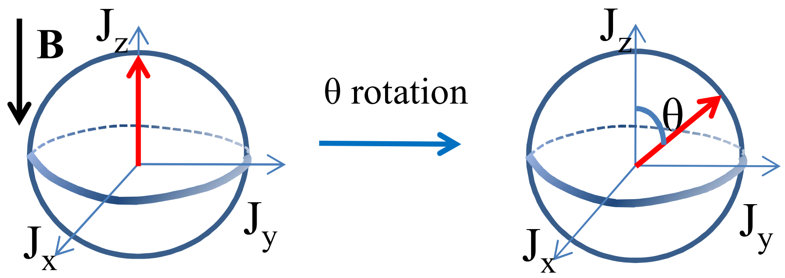
$$p_{m_s}(t) = p_{m_s}(0) + \sin\theta^4 \alpha_{m_s}(\theta) V_{eff}^2 t^2$$

$$\hat{H} = \sum_{i>j} V_{i,j}\left(\hat{S}_i^z \hat{S}_j^z - \frac{1}{2}\left(\hat{S}_i^x \hat{S}_j^x + \hat{S}_i^y \hat{S}_j^y\right)\right) \qquad V_{eff}^2 = \frac{1}{N}\sum_{i>j} V_{i,j}^2$$

while mean field theory predict zero dynamics for θ=π/2 without gradients

1- Excite the spins

Spin =3 for chromium

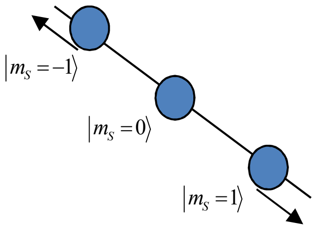


$$\Psi_{initial} = \left|-3_z, -3_z, \ldots, -3_z, -3_z\right\rangle \qquad \Psi_{(t=0)} = \left|-3_\theta, -3_\theta, \ldots, -3_\theta, -3_\theta\right\rangle$$

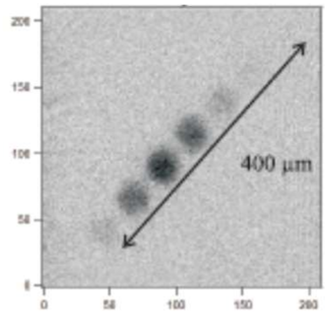
2- Free evolution under the effect of interactions $\Psi_{(t=0)} \rightarrow \Psi(t)$

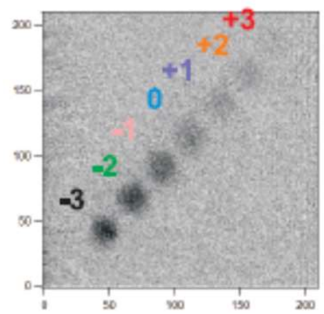
3- Measurement of Spin populations

Stern-Gerlach separation:
(magnetic field gradient)



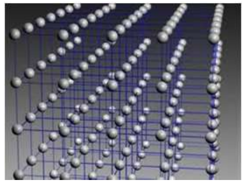
an evolution of spin populations prove the effect of interactions





Out of equilibrium dynamics characterized by the change of the populations of the Zeeman components

Spin dynamics in lattice: comparison with simulations



10000 atoms!

NO exact simulation available beyond 15 atoms: problem with border effects!

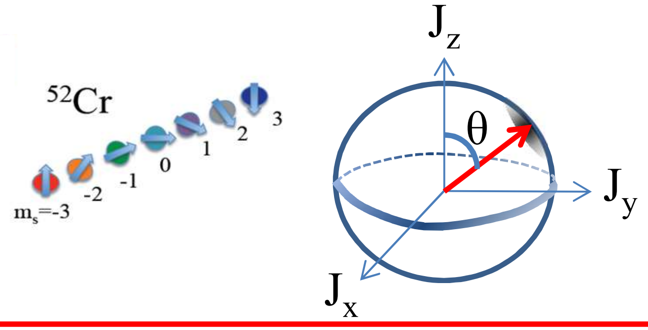
Mean field simulations

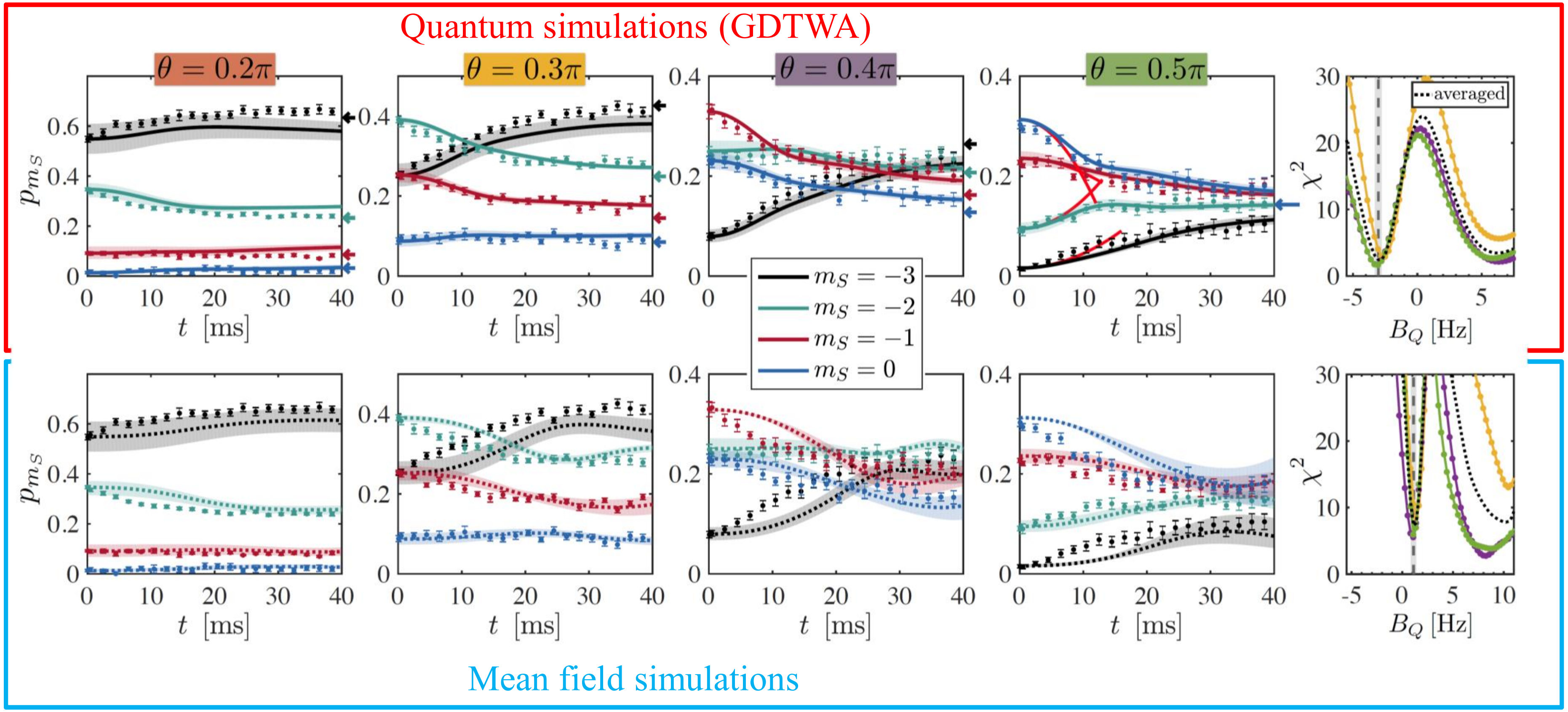
Quantum simulations (Generalized Dichotomized Truncated Wigner Approximation) developed by J. Schachenmayer

Short time exact results: $p_{m_S}(t) = p_{m_S}(0) + \sin\theta^4 \alpha_{m_S}(\theta) V_{eff}{}^2 t^2$

$$\hat{H} = \sum_{i>j} V_{i,j}\left(\hat{S}_i^z \hat{S}_j^z - \frac{1}{2}\left(\hat{S}_i^x \hat{S}_j^x + \hat{S}_i^y \hat{S}_j^y\right)\right) \qquad V_{eff}{}^2 = \frac{1}{N}\sum_{i>j} V_{i,j}{}^2$$

Spin dynamics in lattice: comparison with simulations

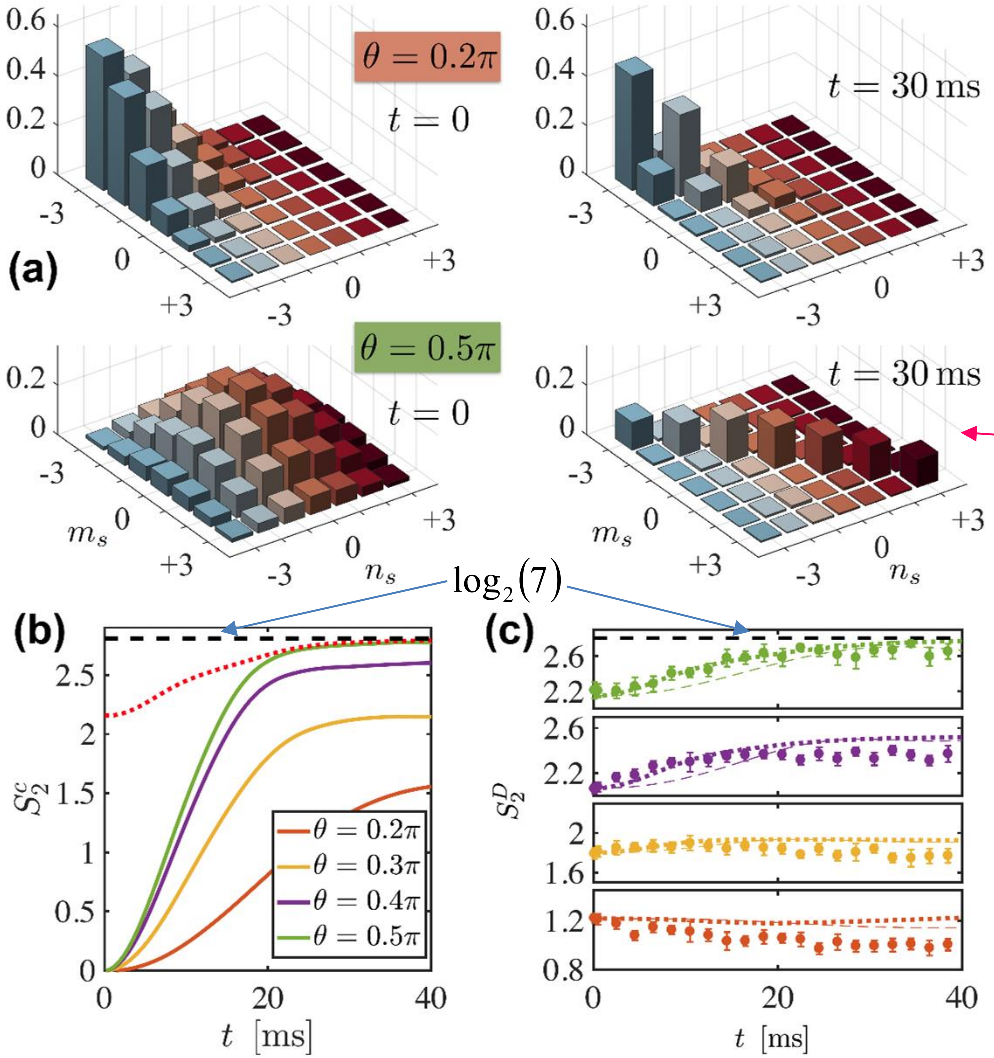




The quantum simulations agree well with data: a very good test for GDTWA for large atom numbers

Spin dynamics in lattice: indirect proof of quantum correlations buildup with comparison to simulations

the quantum state of the full system is pure, but the reduced single spin density matrices can assume a mixed character due to the build up of entanglement between the spins



(a) Absolute values of the central spin density-matrix elements

Off-diagonal single-site coherences are destroyed as the spins become entangled during the quantum dynamics

Close to a maximally mixed state

(b) Second-order Renyi entropy

$$S_2^c = -\log_2\left(Tr[\hat{\rho}_c{}^2]\right)$$

a measure of entanglement if the system is pure

(c) Diagonal entropy

$$S_2^D \approx -\log_2\left(Tr[diag(\hat{\rho}_S)^2]\right) = -\log_2\left(\sum_{m_s} P_{m_s}{}^2\right)$$

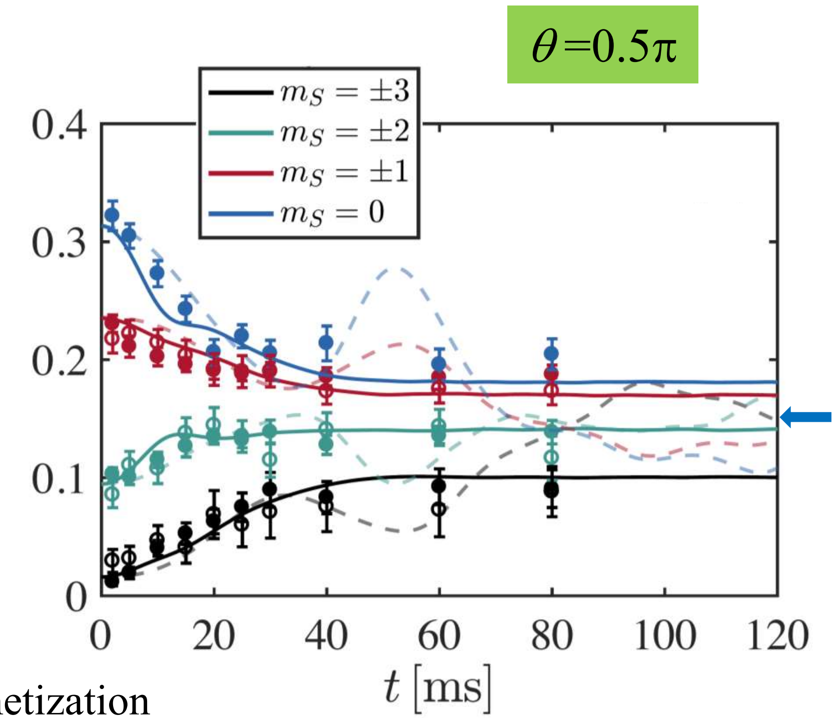
averaged single particle density matrix

Spin dynamics in lattice: Quantum Thermalization

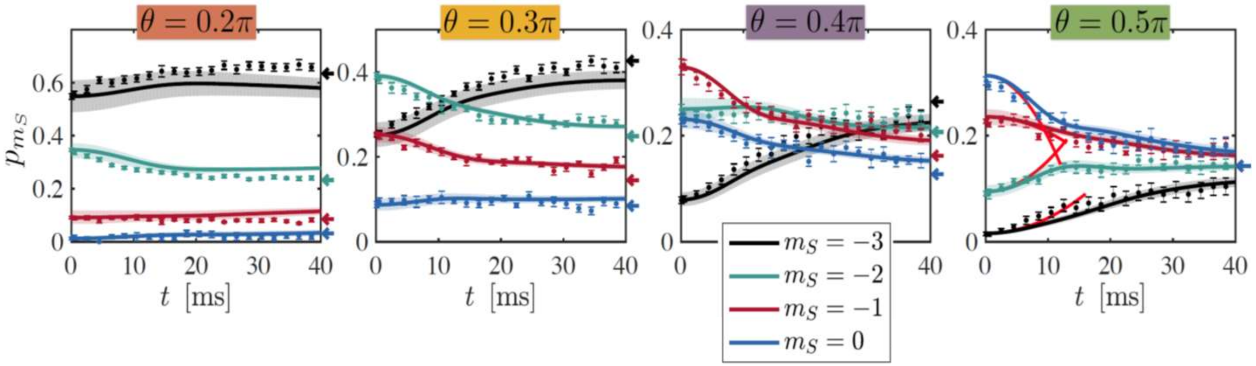
1- Our data show that spin dynamics stops in about 60-80 ms

in agreement with quantum simulations (solid lines)

while mean field simulations show revivals at this time scale (dashed lines)



2- Asymptotic experimental populations are close to population distributions maximizing entropy at fixed magnetization



A long-range interacting many particle isolated system which internally thermalizes through entanglement build-up, and develops an effective thermal-like behavior through a mechanism which is purely quantum and conservative

Models for Quantum Thermalization

$$\beta = \frac{1}{k_B T} = 0 \qquad \boxed{P_{m_s} = \frac{1}{7}}$$

0th order

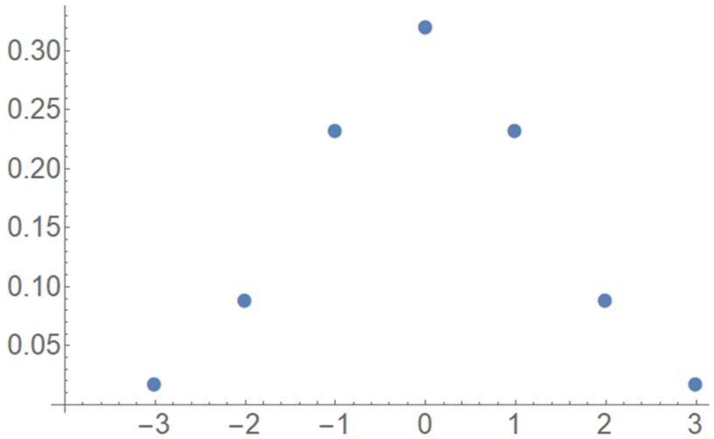
Goal: predict thermalized spin populations for **finite** temperature

One body physics quadratic energy term:
easy exact calculation

$$E(m_s) = B_Q m_s^{\,2}$$

$E = E_0$ Initial energy $\qquad E_0 = \frac{3}{2} B_Q \qquad \theta = \frac{\pi}{2}$

$$P_{m_s} = A \exp[-\beta\, E_{m_s}] \qquad \beta B_Q \approx 0.32$$



Very different with experimental values!

Dipolar interactions:
Analytical model (Ana Maria Rey)

"Canonical approach" $\qquad \hat{\rho} = \exp[-\beta \hat{H}]$

Perturbative approach, small β : $\qquad \hat{\rho} \approx \hat{I} - \beta \hat{H}$

$$\beta = \frac{\overline{H} - E_0}{\Delta H^2} \qquad \langle \hat{P}_{m_s} \rangle = \overline{P}_{m_s} - \beta\left(\overline{H P_{m_s}} - \overline{P}_{m_s}\overline{H}\right)$$

$\overline{H} = Tr[\hat{H}]/Tr[\hat{I}] \qquad \Delta H^2 = Tr[\hat{H}^2]/Tr[\hat{I}] - \overline{H}^2 \qquad E_0 =$ Initial energy

Pure dipolar Hamiltonian:

$$\hat{H} = \sum_{i>j} V_{i,j}\left(\hat{S}_i^z \hat{S}_j^z - \frac{1}{2}\left(\hat{S}_i^x \hat{S}_j^x + \hat{S}_i^y \hat{S}_j^y\right)\right)$$

$$\overline{H} = 0 \qquad \Delta H^2 = 24 V_{eff}^{\,2} \qquad E_0 = -\frac{9}{2} V_a \qquad \theta = \frac{\pi}{2}$$

$$V_{eff}^{\,2} = \frac{1}{N}\sum_{i>j} V_{i,j}^{\,2} \qquad V_a = -\frac{1}{N}\sum_{i>j} V_{i,j}$$

From the 3D lattice structure:

$$V_a = -0.56 \text{ Hz} \qquad V_{eff} = 4.33 \text{ Hz}$$

$$\beta = \frac{9}{48}\frac{V_a}{V_{eff}^{\,2}} \longrightarrow T \approx -9 \text{ nK} \qquad \boxed{P_{m_s} = \frac{1}{7}}$$

1st order

Our Model for Quantum Thermalization

Previous slide suggest that the quadratic effect acts as a perturbative effect on dipolar Hamiltonian

Analytical model (Ana Maria Rey) at 1st order for: $\hat{H} = \sum_{i>j} V_{i,j}\left(\hat{S}_i^z \hat{S}_j^z - \frac{1}{2}\left(\hat{S}_i^x \hat{S}_j^x + \hat{S}_i^y \hat{S}_j^y\right)\right) + B_Q \sum_i \hat{S}_i^{z\,2}$

$$E_0 = \frac{3}{2} B_Q - \frac{9}{2} V_a \qquad \overline{H} = 4B_Q \qquad \Delta H^2 = 12 B_Q^{\ 2} + 24 V_{eff}^{\ \ 2}$$

$$\beta = \frac{5B_Q + 9V_a}{24 B_Q^{\ 2} + 48 V_{eff}^{\ \ 2}}$$

$$P_{m_s} = \frac{1}{7}\left(1 + \beta B_Q (4 - m_s^{\ 2})\right)$$

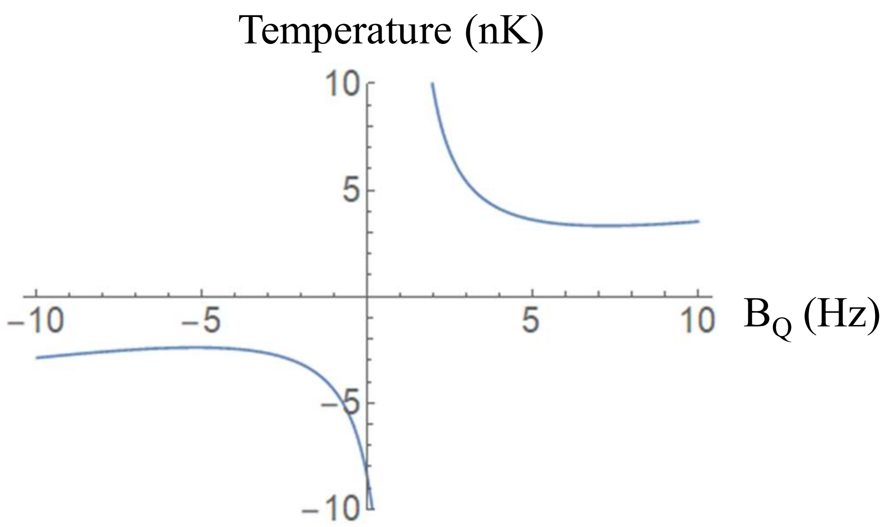
1st order

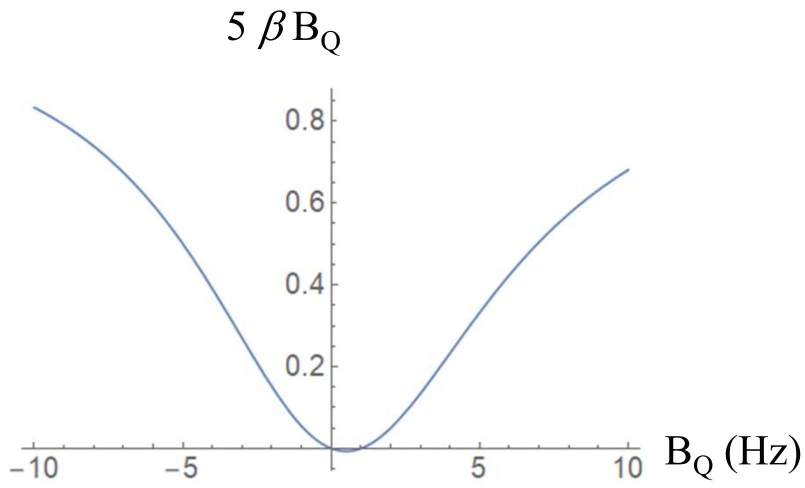
Validity of the perturbative approach:

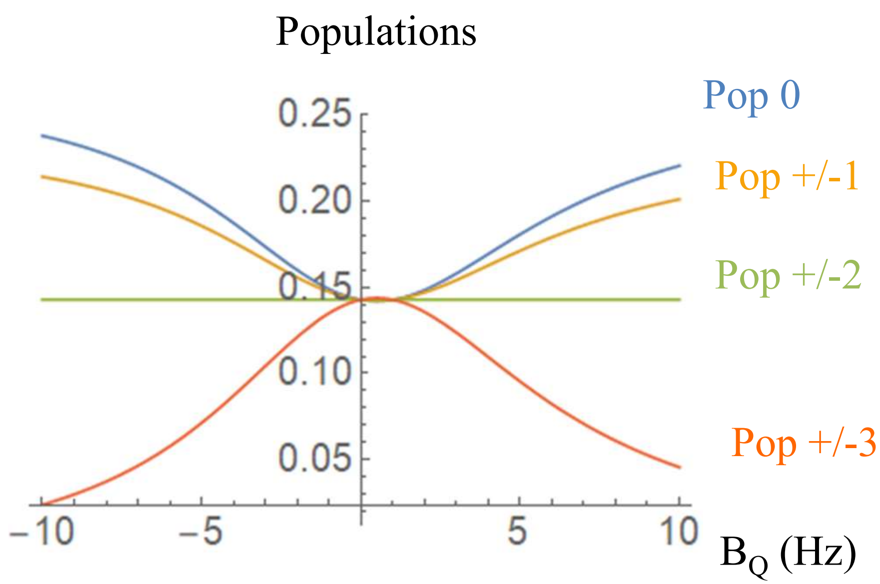
$$\left|B_Q\right| << V_{eff} \qquad \left|B_Q V_a\right| << V_{eff}^{\ \ 2}$$

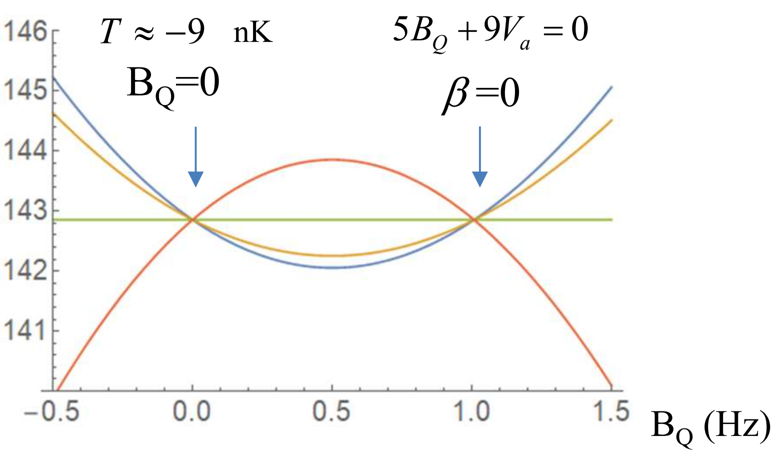
Note: B_Q is given by the full dynamics analysis

Our Model for Quantum Thermalization: results









Our Model for Quantum Thermalization: comparison with experiments

Prediction of the model:

$$P_{m_s} = \frac{1}{7}\left(1 + \beta B_Q (4 - m_s^{\,2})\right)$$

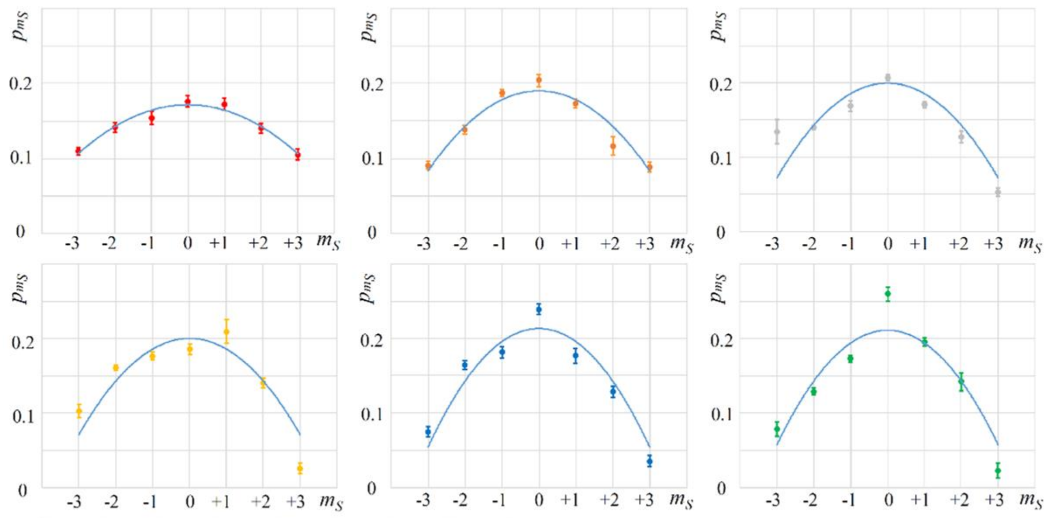
Parabolic shape

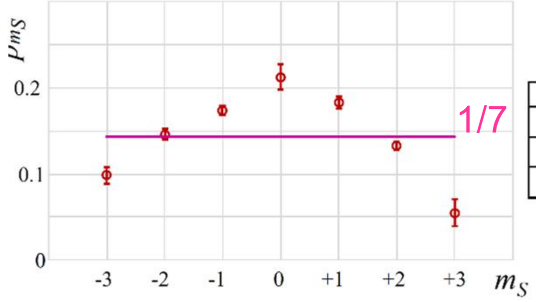
Stationnarity value for m_S= +/-2

One P_{m_S} independent of B_Q only for S=1/2, 3, 48, 121/2

$$P_{m_s} = \frac{1}{2S+1}\left(1 + \beta B_Q \left(\frac{S(S+1)}{3} - m_s^{\,2}\right)\right)$$

Comparison with experimental results





Data	(red)		(orange)		(grey)		(yellow)		(blue)		(green)	
χ	0.7		1.5		2.7		3.5		2.8		3.4	
B_Q (Hz)	4.2	-2.9	5.9	-4.2	6.9	-4.9	7.0	-5.0	8.7	-6.2	8.3	-6.0
T (nK)	4.0	-2.7	3.4	-2.4	3.3	-2.4	3.3	-2.4	3.4	-2.4	3.4	-2.4

Quantum Thermalization at a few nK

Other experiments:

Greiner: few ½ spins, superexchange processes
B. Lev: Dy atoms, thermalization through collisions

Collective Spin Length measurement in Optical Lattice: an entanglement witness?

$$\vec{J} = \sum_N \vec{s}_i \qquad \left|\vec{J}\right| = \left(\langle \hat{J}_x \rangle^2 + \langle \hat{J}_y \rangle^2 + \langle \hat{J}_z \rangle^2 \right)^{1/2} \qquad 0 \le \ell = \frac{\left|\vec{J}\right|}{N} \le 3$$

ℓ is the contrast of an atomic interference sequence (Ramsey type experiment)

Many reasons for ℓ to change:

pure dipolar dynamics

$$\ell(t) \underset{short\ time}{=} 3 - \frac{81}{16}\frac{V_{eff}^2}{\hbar^2}t^2$$

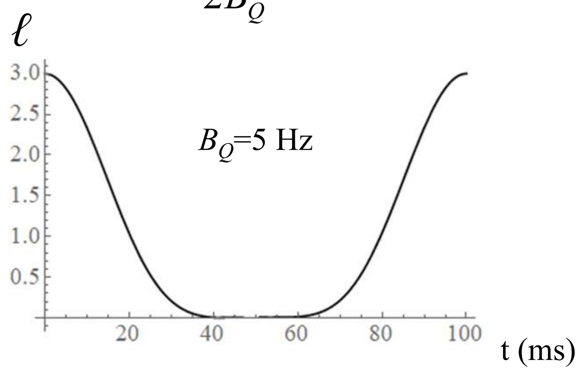
$V_{eff} \approx 4$ Hz $\qquad \tau_d \approx 15$ ms

Dipolar interactions lead to zero spin length!

quadratic field

damping and revival

$$\tau_Q \approx \frac{h}{2B_Q}$$



Not a pure dipolar dynamics…

magnetic inhomogeneities

$\ell(t) \to 0$

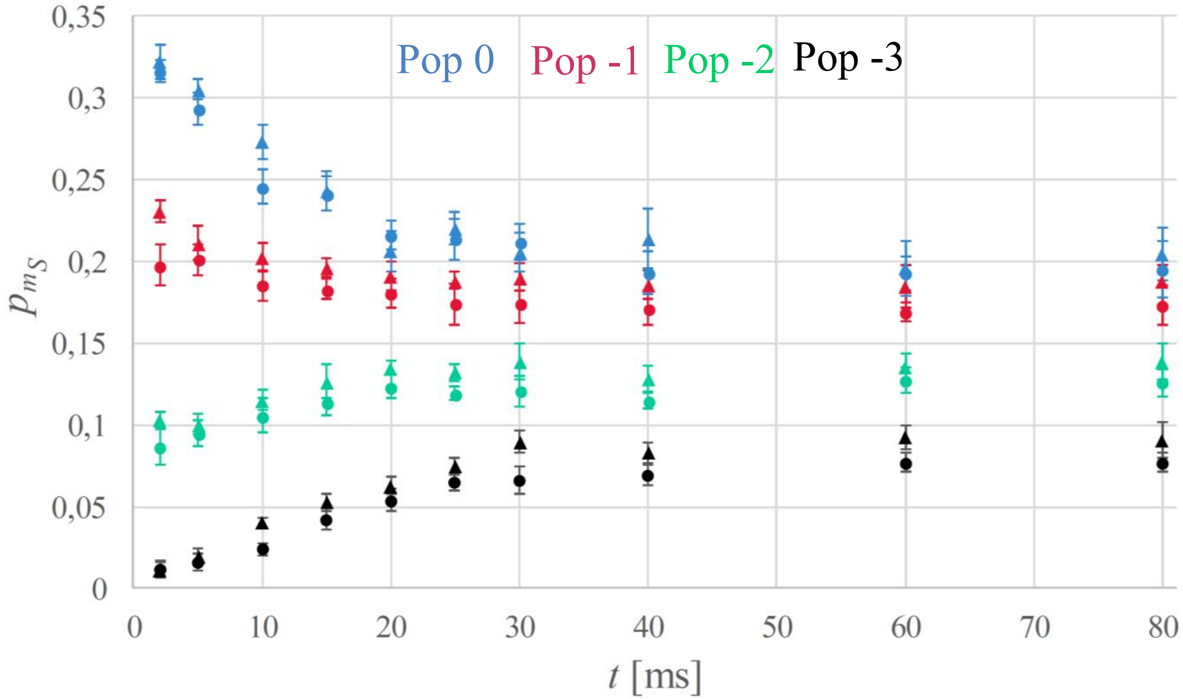
$$\tau_b \approx \frac{h}{2g\mu_B b R_{lattice}}$$

$b \approx 0{,}1$ G.cm^{-1}

$\tau_b \approx 3$ ms

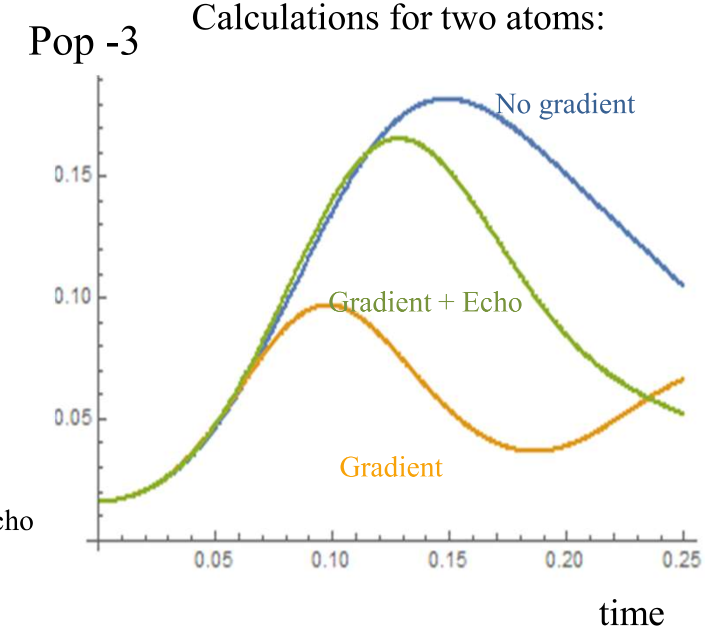
Echo type experiment necessary

Collective Spin Length measurement in Optical Lattice: does an echo change the dynamics?

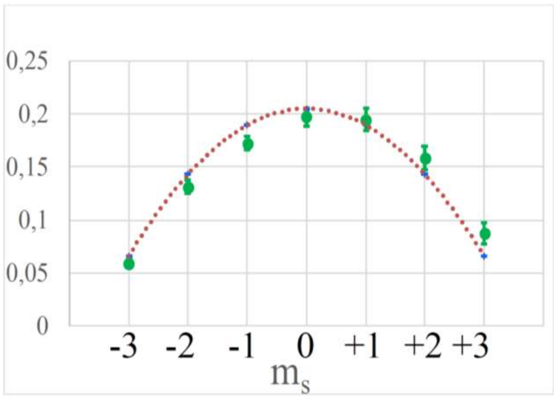


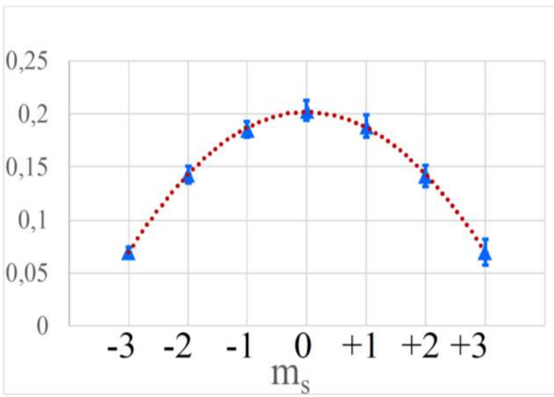
The two dynamics look very close
Is it a surprise?

Calculations for two atoms:

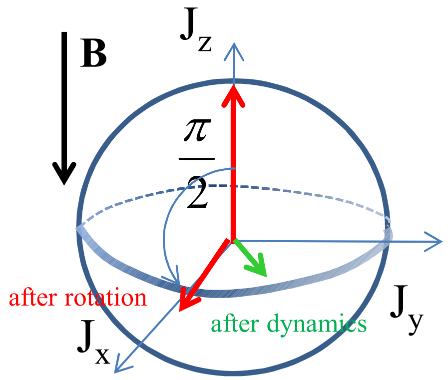


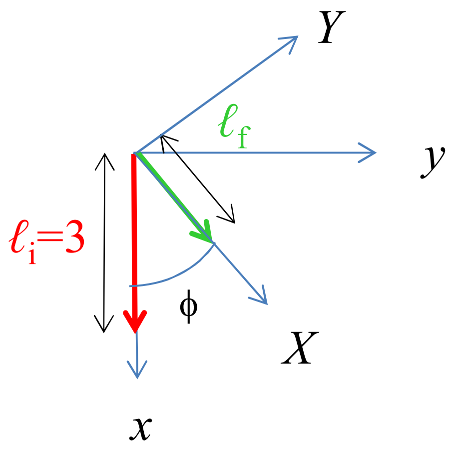
● $\frac{\pi}{2} - t - (SG)_z$ Dynamics without echo ▲ $\frac{\pi}{2} - \frac{t}{2} - \pi - \frac{t}{2} - (SG)_z$ Dynamics with Echo





Collective Spin Length measurement in Optical Lattice: procedure





ϕ is due to dephasing
= magnetic noise

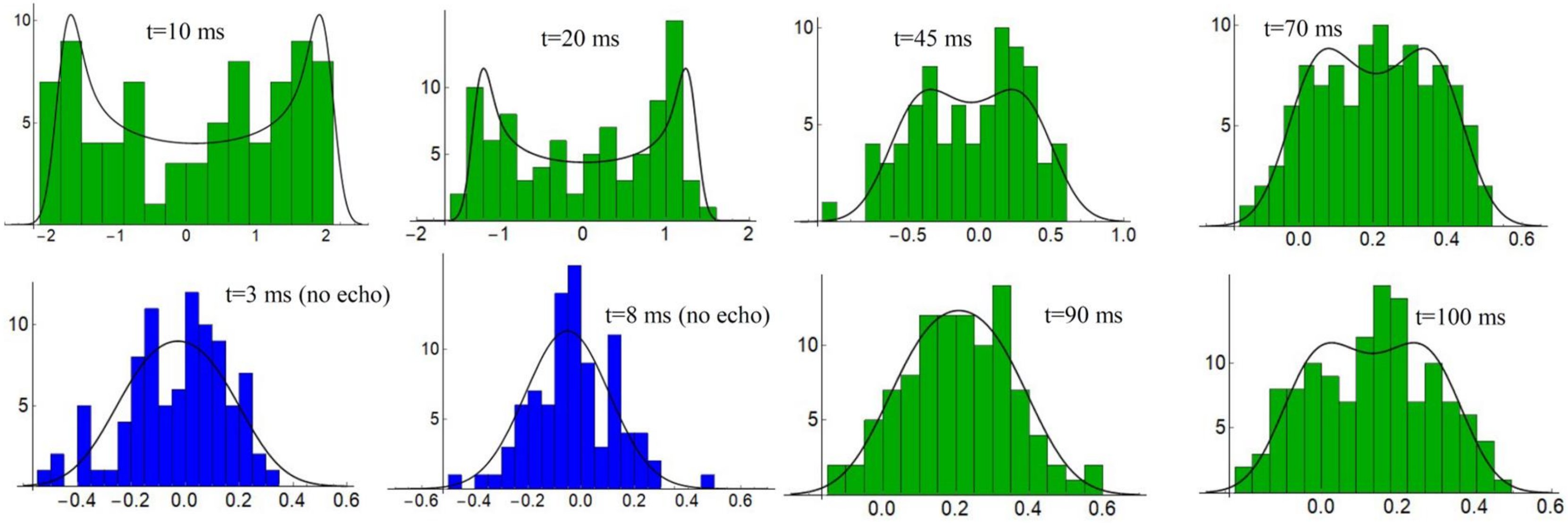
ϕ is random after a few ms

$$\langle J_X \rangle = \ell, \langle J_Y \rangle = \langle J_z \rangle = 0$$

		quantity measured:	$\vec{J}$ component measured:
Experiment for measuring spin dynamics:	$\frac{\pi}{2} - t - (SG)_z$	M_z	J_z
Experiment for measuring spin length:	$\frac{\pi}{2} - t - \frac{\pi}{2} - (SG)_z$ Ramsey type experiment	M_z	$J_\phi = \cos\phi\, J_X + \sin\phi\, J_Y$ ϕ is random
Experiment for measuring spin length **and** cancel effect of magnetic inhomogeneities:	$\frac{\pi}{2} - \frac{t}{2} - \pi - \frac{t}{2} - \frac{\pi}{2} - (SG)_z$ Echo type experiment	M_z	

We measure distributions of M_z $-3 < M_z < +3$

Collective Spin Length measurement in Optical Lattice: data analysis (1)



Assume Classical Spin:

$$J_\phi = \cos\phi \, J_X + \sin\phi \, J_Y \Rightarrow M_z = \ell \cos\phi$$

$$\frac{dN}{dM_z} = \frac{1}{\pi\ell} \frac{1}{\sqrt{1 - \frac{M_z^{\ 2}}{\ell^2}}}$$

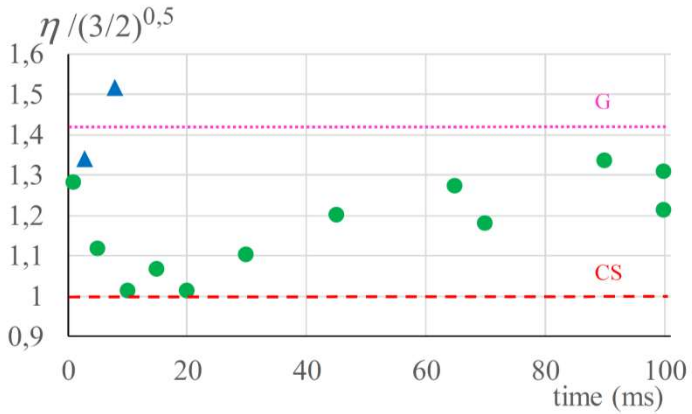
Assume ℓ=0, and Gaussian noise:

$$\frac{dN}{dM_z} = \frac{1}{\sqrt{\pi}\sigma} \exp\left(- \frac{M_z^{\ 2}}{\sigma^2} \right)$$

two extreme proba-bility distributions

Ratio of distribution moments:

$$\eta = \frac{\sqrt{M_4}}{M_2}$$



1st method to derive ℓ: fit probability distributions with a convolution of the two « extreme » distributions

Collective Spin Length measurement in Optical Lattice: data analysis (2)

we measure the collective spin component $J_\phi = \cos\phi\, J_X + \sin\phi\, J_Y$ and collect values of Mz

$$\left\langle \hat{J}_\phi{}^2 \right\rangle = \left\langle \cos^2\phi\, \hat{J}_X{}^2 + \sin^2\phi\, \hat{J}_Y{}^2 + \cos\phi\ \sin\phi\, \hat{J}_X\hat{J}_Y + \hat{J}_Y\hat{J}_X \right\rangle$$

ϕ random → $\left\langle \hat{J}_\phi{}^2 \right\rangle = \left\langle \frac{1}{2}\hat{J}_X{}^2 + \frac{1}{2}\hat{J}_Y{}^2 \right\rangle$

ϕ and **J** uncorrelated

$$Var(\hat{J}_X) = \left\langle \hat{J}_X{}^2 \right\rangle - \left\langle \hat{J}_X \right\rangle^2 \qquad \left\langle \hat{J}_X \right\rangle = N\,\ell \qquad Var(\hat{J}_Y) = \left\langle \hat{J}_Y{}^2 \right\rangle$$

$$\left\langle \hat{J}_\phi{}^2 \right\rangle = \frac{1}{2}N^2\ell^2 + \frac{Var(\hat{J}_X) + Var(\hat{J}_Y)}{2}$$

$$\vec{j} = \frac{\vec{J}}{N} \qquad \boxed{\left\langle \hat{j}_\phi{}^2 \right\rangle = \frac{1}{2}\ell^2 + \frac{Var(\hat{j}_X) + Var(\hat{j}_Y)}{2}}$$

Quantum noise ~ 1 /N

Real life:

$$\left\langle \hat{j}_\phi{}^2 \right\rangle_{exp} = \frac{1}{2}\ell^2 + \frac{Var(\hat{j}_X) + Var(\hat{j}_Y)}{2} + \sigma_{exp}{}^2$$

$$\approx \frac{1}{2}\ell^2 + \sigma_{exp}{}^2$$ Technical noise dominates Quantum noise…

2nd method to derive ℓ:

From:

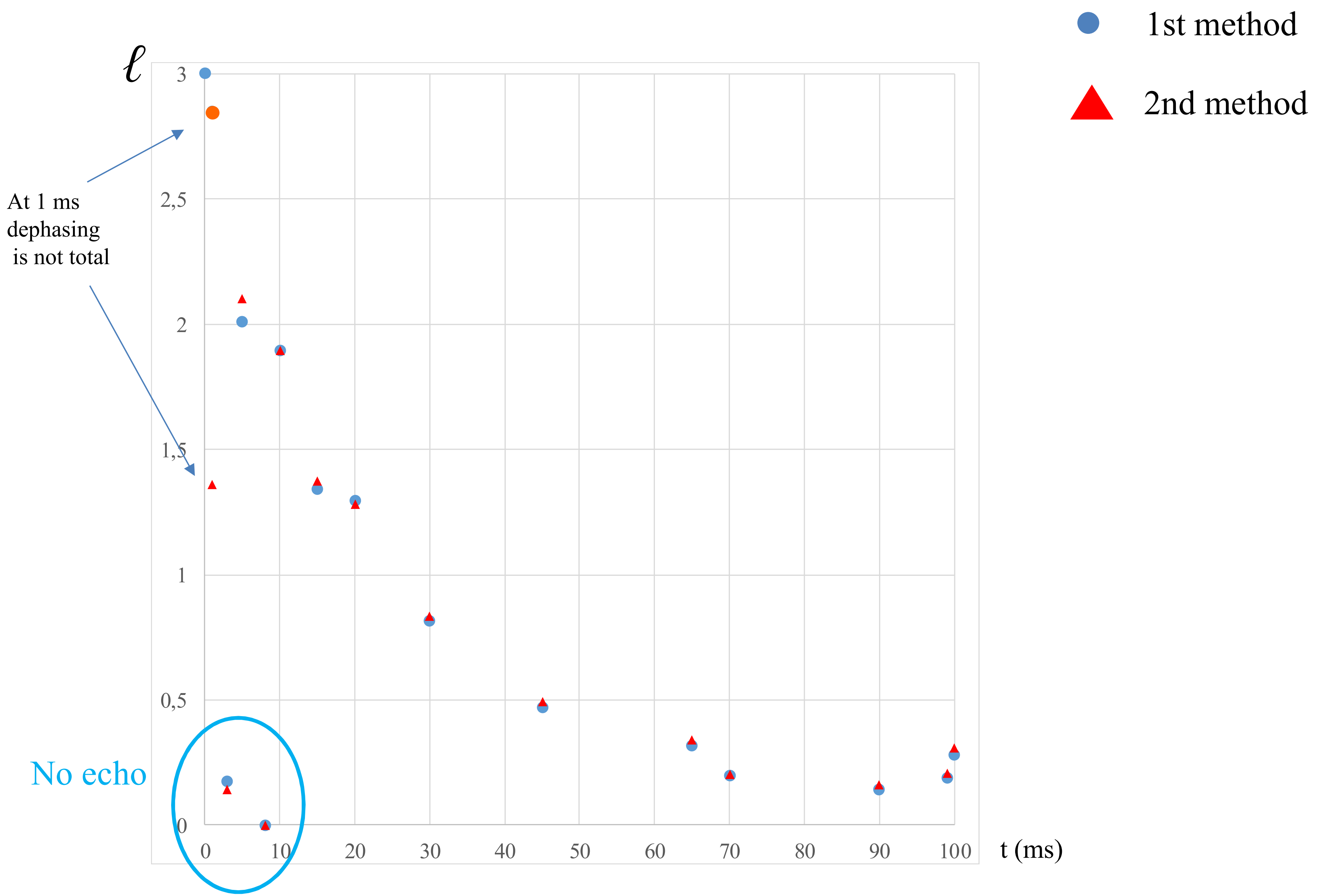
$$\left\langle \hat{j}_\phi{}^2 \right\rangle_{exp} = \frac{1}{N_S}\sum_{Ns}\left(\frac{M_z}{N}\right)^2$$

N → Atom number

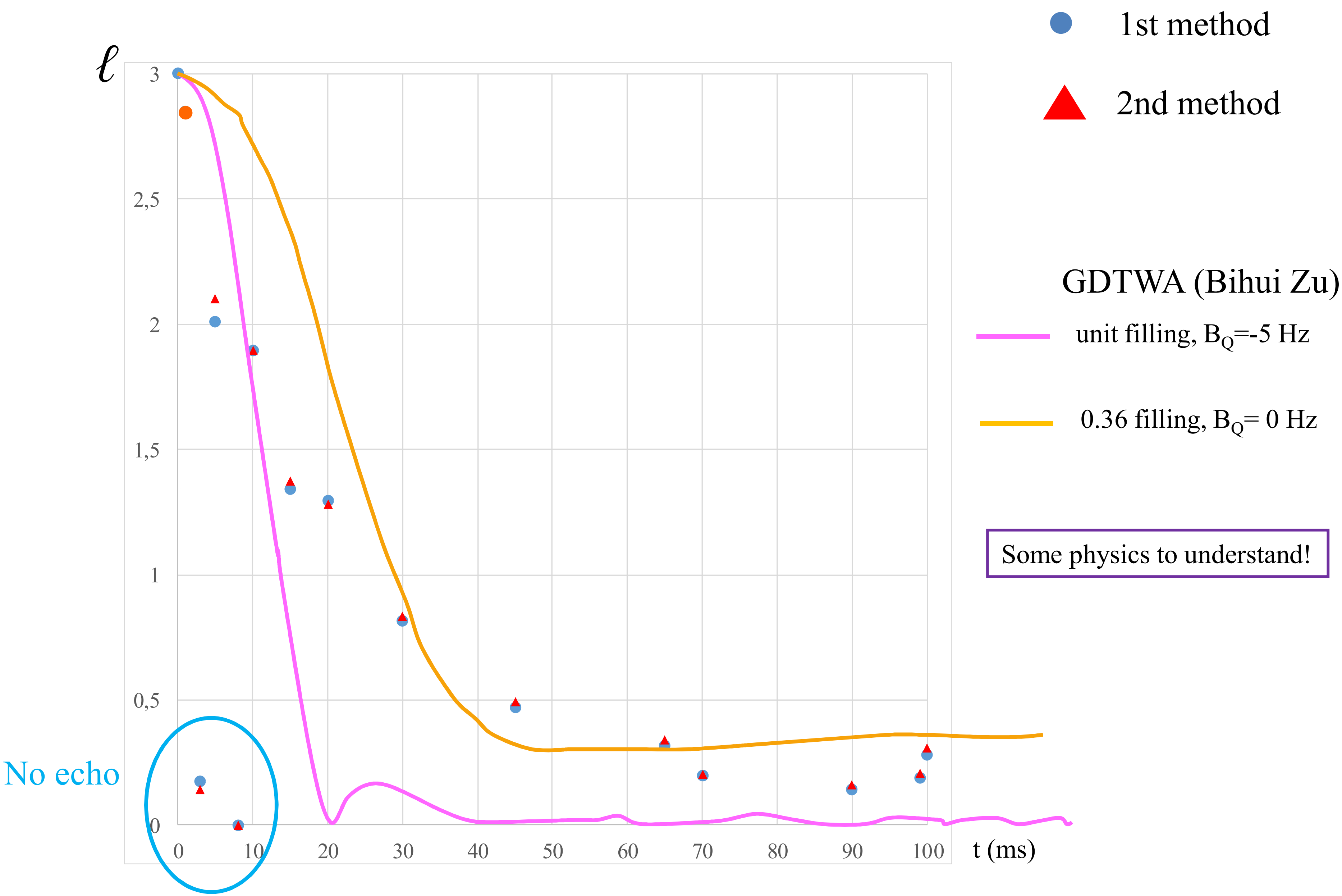
N_S → Number of sample

And: σ_{exp}

Collective Spin Length measurement in Optical Lattice: results



Collective Spin Length measurement in Optical Lattice: comparison with simulations



Measurements of spin fluctuations

$$\left\langle \hat{j}_\phi^{\ 2} \right\rangle_{exp} = \frac{1}{2}\ell^2 + \frac{Var(\hat{j}_X) + Var(\hat{j}_Y)}{2} + \sigma_{exp}^{\ 2}$$

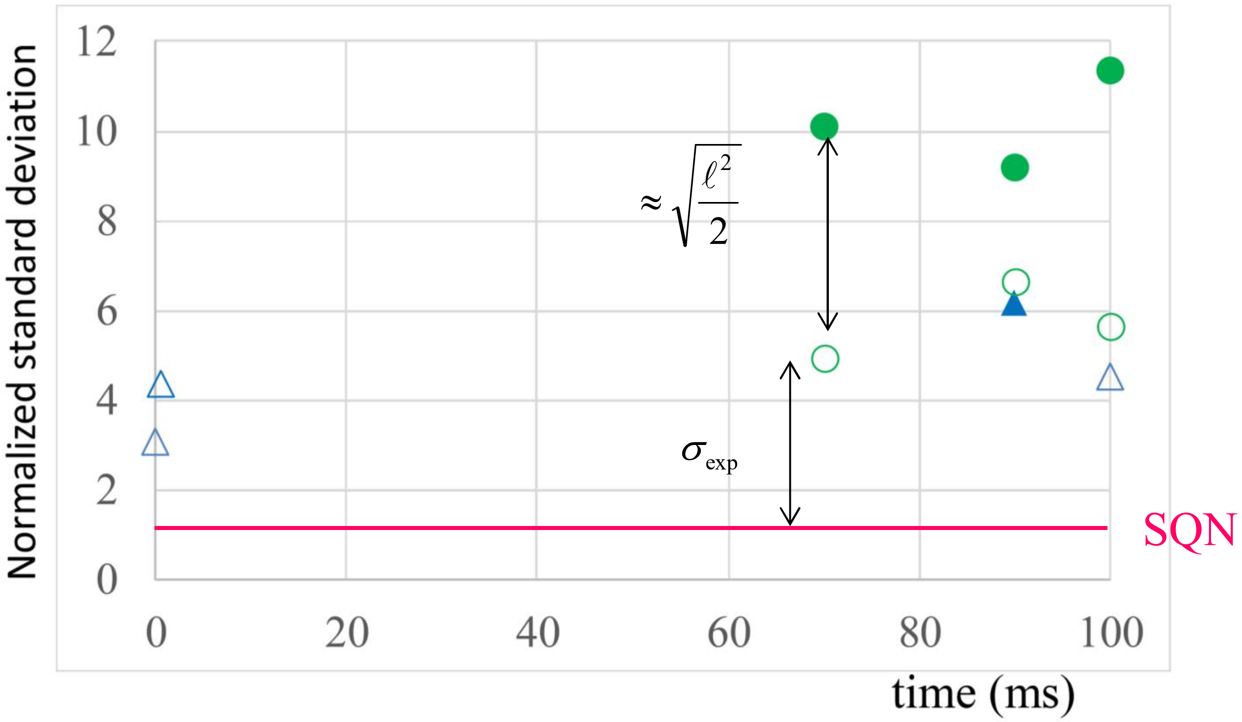
$$\left\langle \hat{j}_z^{\ 2} \right\rangle_{exp} = \frac{3}{2N} + \sigma_{exp}^{\ 2}$$

● Echo Ramsey

○ Echo No Ramsey

▲ No Echo Ramsey

△ No Echo Ramsey



Spin dynamics in lattice: quest for entanglement witnesses

Prediction (Ana Maria Rey):
PRL **110**, 075301 (2013)

θ small → classical precession
θ large → entanglement grows

Interpretation: dynamics comes from the difference to the Heisenberg Hamiltonian as there is no dynamics under H_{Heis}

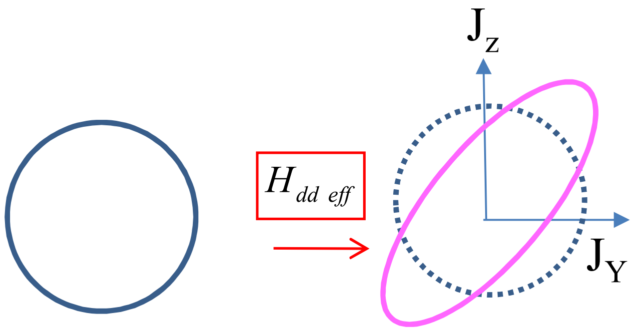
$$H_{dd} = S_{1z}S_{2z} - \frac{1}{4}\left(S_{1+}S_{2-} + S_{1-}S_{2+}\right)$$

$$H_{Heis} = \vec{S}_1.\vec{S}_2 = S_{1z}S_{2z} + \frac{1}{2}\left(S_{1+}S_{2-} + S_{1-}S_{2+}\right)$$

$$H_{dd} = -\frac{1}{2}H_{Heis} + \frac{1}{2}S_{1z}S_{2z}$$

$$S_{1z}S_{2z} \underset{t\to 0}{\approx} S_z^2$$

Squeezing



Squeezing is nice, but it is not an entanglement witness for spin s > ½ !

A. S. Sørensen and K. Mølmer, Phys. Rev. Lett. **86**, 4431 (2001)

Entanglement witnesses from measurements of collective spin components of order 2

G. Vitagliano et al, Phys. Rev. Lett. **107**, 240502 (2011) → all of them

$$\xi^2_{os} = (N-1)\frac{\left(\tilde{\Delta}\hat{J}_U\right)^2 + N\,j^2}{\left\langle \tilde{\hat{J}}_X \right\rangle^2 + \left\langle \tilde{\hat{J}}_V \right\rangle^2} \geq 1$$

$$\tilde{\Delta}J_l^{\,2} = \left\langle \tilde{\hat{J}}_m^{\,2} \right\rangle - \left\langle \hat{J}_m^{\,2} \right\rangle$$

$$\left\langle \tilde{\hat{J}}_m^{\,2} \right\rangle = \left\langle \hat{J}_m^{\,2} \right\rangle - \left\langle \sum_{i=1}^{N} \hat{j}_{i\ m}^{\,2} \right\rangle$$

$\hat{J}_U \perp \hat{J}_V$

in the Yz plane

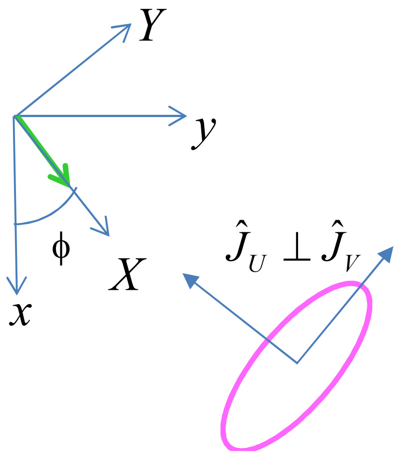
can we measure it?

$$\hat{J}_z^{\,2} \rightarrow \left(\sum_i m_{si}\right)^2$$

$$\sum_i \hat{j}_{i\ z}^{\,2} \rightarrow \sum_i m_{si}^{\,2}$$

1- control ϕ

2- rotate by θ_S

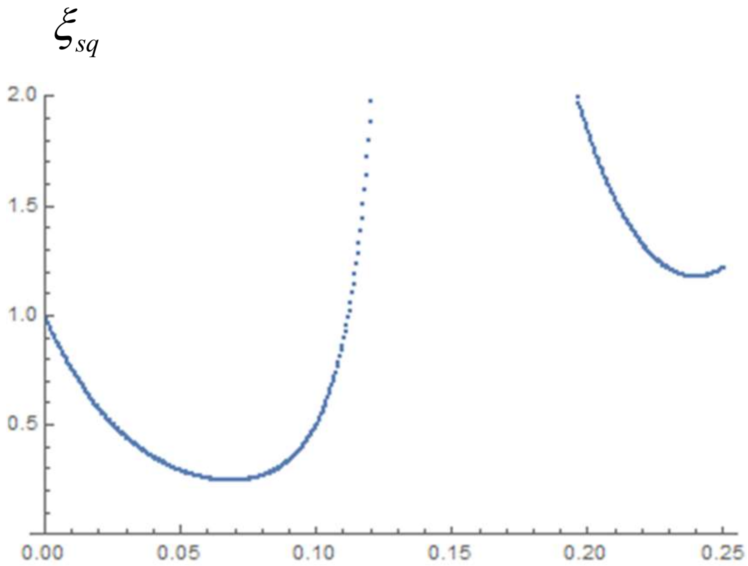


Spin squeezing criteria: an entanglement witness for dipolar dynamics?

,

Results for two atoms $\frac{1}{V_{dd}}\hat{H}_{dd} = \hat{s}_1\hat{s}_2 - \frac{1}{4}\left(\hat{s}_1^{+}\hat{s}_2^{-} + \hat{s}_1^{-}\hat{s}_2^{+}\right)$ $V_{dd} \propto \frac{1}{R^3}$

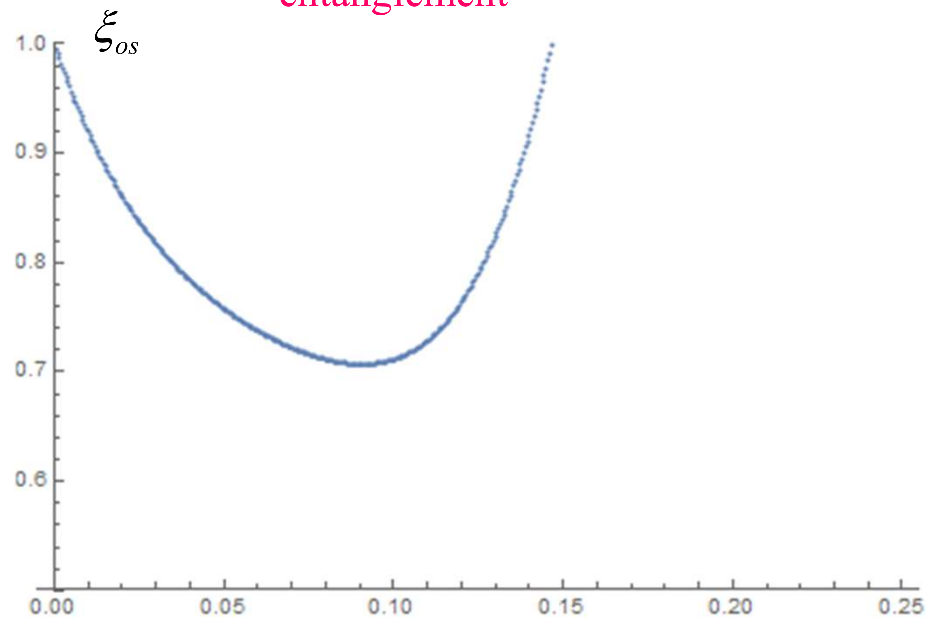
squeezing



$t = \frac{\hbar}{V_{dd}}$

$$\xi_{sq} = \frac{\Delta J_{\perp}^{\ 2}}{\frac{\langle J_X \rangle^2}{2N\ j}}$$

entanglement



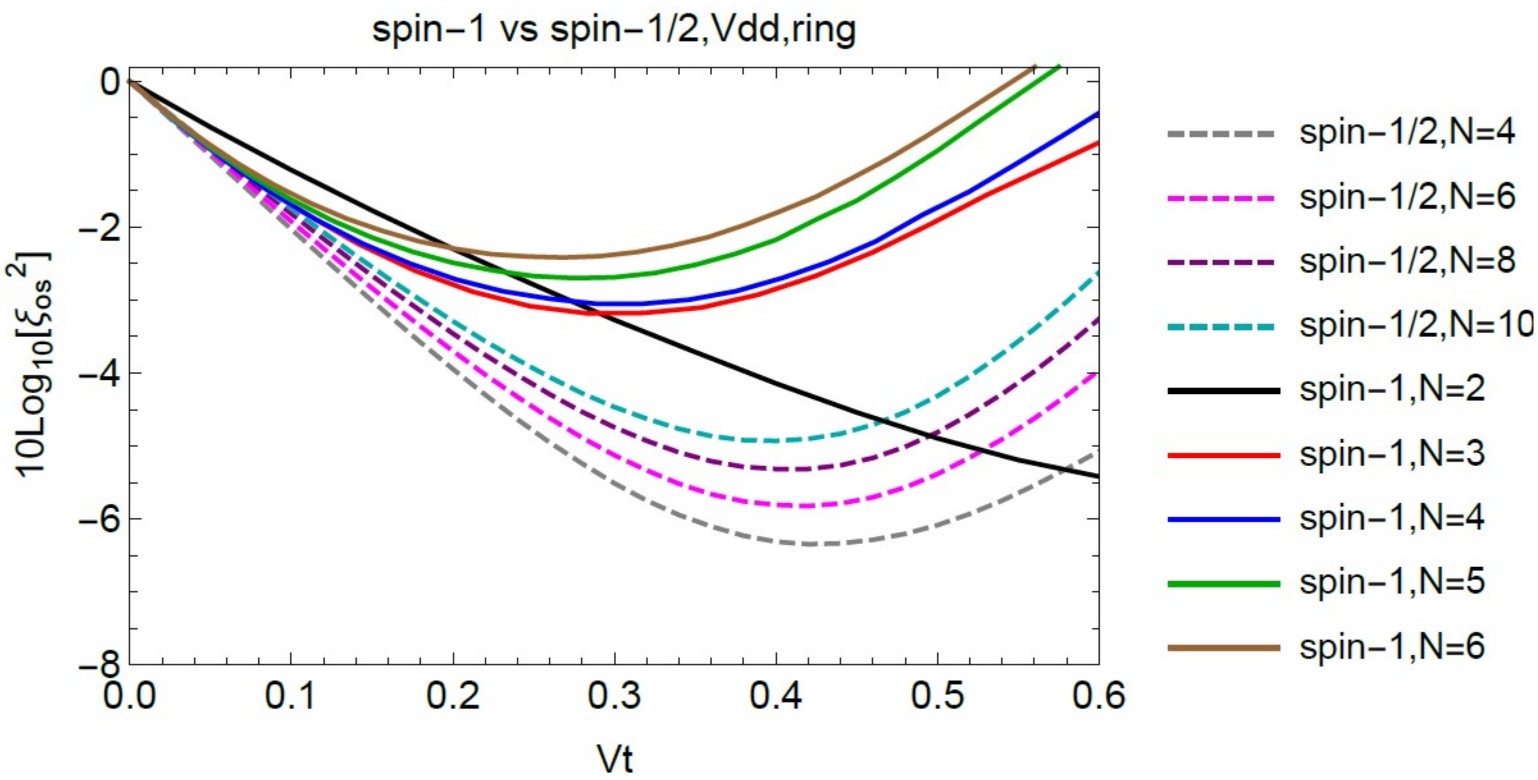
$t = \frac{\hbar}{V_{dd}}$

$$\xi^2_{os} = (N-1)\frac{\left(\tilde{\Delta}\hat{J}_U\right)^2 + N\ j^2}{\left\langle \tilde{\hat{J}}_X \right\rangle^2 + \left\langle \tilde{\hat{J}}_V \right\rangle^2}$$

enough squeezing is obtained to prove entanglement, but…

Spin squeezing criteria: an entanglement witness for dipolar dynamics?

enough squeezing is obtained to prove entanglement, but there is a bad scaling…



Bihui Zu

Bipartite measurements: a possible entanglement witness?

witness which could be adapted: Tommaso Roscilde

$$2\ \Delta\left(J_y^A - g_y J_y^B\right)\Delta\left(J_z^A - g_z J_z^B\right) \geq \left(\left|\left\langle J_x^A \right\rangle\right|_{\mathrm{inf}} + \left|g_y g_z\right| \left|\left\langle J_x^B \right\rangle\right|_{\mathrm{inf}}\right)$$

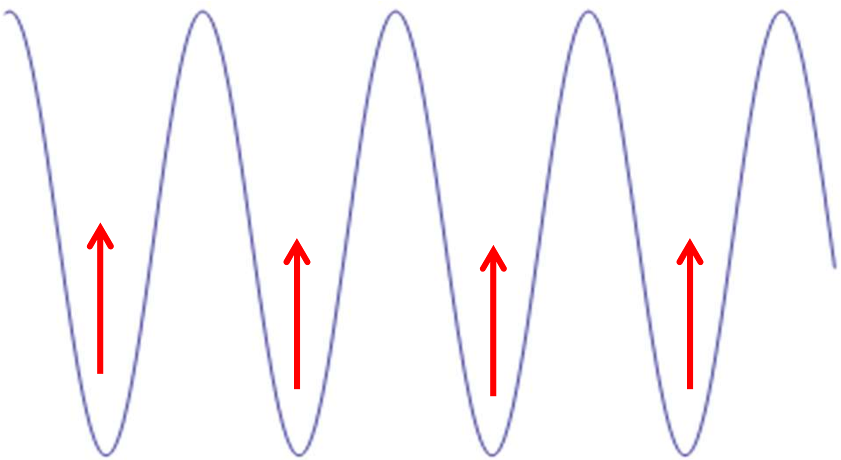
$$g_y = \frac{\left\langle J_y^A J_y^B \right\rangle - \left\langle J_y^A \right\rangle \left\langle J_y^B \right\rangle}{\left(\Delta J_y^B\right)^2}$$

$$\left|\left\langle J_x^A \right\rangle\right|_{\mathrm{inf}} = \sum_{J_x^B} \wp(J_x^B) \left| \sum_{J_x^A} \wp(J_x^A \middle| J_x^B) J_x^A \right|$$

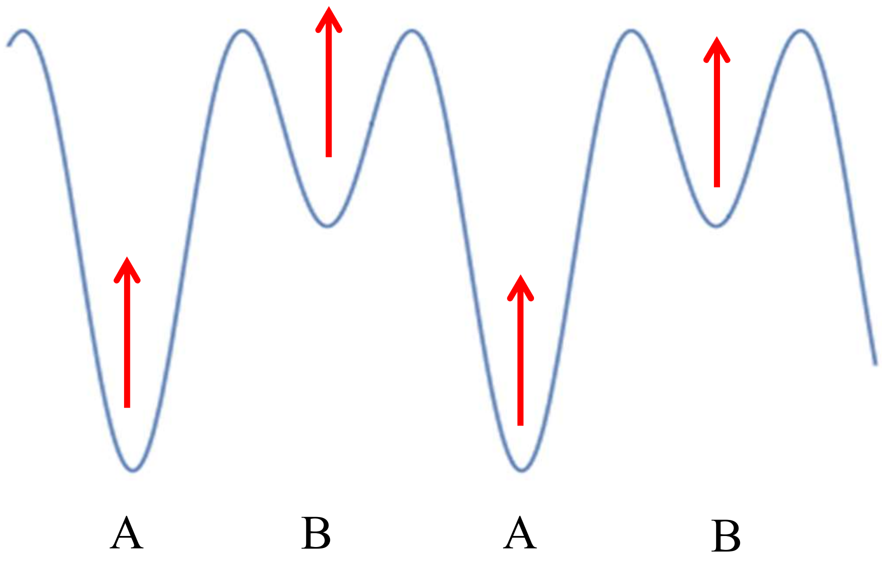
$$\Psi_{(t=0)} = \left| -2_z, -2_z, \ldots, -2_z, -2_z \right\rangle \qquad \Delta\left(J_z^A + J_z^B\right) = 0$$

Bipartite measurements: realization with bichromatic lattice

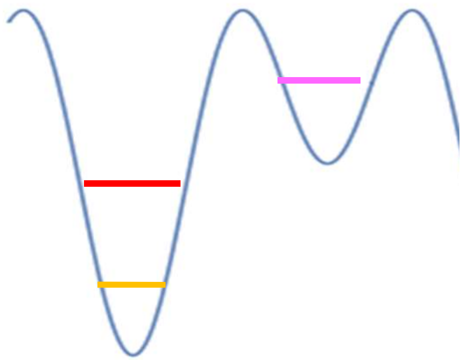
1- spin dynamics in single color lattice

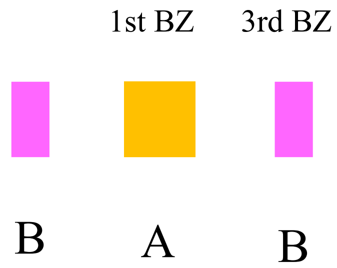


2- create the double color lattice

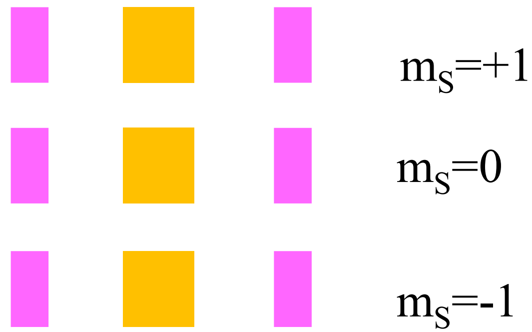


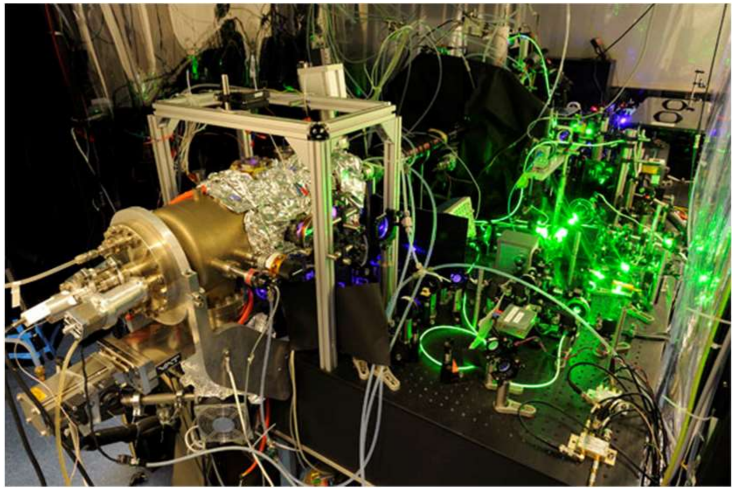
3- combine band mapping

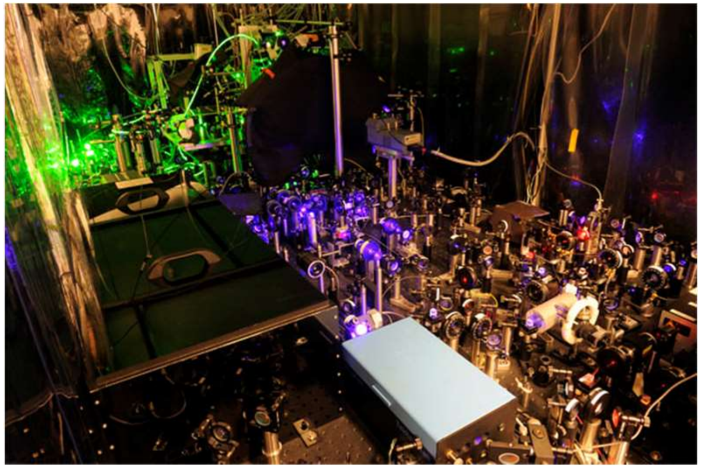


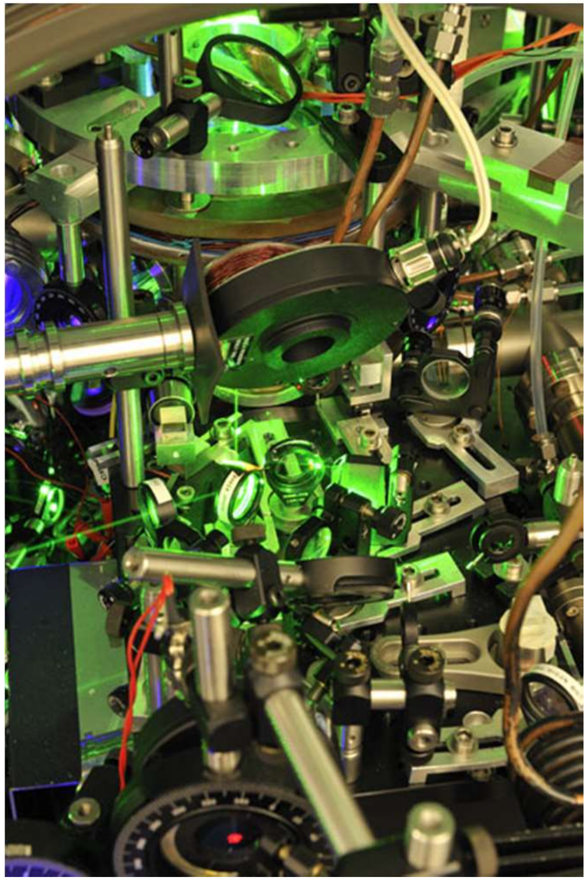


+ Stern Gerlach









thank you for your attention!

We are looking for a post doc!
We have money for two years!

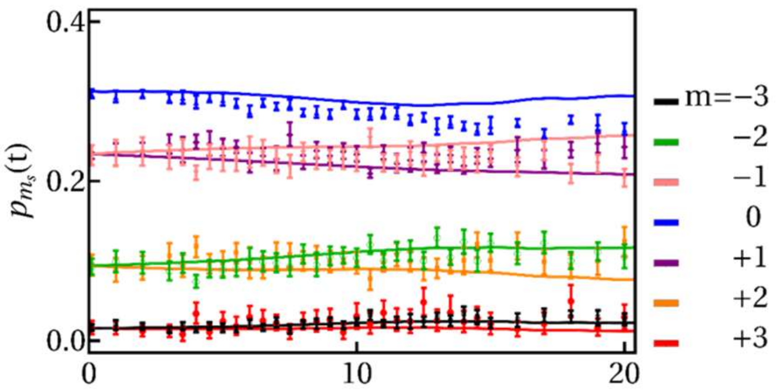
What happens when the same experiment is made with a BEC?

$$\Psi_{initial} = \left|-3_z, -3_z, \ldots, -3_z, -3_z\right\rangle \xrightarrow{\theta \text{ rotation}} \Psi_{(t=0)} = \left|-3_\theta, -3_\theta, \ldots, -3_\theta, -3_\theta\right\rangle$$

Magnetic gradients below 4 mG.cm^{-1} in all three directions

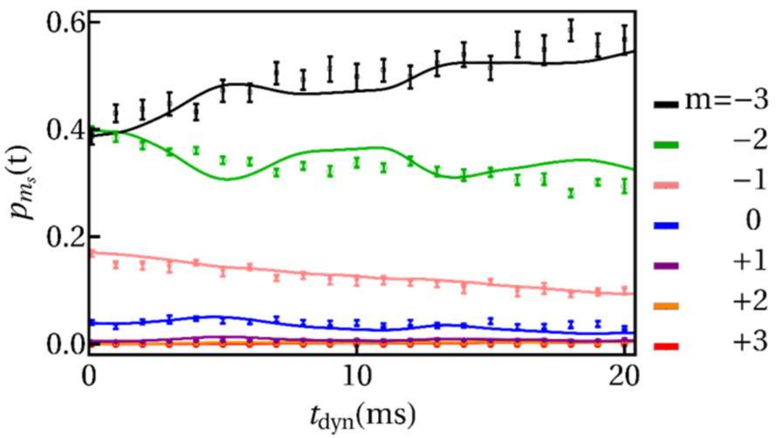
$\vec{\nabla} B = 0$

$\theta = \dfrac{\pi}{2}$



$\vec{\nabla} B = 0$

$\theta = \dfrac{\pi}{4}$

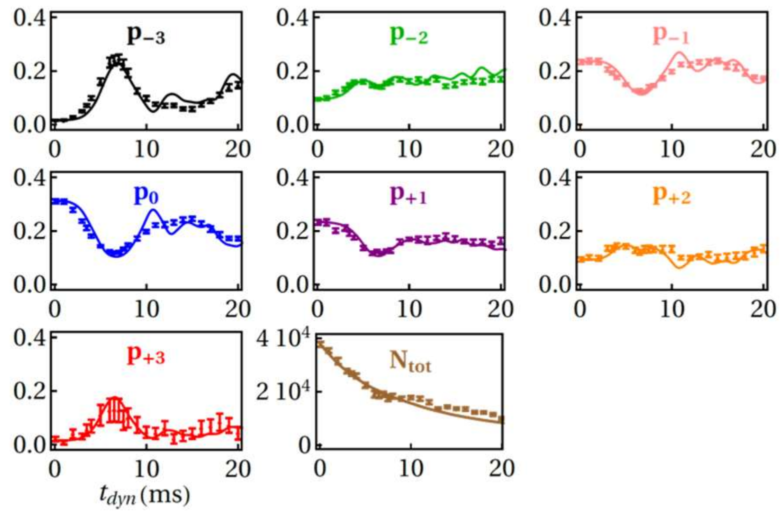


Mean field predictions:
Without magnetic gradients spin dynamics is triggered by dipole interactions unless $\theta=\pi/2$
Kawaguchi, Saito and Ueda, PRL **98** 110406 (2007)

Magnetic gradient 45 mG.cm^{-1}

$\vec{\nabla} B \neq 0$

$\theta = \dfrac{\pi}{2}$



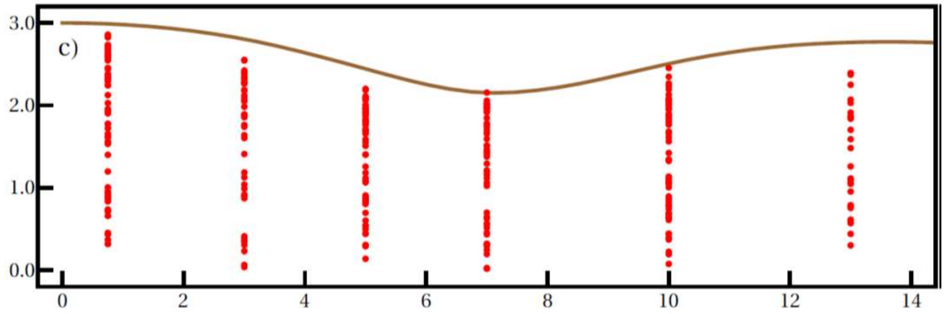
comparison with GPE excellent:
non beyond mean field effect

S. Lepoutre et al, Phys. Rev A **97** 023610 (2018)

Spin dynamics in a bulk chromium BEC: preservation of a ferromagnetic state

Experimental results after a Ramsey type experiment $\pi/2 - t - \pi/2$

Final magnetisation

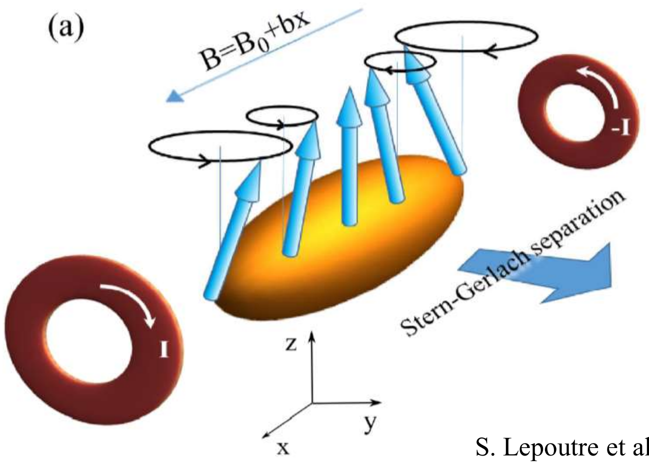


t_{dyn} (ms)

The experimental measurement demonstrate that the norm of the collective spin remains high
This shows not only preservation of ferromagnetism but as well that the spins remain almost parallel

S. Lepoutre et al, Phys. Rev A **97** 023610 (2018)

Trapped magnon modes !



Eigenmodes frequencies:

$$2\pi\nu_{i,j,k} = \frac{\hbar}{M}\left(\frac{i}{\sigma_x^2} + \frac{j}{\sigma_y^2} + \frac{k}{\sigma_z^2}\right)$$

Validity criteria:

$$\frac{1}{M}\left(\frac{g\mu_B b}{\omega}\right)^2 << n\, c_1 \qquad c_1 = \frac{a_6 - a_4}{11}$$

Magnetic gradient — **Trap frequency** — **density**

Natural timescale: $\tau = \left(\frac{2Mw}{g\mu_B b'}\right)^{1/2}$

S. Lepoutre et al, Phys. Rev. Lett. **121**, 013201 (2018)

Adiabatic production of the ground state of an Hamiltonian: principle

$$\hat{H}_{\mathrm{XXZ}} = \sum_{i>j}^{N} J_{i,j}\left(\Delta \hat{s}_i^z \hat{s}_j^z + \left(\hat{s}_i^x \hat{s}_j^x + \hat{s}_i^y \hat{s}_j^y\right)\right) + \mu\, B_z \sum_{i=1}^{N} \hat{s}_i^z + \mu\, B_x \sum_{i=1}^{N} \hat{s}_i^x$$

Theoretical model

longitudinal transverse

$$\hat{H}_{\mathrm{tot}\ \ \text{rotating frame}} = \sum_{i>j}^{N} V_{i,j}\left(\hat{s}_i^z \hat{s}_j^z - \frac{1}{2}\left(\hat{s}_i^x \hat{s}_j^x + \hat{s}_i^y \hat{s}_j^y\right)\right) + \hbar\delta \sum_{i=1}^{N} \hat{s}_i^z + \hbar\Omega_{RF} \sum_{i=1}^{N} \hat{s}_i^x$$

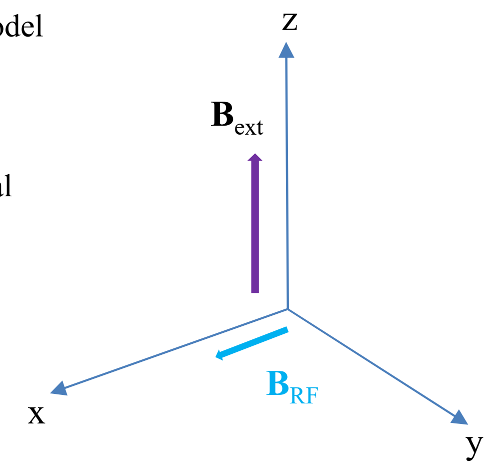
Experimental realization

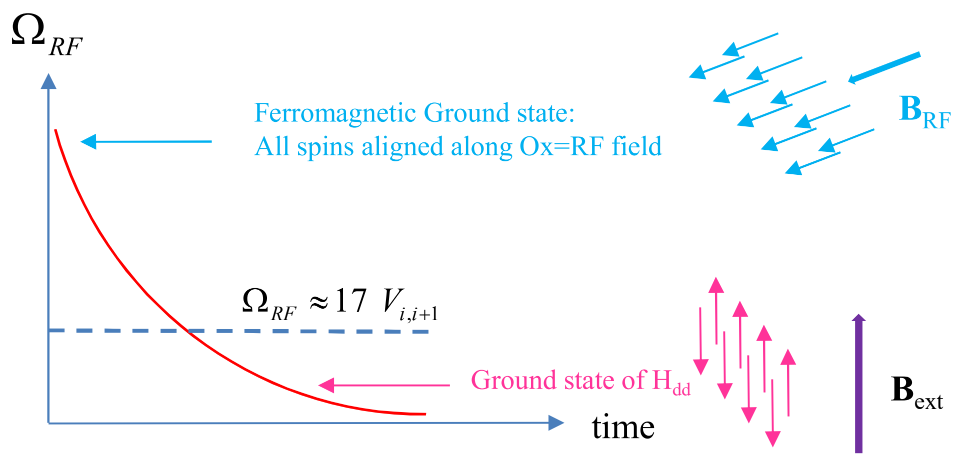
$$V_{i,j} = \frac{\mu_0}{4\pi}(g\mu_B)^2 \frac{1-3\cos^2\theta_{ij}}{r_{ij}^{\ 3}}$$

H_{dd} = Dipolar Hamiltonian

$$\delta = \omega_{\text{Larmor}} - \omega_{RF}$$

Ω_{RF} : RF Rabi frequency

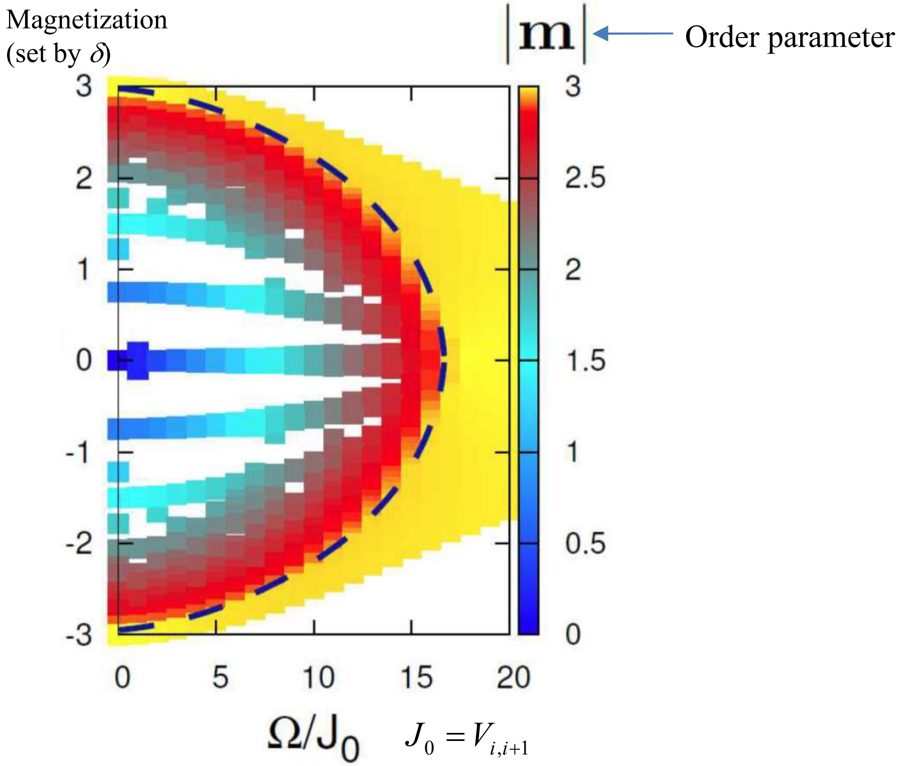




Principle of the experiment:
Perform an adiabatic passage
with an easy knob (RF amplitude)
from a polarized state to a non trivial
ground state of the dipolar Hamiltonian

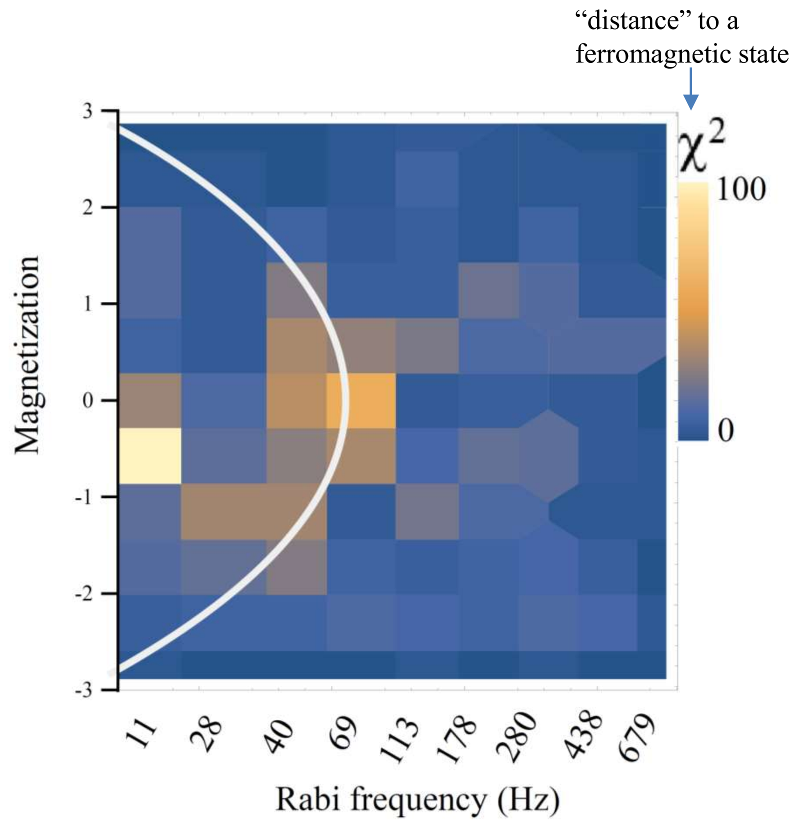
Adiabatic production of the ground state of an Hamiltonian: results (preliminary!)

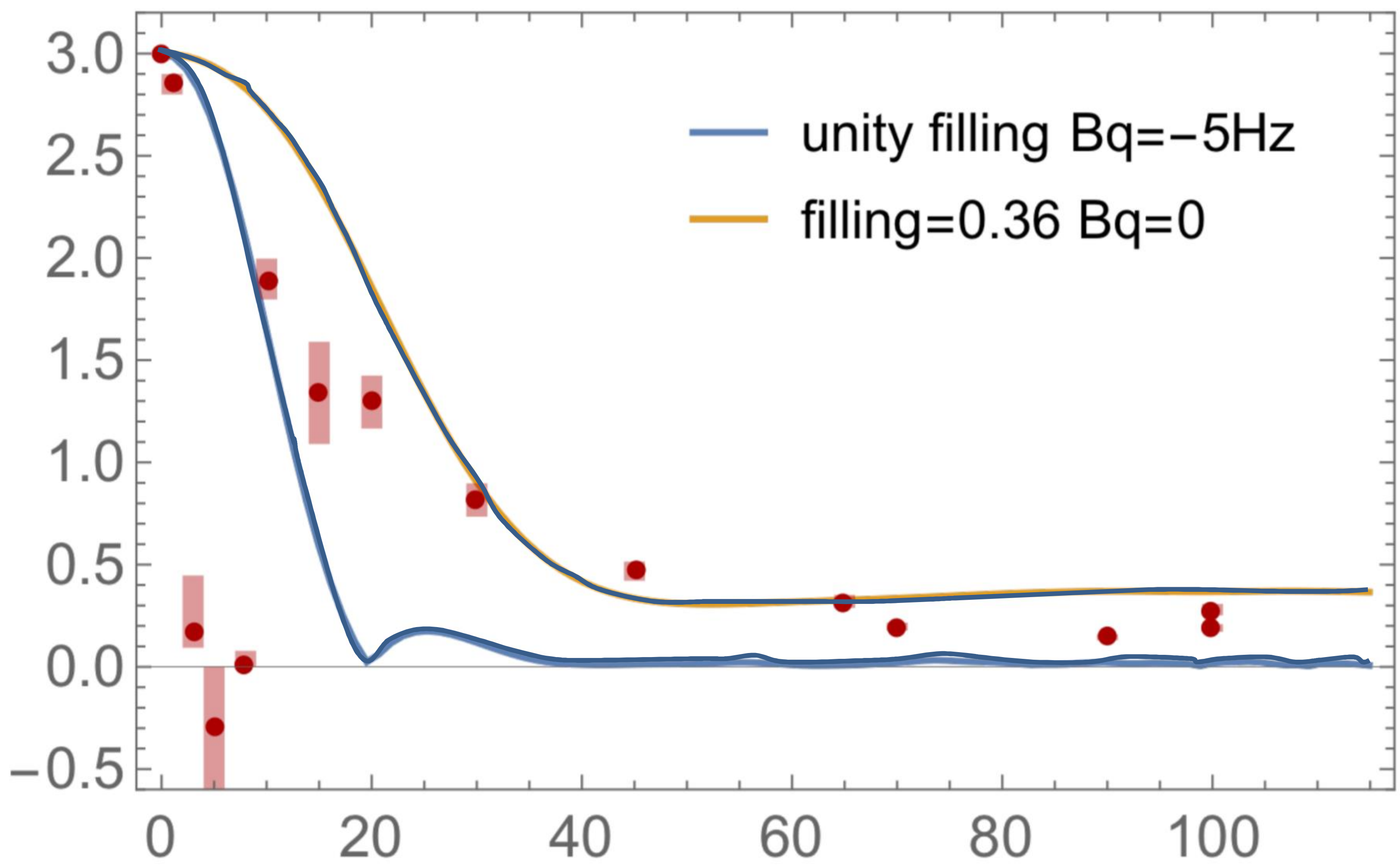
Theoretical prediction (Tommaso Roscilde)
mean field calculations



Ω_{RF} : RF Rabi frequency

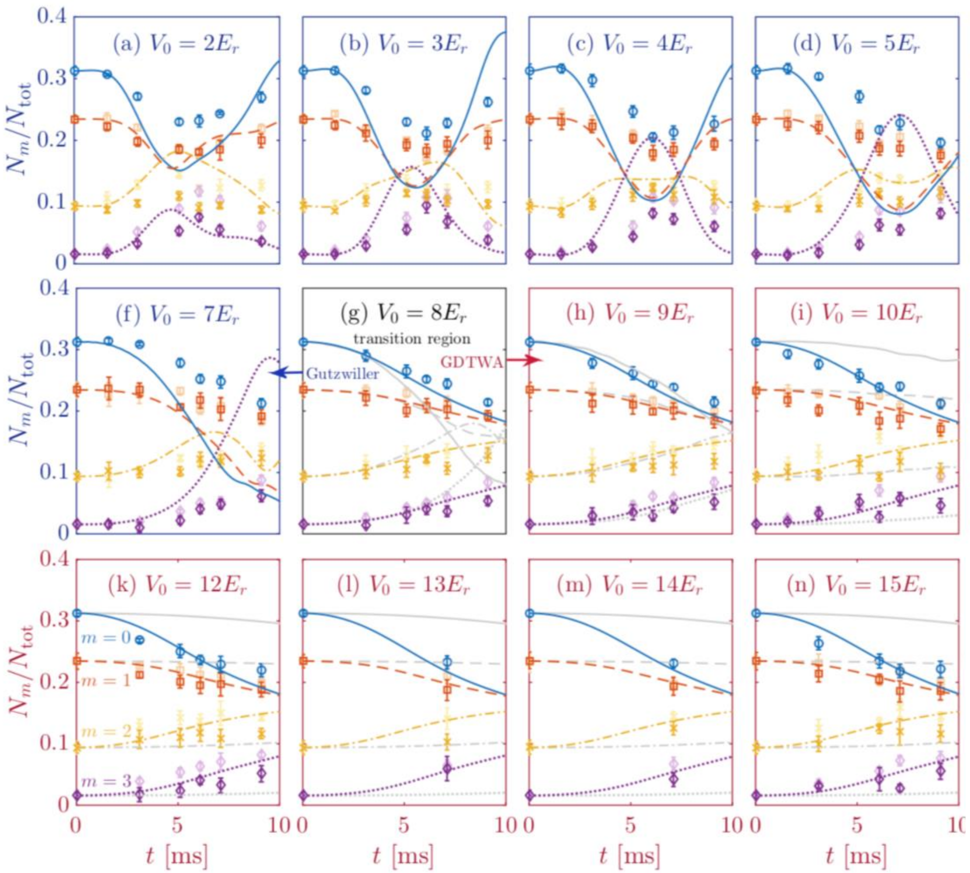
Experimental measurements

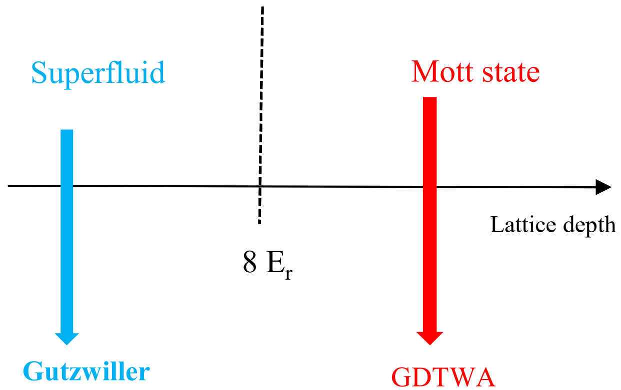



unity filling Bq=−5Hz
filling=0.36 Bq=0
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
0
20
40
60
80
100

Spin dynamics in lattice as a function of lattice depth

Competition between dipolar interactions, tunneling and tunneling assisted superexchange





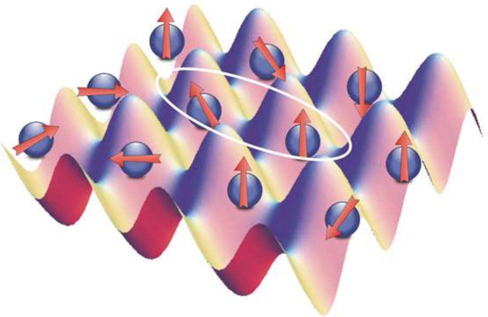
The **Gutzwiller** method aim to describe bosons in an optical lattice.
Our work is the first to consider the extension of this method to describe spin-3 bosons with dipole-dipole interactions.
It treats onsite terms exactly and inter-site couplings (due to tunneling and interactions) at the meanfield level.

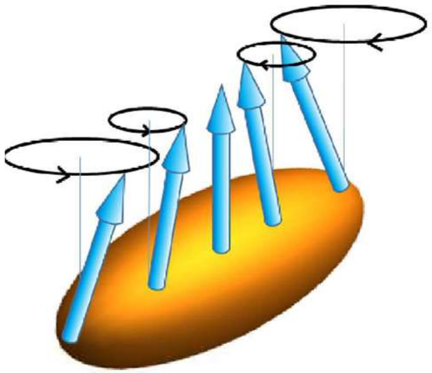
Petra Fersterer et al, PRA (2019)

Spin dynamics in quantum gases: summary of our results

In deep optical lattices
Mott insulating state, one atom per site

In a bulk BEC = superfluid





The norm of the collective spin goes rapidly to zero

The BEC remain almost ferromagnetic

Lepoutre et al, Phys. Rev. A **97**, 023610 (2018)

Quantum correlations build up, entanglement grows

Spin dynamics well described by mean field simulations
(Kaci Kechadi, Paolo Pedri at LPL)

Spin dynamics lead to quantum thermalization

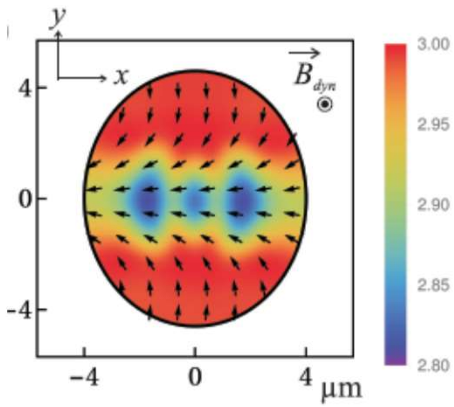
Collective Spin Modes of a Trapped Quantum Ferrofluid
(trapped magnon modes)

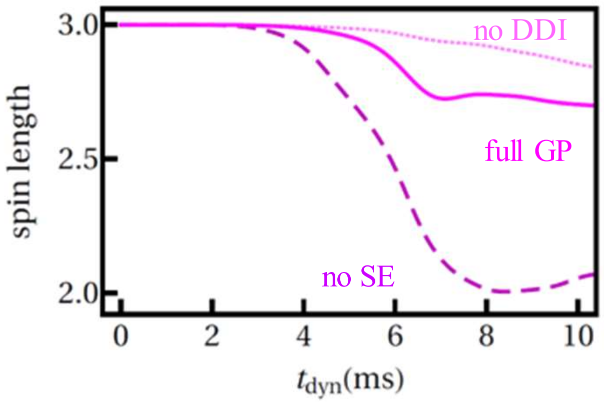
Lepoutre et al, Phys. Rev. Lett. **121**, 013201 (2018)

Lepoutre et al, arXiv:1803.02628 (2018)

Spin dynamics in a bulk chromium BEC: preservation of a ferromagnetic state

GPE results
for local spin length

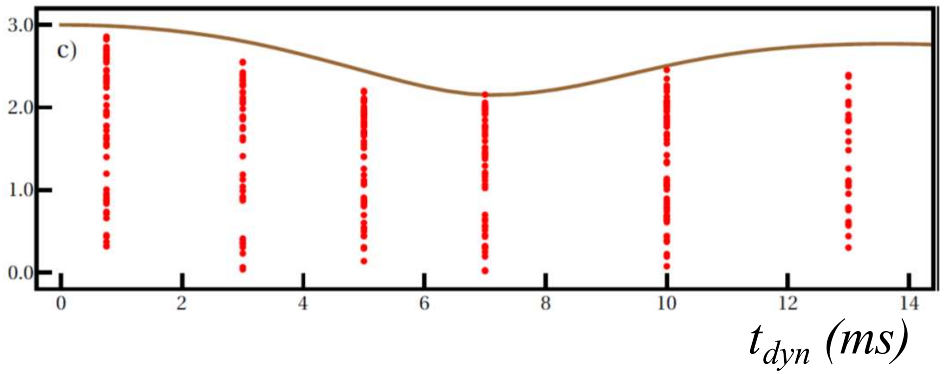




Final
magnetisation

Experimental results

$\pi/2 - t - \pi/2$



The experimental measurement demonstrate that the norm of the collective spin remains high
This shows not only preservation of ferromagnetism but as well that the spins remain almost parallel

S. Lepoutre et al, Phys. Rev A **97** 023610 (2018)

Spin dynamics in a bulk chromium BEC: ferrofluid model predict spin collective modes

Hydrodynamic equation - Kudo and Kawaguchi, Phys Rev A **82**, 053614 (2010)

$$\frac{\partial \vec{S}}{\partial t} = -\vec{S} \times \left[-\frac{\hbar}{2M} \left(\vec{a}.\vec{\nabla} \right) \vec{S} - \frac{\hbar}{2M} \nabla^2 \vec{S} + \frac{g\mu_B}{\hbar} \vec{B}(\vec{r}) \right] \qquad \vec{a} = \vec{\nabla}(n_{tot})/n_{tot}$$

Spin remains almost ferromagnetic:

$$\vec{S}(\vec{r}) = \left\{ f, g, \sqrt{1 - f^2 - g^2} \right\}$$

$$f = P(\vec{r}) \sin(\omega t) \qquad g = P(\vec{r}) \cos(\omega t)$$

Assume a Gaussian density:

$\Rightarrow P(\vec{r})$ Hermit polynomials

Eigenmodes frequencies:

$$2\pi\nu_{i,j,k} = \frac{\hbar}{M} \left(\frac{i}{\sigma_x^2} + \frac{j}{\sigma_y^2} + \frac{k}{\sigma_z^2} \right)$$

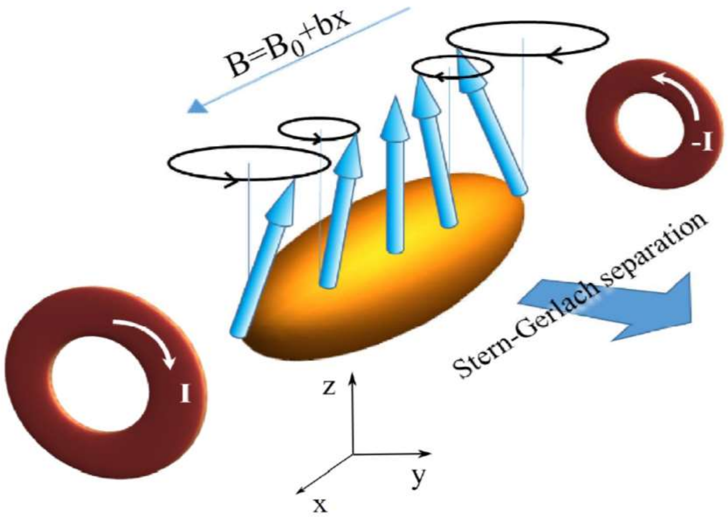
$$2\pi\nu \approx \omega \times \frac{\hbar\omega}{\mu} \ll \omega$$

Excite spin modes with a magnetic gradient:

$$\vec{B}(\vec{r}) = bx\vec{u}_x$$

$$f(x,t) = M\sigma_x^2 g\mu_B b/\hbar^2 \left(1 - \cos 2\pi\nu t\right) x$$

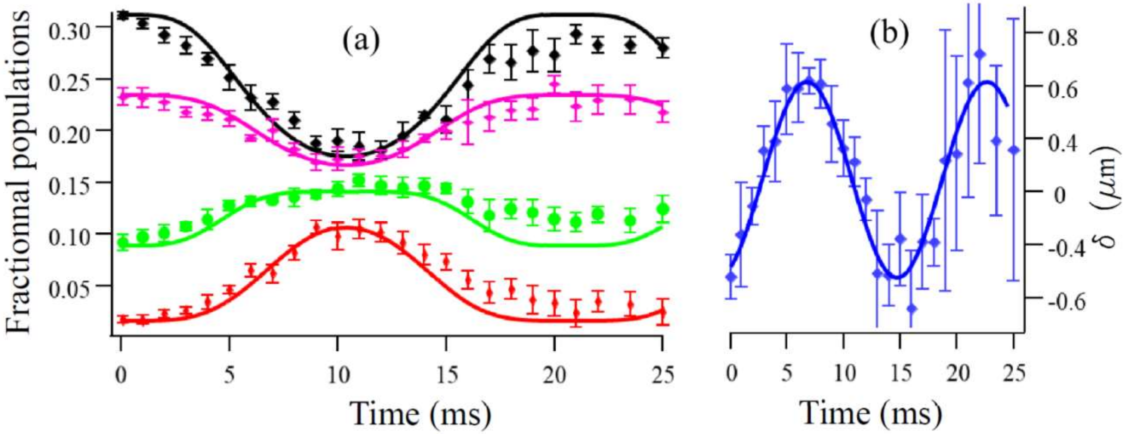
Trapped magnon modes !



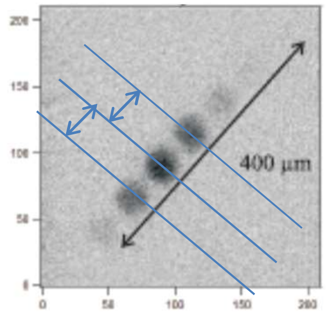
Spin dynamics in a bulk chromium BEC: observation of trapped magnon modes

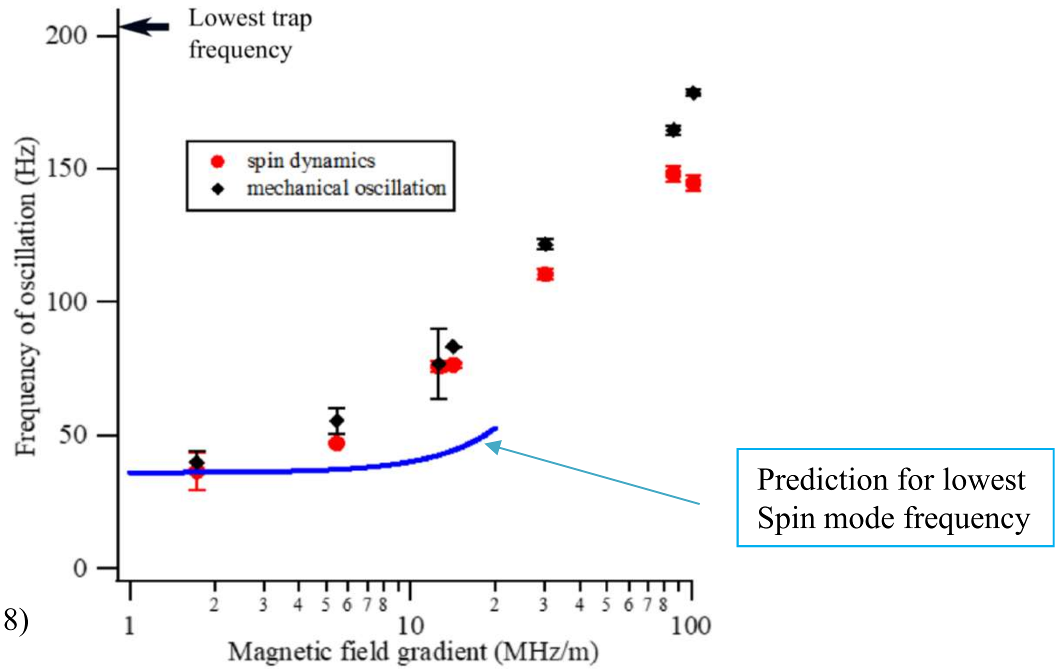
Comparison with experiment:

evolution of spin components populations **and** spin components peak density positions are derived from the ferrofluid model



$\delta(t)$ is the spin components separation after TOF





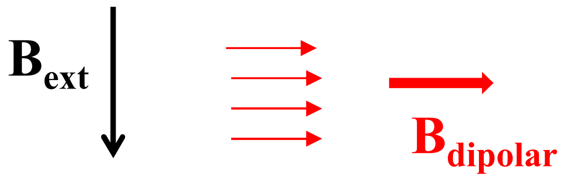
Lepoutre et al, Phys. Rev. Lett. **121**, 013201 (2018)

Spin dynamics in a bulk chromium BEC: triggering spin dynamics

Van der Waals interactions cannot trigger spin dynamics as the initial state is ferromagnetic and is therefore an eigenstate of H_{VdW}

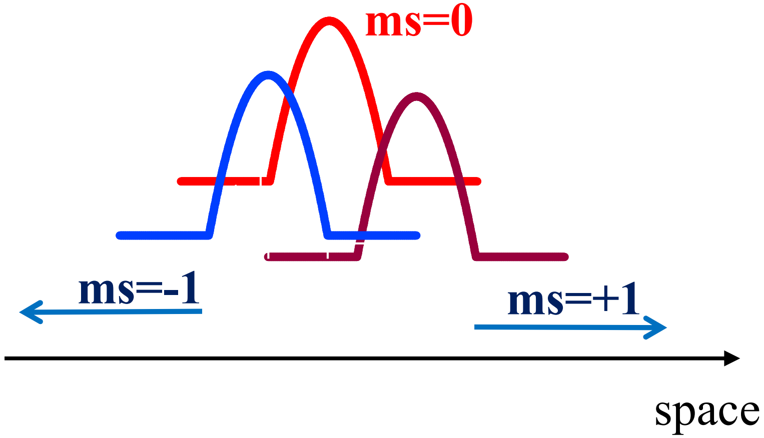
$$\Psi_{(t=0)} = \left| -3_\theta, -3_\theta, \ldots., -3_\theta, -3_\theta \right\rangle$$

Dipolar interactions
mean field predictions: spin dynamics is triggered by DDIs unless $\theta = \pi/2$
Kawaguchi, Saito and Ueda, PRL **98** 110406 (2007)



$B_{dipolar}$ is parallel to the spins if $\theta = \pi/2$

Magnetic filed gradients can trigger spin dynamics as they can locally break the initial ferromagnetic character of the ground state

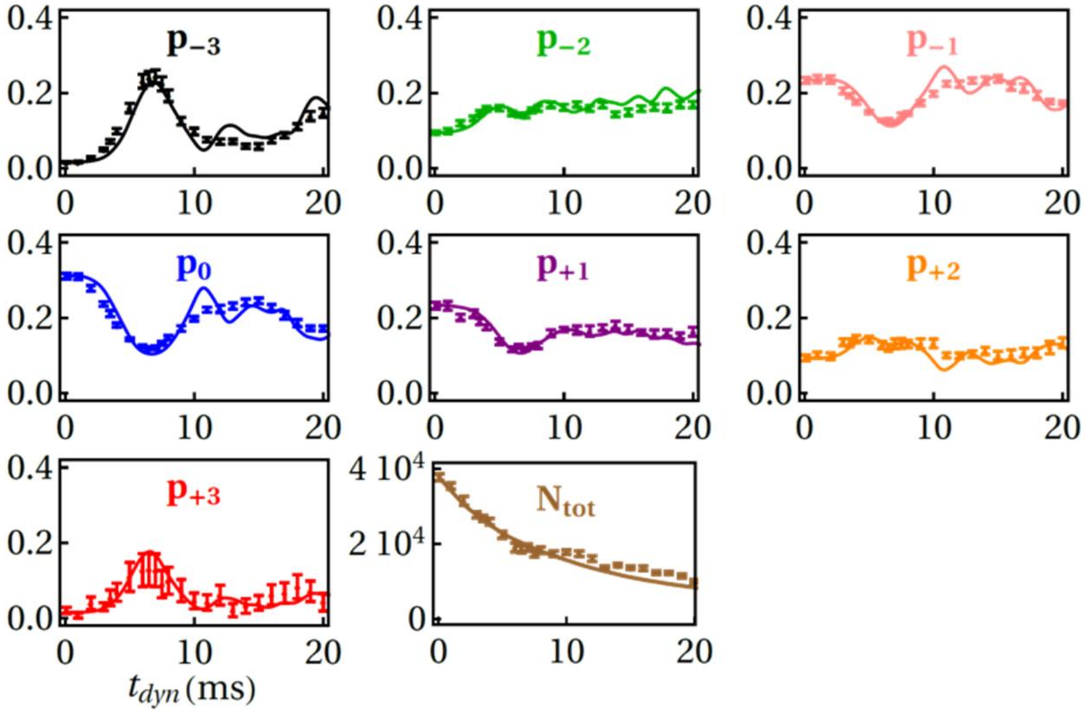


Spin dynamics in a bulk chromium BEC: comparison with GPE

$$\vec{\nabla} B \neq 0$$

$$\theta = \frac{\pi}{2}$$

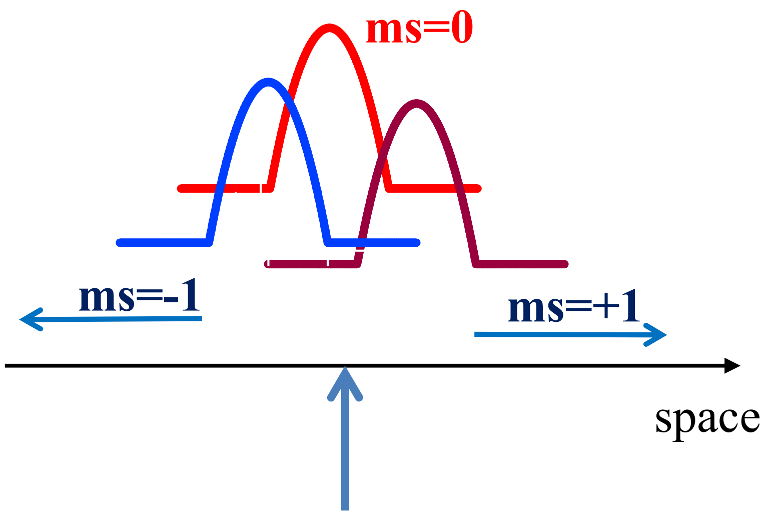
Magnetic gradient
45 mG.cm^{-1}



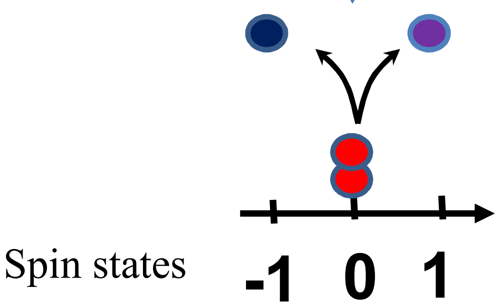
Losses due to dipolar relaxation

S. Lepoutre et al, Phys. Rev A **97** 023610 (2018)

Spin dynamics in a bulk chromium BEC: simple model to interpret protection of ferromagnetism



Local structure of spinor
- modified by gradient
- restored by spin-exchange



The spin-dependent interactions can undo the population imbalance that the magnetic field gradient creates !

Assuming infinite strong spin dependent interactions one obtains:

Magnetic gradient

$$\frac{p_{m_s}(t)}{p_{m_s}(0)} = 1 + \left(\frac{g\mu_B b}{2Mw}\right)^2 \left(m_s^2 - \sum_{m_{s'}} m_{s'}^2 p_{m_s'}(0)\right) t^4$$

BEC radius

Universal behavior: interactions adiabatically eliminated

Validity criteria:

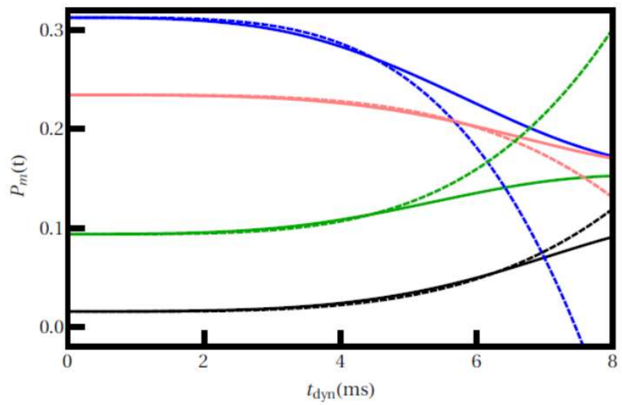
$$\frac{1}{M}\left(\frac{g\mu_B b}{\omega}\right)^2 << n\, c_1 \qquad c_1 = \frac{a_6 - a_4}{11}$$

Trap frequency **density**

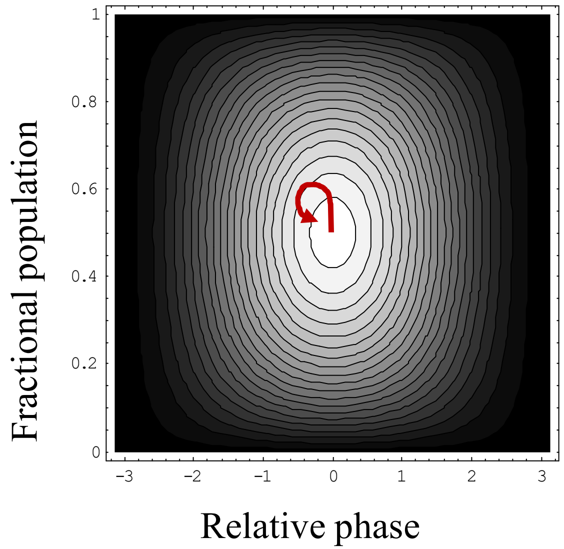
Natural timescale:

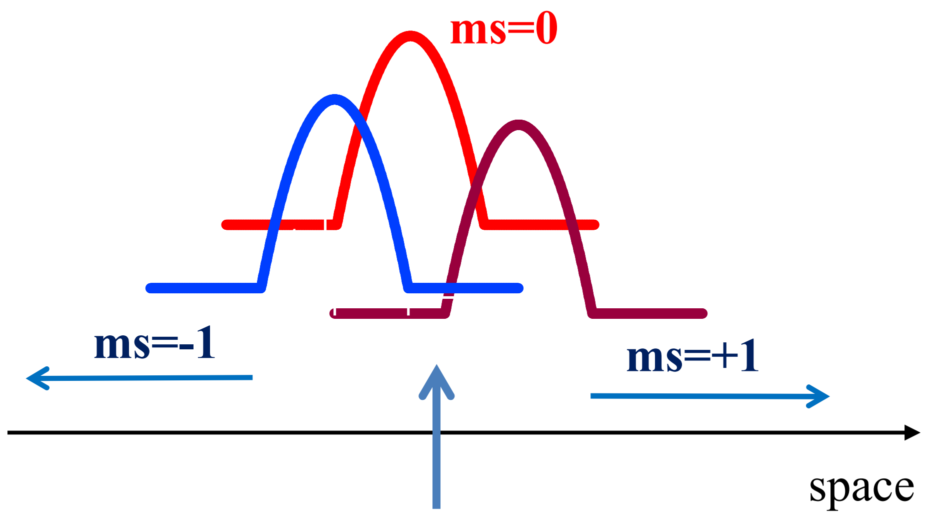
$$\tau = \left(\frac{2Mw}{g\mu_B b'}\right)^{1/2}$$

This model is in good agreement at short time with GP simulation when DDI are neglected.

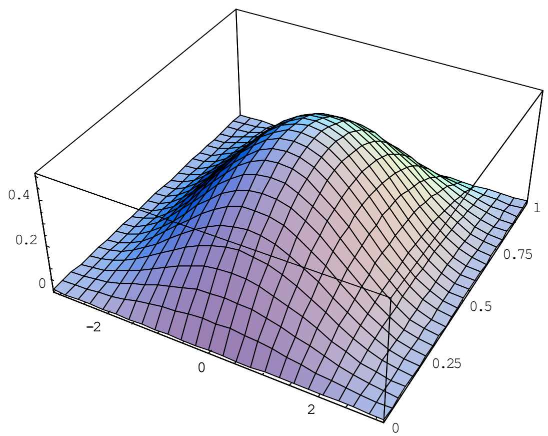


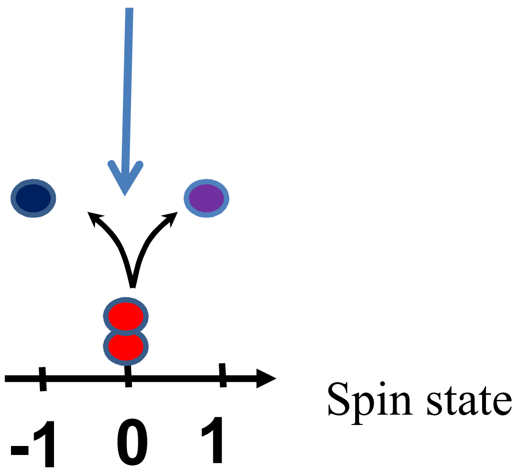
Interpretation: locally, spinor is at a maximum of the interaction energy. Magnetic field gradients cannot change the spinor structure without violating energy conservation

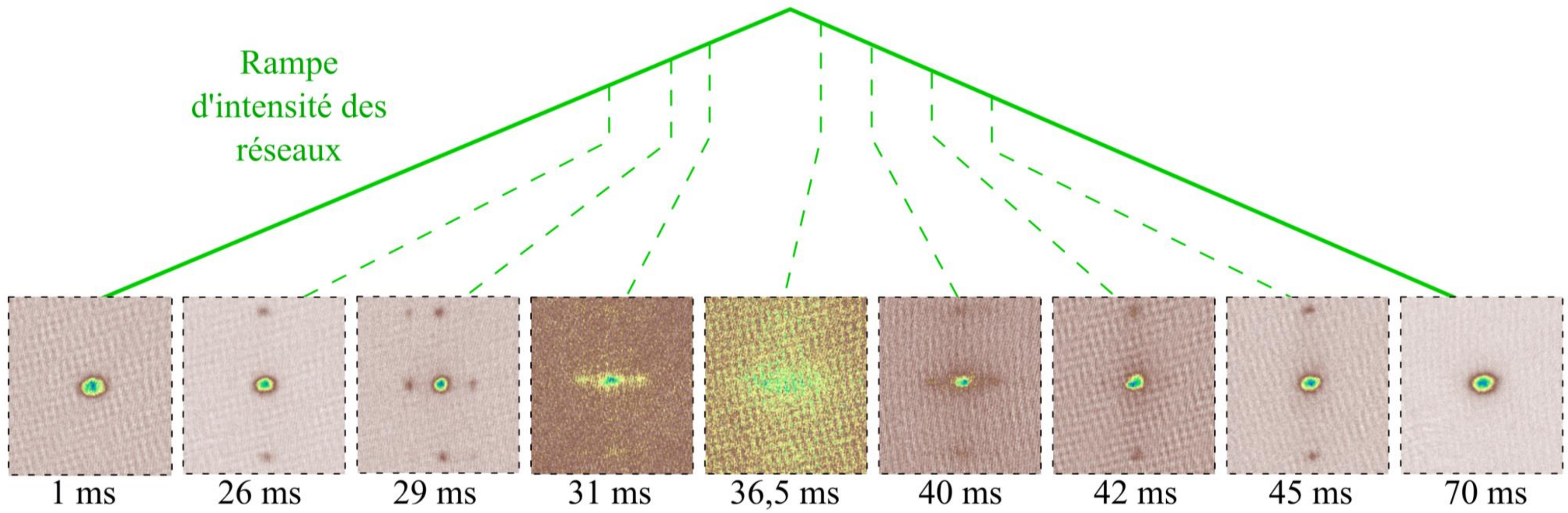




Local structure of spinor
- modified by gradient
- restaured by spin-exchange



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