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Quantum thermalization of long-range interacting spins

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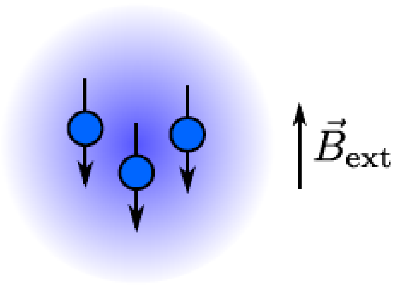
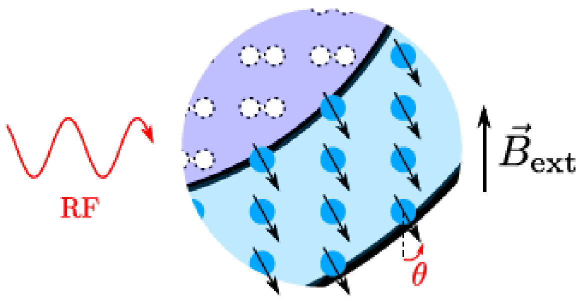
Quantum Thermalization (Isolated System)

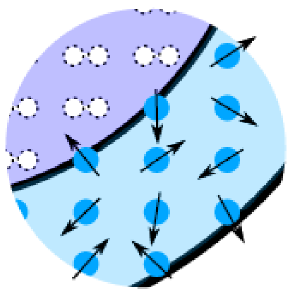
Initial state



Quench



Evolution to…



Stationnary state

…

Thermal-like

Quantum many-body physics of pure states → **Thermodynamics (mixed states)**

Eigenstate Thermalization Hypothesis Deutsch, Srednicki, Olshanii

…and the Eigenstate Thermalization Hypothesis

(discussion from Rigol et al. 2009)

$$|\psi(0)\rangle = \sum_\alpha C_\alpha |\Psi_\alpha\rangle$$

$$|\psi(t)\rangle = e^{-i\hat{H}t}|\psi(0)\rangle = \sum_\alpha C_\alpha e^{-iE_\alpha t}|\Psi_\alpha\rangle$$

$$\langle\hat{A}(t)\rangle \equiv \langle\psi(t)|\hat{A}|\psi(t)\rangle = \sum_{\alpha,\beta} C_\alpha^\star C_\beta e^{i(E_\alpha - E_\beta)t} A_{\alpha\beta}$$

$$A_{\alpha\beta} = \langle\Psi_\beta|\hat{A}|\Psi_\alpha\rangle$$

$$\overline{\langle\hat{A}\rangle} = \sum_\alpha |C_\alpha|^2 A_{\alpha\alpha}$$

If a system approaches a steady state, it should be given by $\overline{\langle\hat{A}\rangle}$.
The system should settle to an appropriate statistical mechanical ensemble

$$\sum_\alpha |C_\alpha|^2 A_{\alpha\alpha} = \langle A\rangle_{\text{microcan.}}(E_0)$$

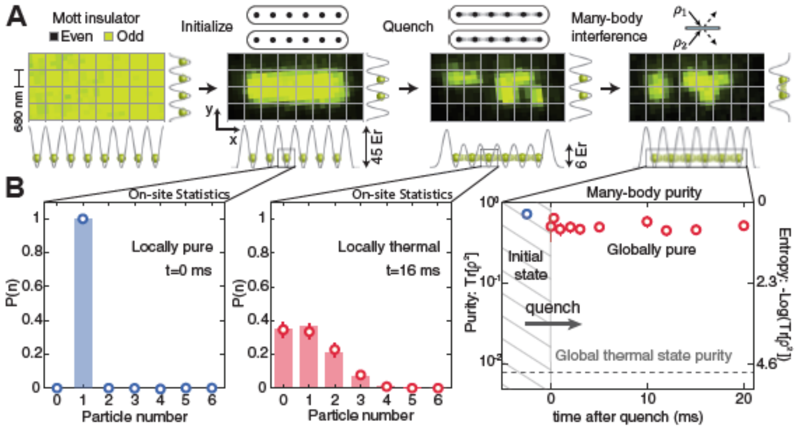
The E.T.H. gives a possibility for this to happen :

$$A_{\alpha\alpha} = \langle A\rangle_{\text{microcan.}}(E_0)$$

State of the art:

- E.T.H. seems verified for non-integrable systems (plaquette simulations)
- Experimentally, Greiner 2016

Small systems …



The system is globally pure
Growth of entanglement
The entanglement entropy assumes the role of the « thermal-like » entropy

Why Eigenstate Thermalization Hypothesis is valid remains mysterious

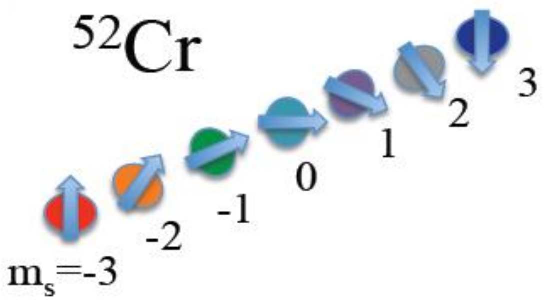
$$A_{\alpha\alpha} = \langle A \rangle_{\text{microcan.}}(E_0)$$

Independent of initial state !!

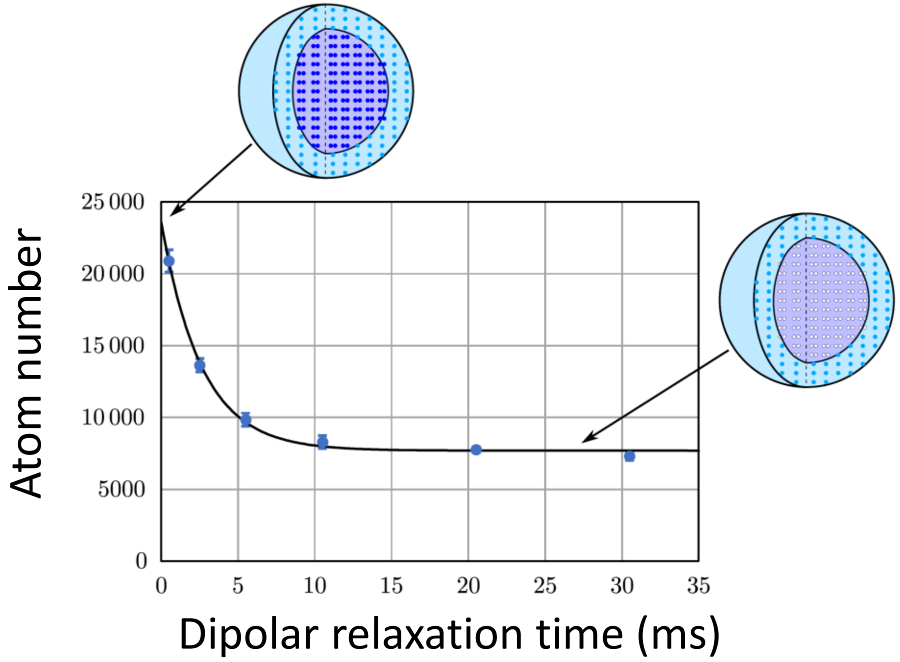
What sets to timescale for thermalization?
Long-range interacting systems may possess peculiar thermodynamic properties

See T. Dauxois et al. 2002

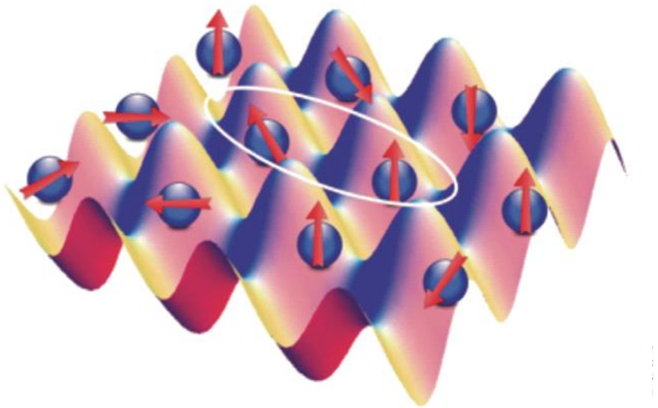
We revisit this physics using a macroscopic ensemble of Cr atoms in optical lattices



1- Prepare a unit-filled sample of Cr atoms in a 3D optical lattice



2- Interactions through long-range dipole-dipole interactions



$$H = c_{dd} \sum_{(i,j)} \left[S_i^z . S_j^z - \frac{1}{4} \left(S_i^+ . S_j^- + S_i^- . S_j^+ \right) \right] \frac{(1 - 3\cos^2\theta_{ij})}{r_{ij}^3}$$

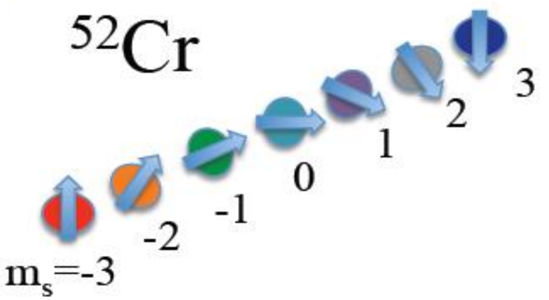
Outline:

1- Theoretical introduction to thermalization

2- Thermalization of the Zeeman populations

3- Thermalization of the collective spin

Lanczos approach to calculate many-body dynamics



$$T = \sum_{i,j} \alpha_{i,j} \left[S_i^z S_j^z - \frac{1}{4}(S_i^+ S_j^- S_j^+ S_i^-) \right] \qquad \alpha_{i,j} = \frac{1 - 3\cos\theta_{i,j}^2}{r_{i,j}^3}$$

$$\Psi_0 = |2,2,2,2,\ldots,2\rangle \xrightarrow{\;T\;} \Psi_1 = \sum_{(i,j)} \alpha_{i,j} |2,\underset{i}{1},2,2,\ldots 2,\underset{j}{3},2\rangle \xrightarrow{\;T\;} (\ldots)$$

$$\left.\begin{aligned} |\psi_1'\rangle &= T\,|\psi_0\rangle \\ &\ldots \\ |\psi_i'\rangle &= T^i\,|\psi_0\rangle \end{aligned}\right\} \rightarrow \text{Orthonormalize} \qquad H = \begin{pmatrix} I_0 & \Gamma_0 & 0 & \cdots & 0 \\ \Gamma_0 & I_1 & \Gamma_1 & \cdots & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ 0 & \cdots & \Gamma_{n-2} & I_{n-1} & \Gamma_{n-1} \\ 0 & \cdots & 0 & \Gamma_{n-1} & I_n \end{pmatrix}$$

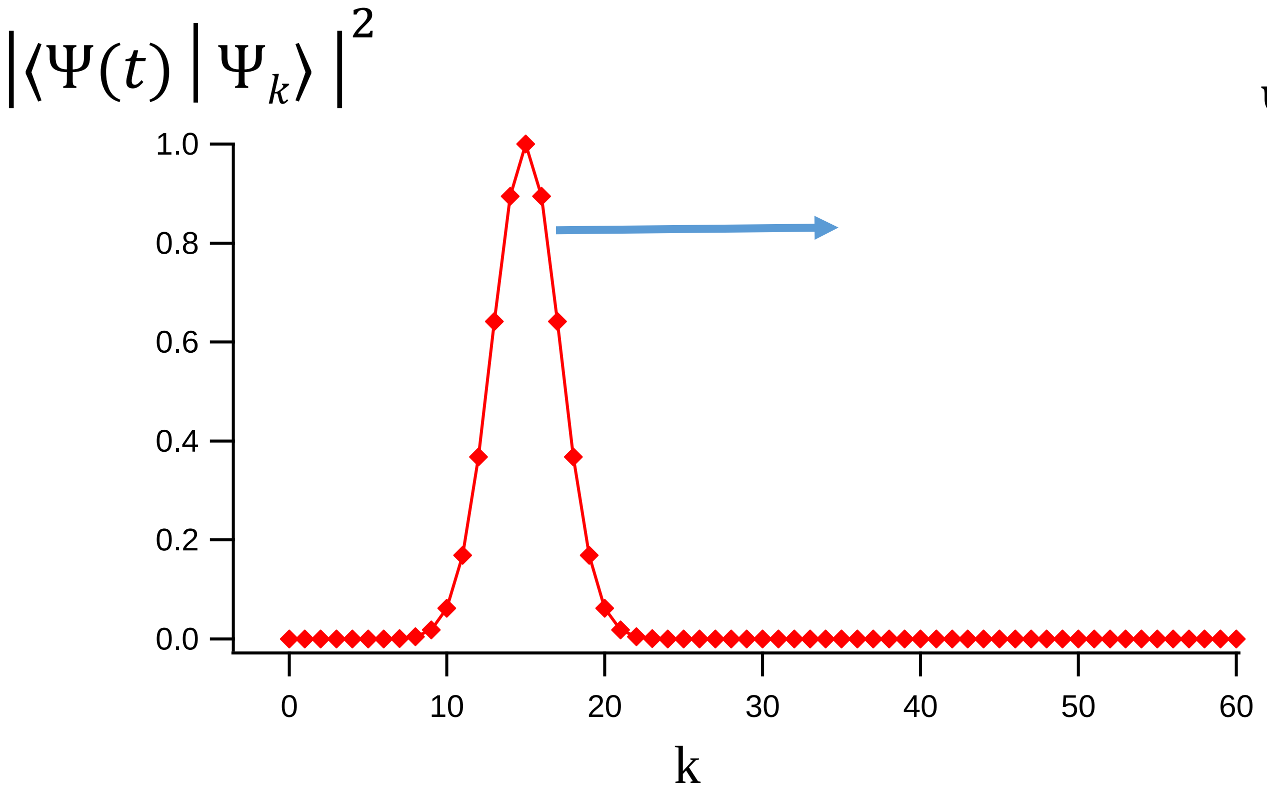
Any many-body problem can be cast into a 1D problem (tri-diagonal matrix)
Very few (Lanczos) states are needed to describe the problem (but may be very hard to calculate…)

Dynamics in the Lanczos basis

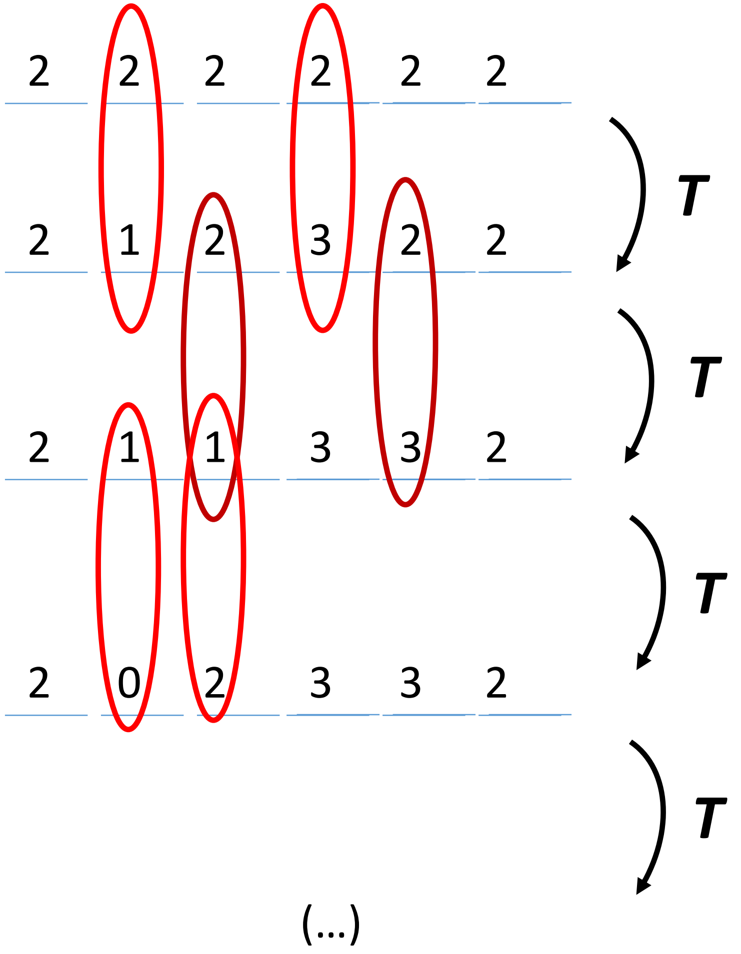
Natural timescale for thermalization given by $\Gamma_0 = \langle \psi_0 | V_{dd} | \psi_1 \rangle = \frac{\sqrt{60}}{4} \sqrt{\sum_{i,j} \alpha_{i,j}^2}$

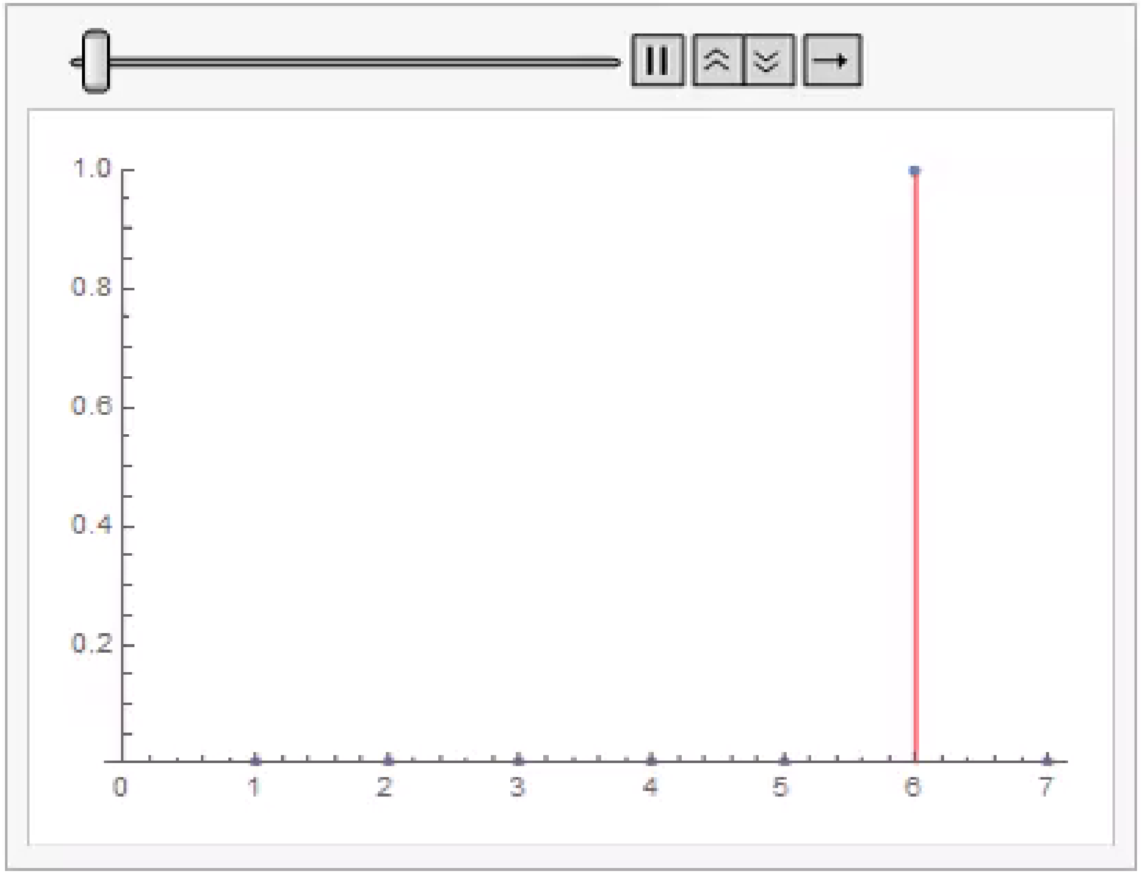
$$\Psi_0 = |2,2,2,2,\ldots,2\rangle$$

$$\Psi_1 = \sum_{(i,j)} \alpha_{i,j} |2,\underset{i}{1},2,2,\ldots 2,\underset{j}{3},2\rangle$$



Lanczos approach provides an intuitive picture for thermalization





Dynamics at constant magnetization can be seen as a random walk

→ the steady state maximizes entropy at a given magnetization

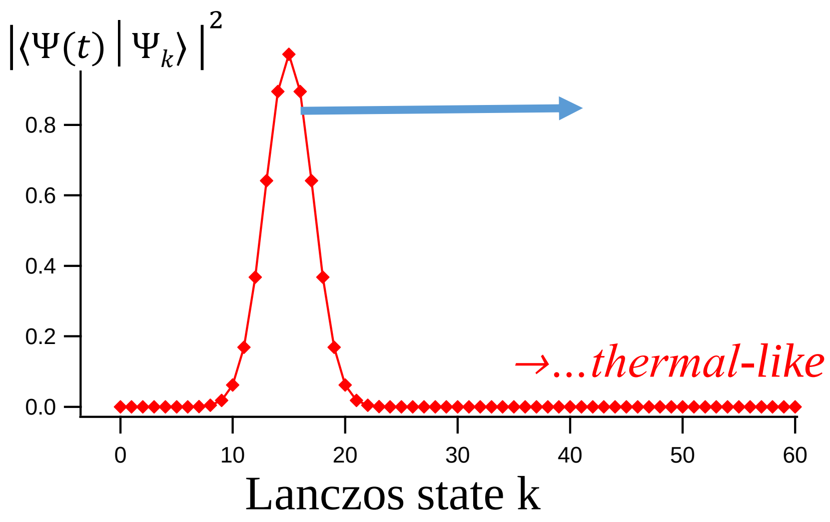
$\rightarrow p_{m_s} \approx e^{\alpha m_s}$

Lanczos approach and E.T.H

$$T = \sum_{i,j} \alpha_{i,j} \left[S_i^z S_j^z - \frac{1}{4}(S_i^+ S_j^- S_j^+ S_i^-) \right]$$

$$\Psi_0 = |2,2,2,2,\ldots,2\rangle \xrightarrow{\ \boldsymbol{T}\ } \Psi_1 = \sum_{(i,j)} |2,\underset{i}{1},2,2,\ldots 2,\underset{j}{3},2\rangle \xrightarrow{\ \boldsymbol{T}\ } (\ldots)$$

Dynamics is captured by the dynamics between Lanczos states (tri-diagonal matrix; 1D motion).



Those are produced by a **random walk** starting from the initial state (therefore maximize entropy)

The Lanczos basis can be used to decompose the many-body eigenstates Φ_j that are needed to describe Ψ_0

$$\Phi_j = \sum b_k^j \Psi_k \rightarrow \textit{provides a justification for ETH ?}$$

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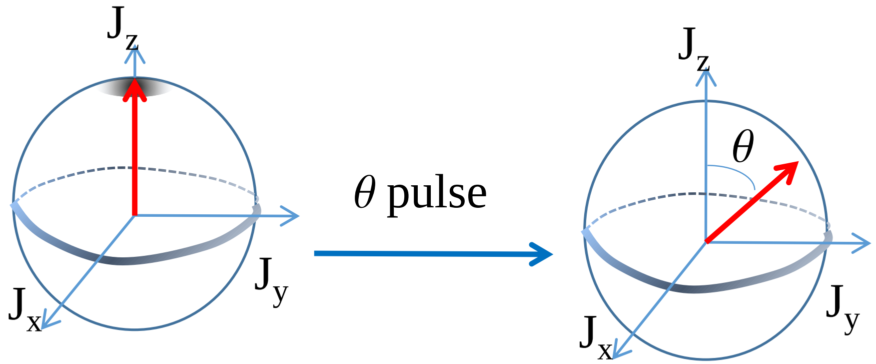
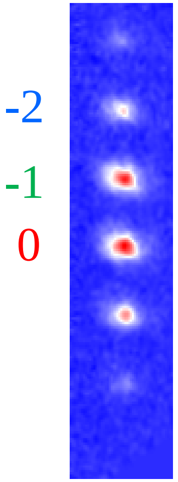
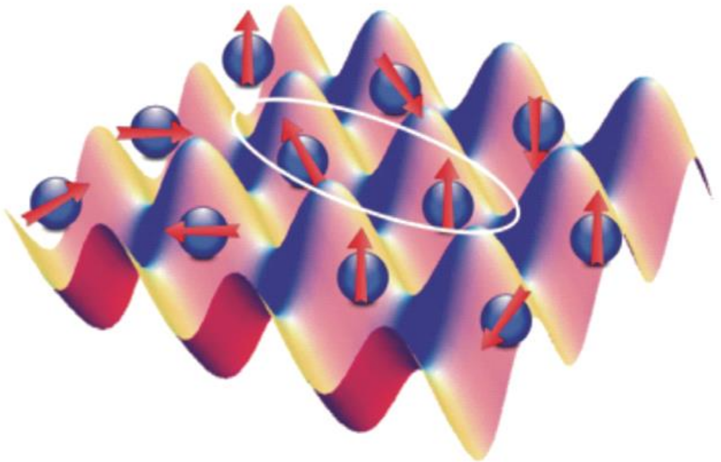
Quantum thermalization with 10^4 magnetic atoms



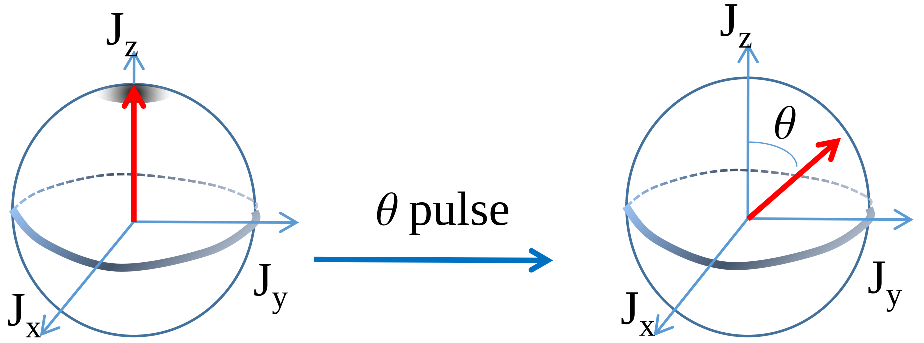


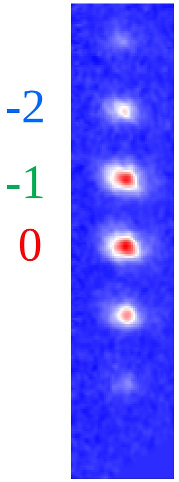
$$\Psi_0 = |3_\theta, 3_\theta, 3_\theta, 3_\theta, \ldots, 3_\theta\rangle$$

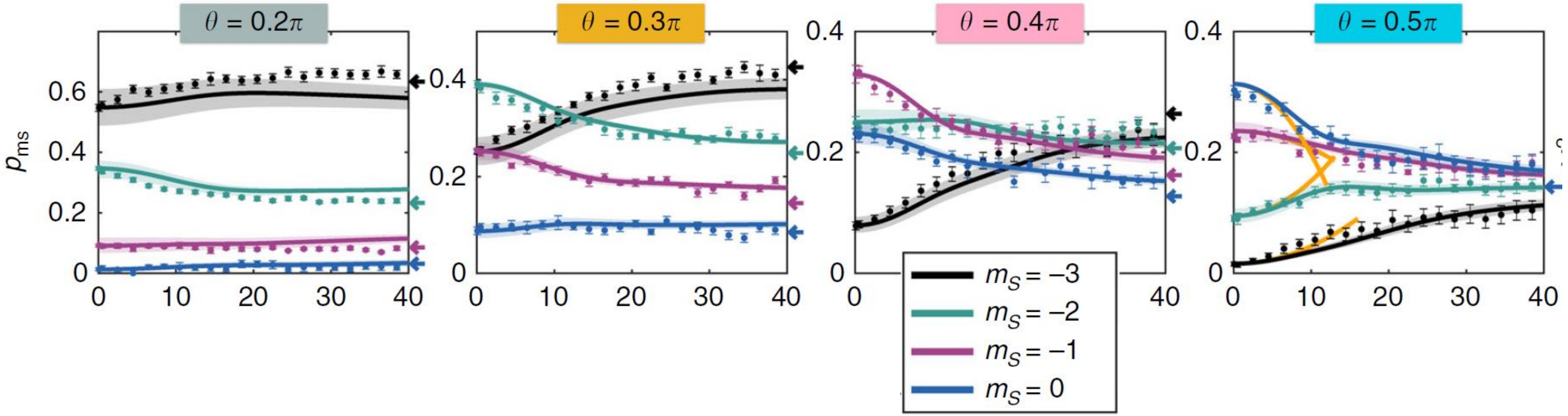


$$H = c_{dd}\sum_{(i,j)}\left[S_i^z.S_j^z - \frac{1}{4}(S_i^+.S_j^- + S_i^-.S_j^+)\right]\frac{(1-3\cos^2\theta_{ij})}{r_{ij}^3}$$

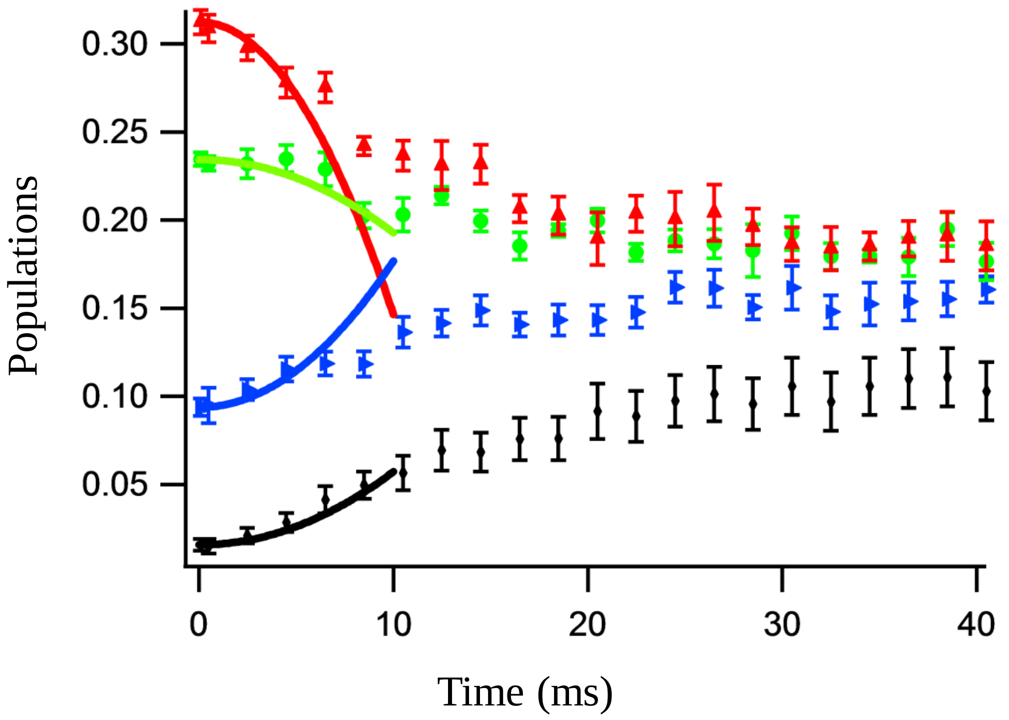
Experimental results

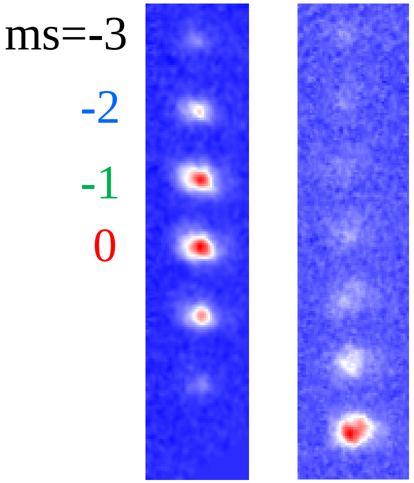






Short term dynamics of the many-body system





Perturbation theory

$$\langle A(t)\rangle = \langle A\rangle - it\langle [A,H]\rangle - \frac{t^2}{2}\langle [[A,H],H]\rangle + \dots$$

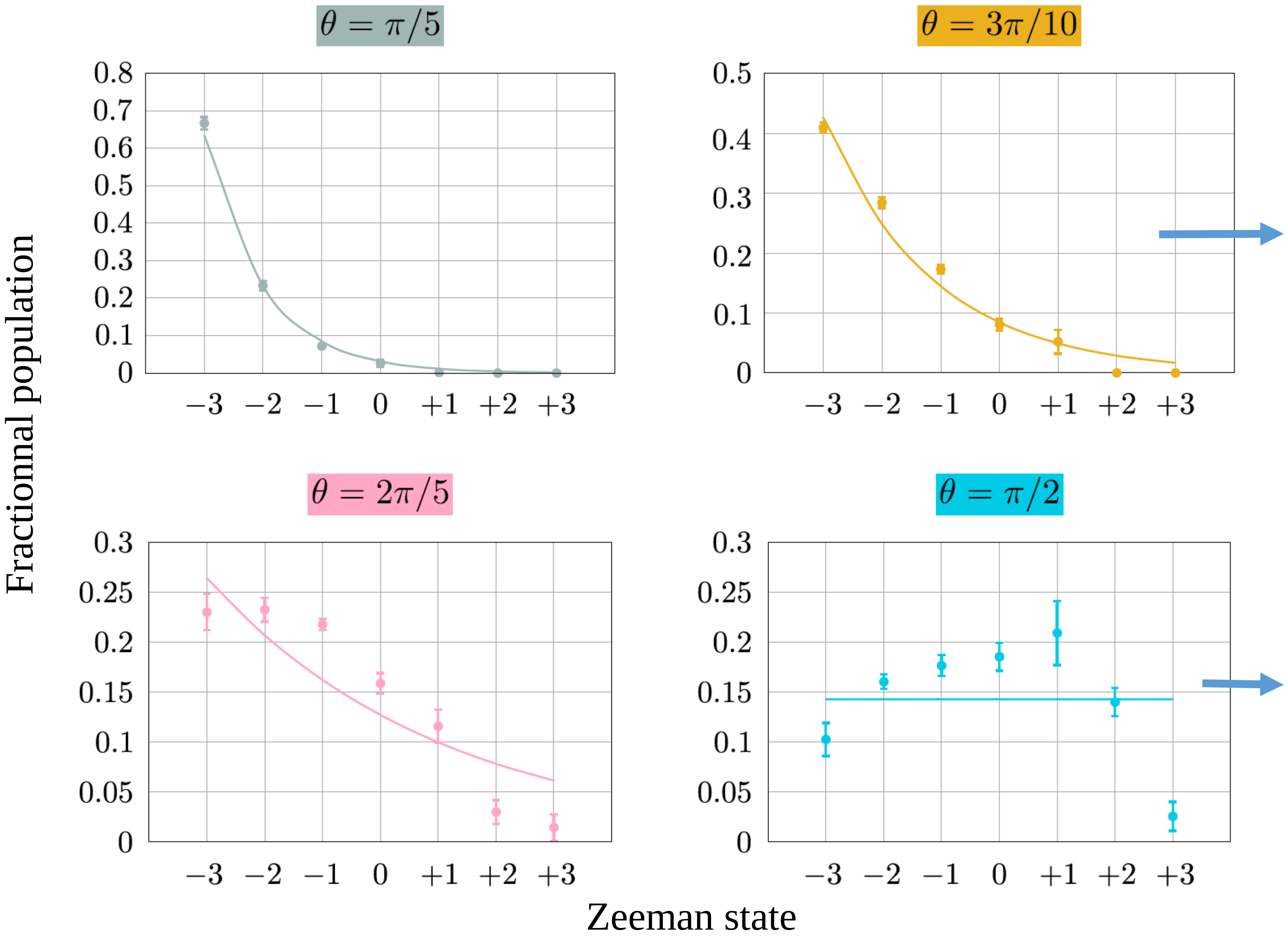
$$\Gamma = \sqrt{\sum_{(i,j)} \left(V_{(i,j)}{}^2\right)}$$

$$p_{m_s}(t) = p_{m_s}(0) + \alpha_m \sum_i V_{dd}^2(\vec{r}_i)t^2$$

$$\alpha_m = 135/512(1,2,-1,-4,-1,2,1)$$

$$\Gamma_0 = \langle \psi_0 | V_{dd} | \psi_1 \rangle = \frac{\sqrt{60}}{4}\sqrt{\sum_{i,j} \alpha_{i,j}^2}$$

Asymptotic behavior



Small angles:
« Thermal-like behavior »
(maximum of entropy)

Large angles:
Need to go beyond.

Take into account energy constraints.

Two contributions for energy

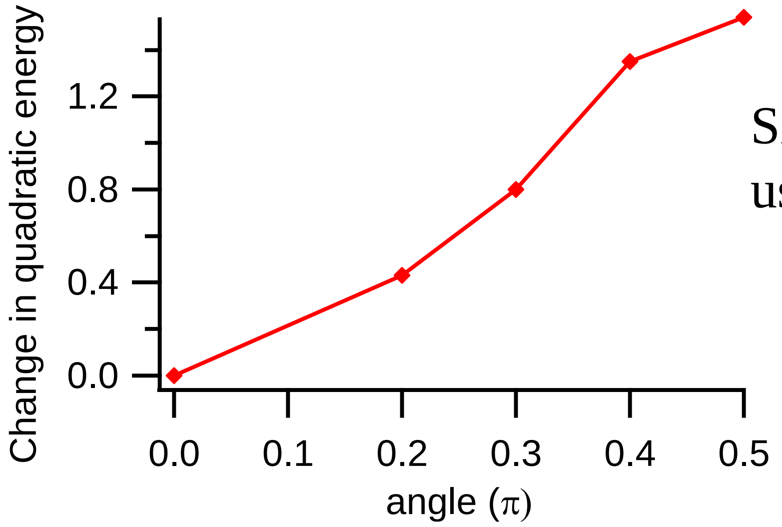
Dipole-dipole interactions

$$\langle \Psi(t) | V_{dd} | \Psi(t) \rangle$$

Difficult to calculate
except at t=0

Tensor light-shift leads to
an effective quadratic Zeeman effect

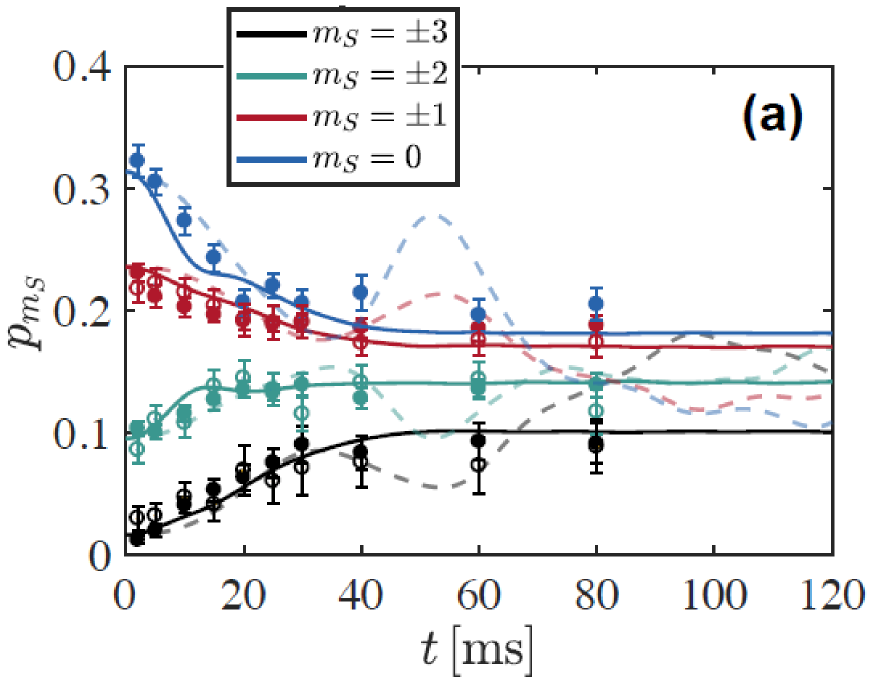
$$E(m_s) = B_Q m_s{}^2$$



Simple to evaluate
using experimental data

$$\sum m_s^2 p_{m_s}$$

→ This explains why simply maximizing entropy is sufficient at small angles



in agreement with GDTWA simulations (solid lines)

while mean field simulations show revivals at this time scale (dashed lines)

Analytic model for quantum thermalization

Look at the thermal state that corresponds to the initial energy

High-temperature expansion (A.M. Rey)

$$\hat{\rho} = \exp[-\beta \hat{H}] \approx Id - \beta \hat{H}$$

Analytically compute <H>
(includes $E(m_s)=B_Q m_s^2$ and dipolar interactions)

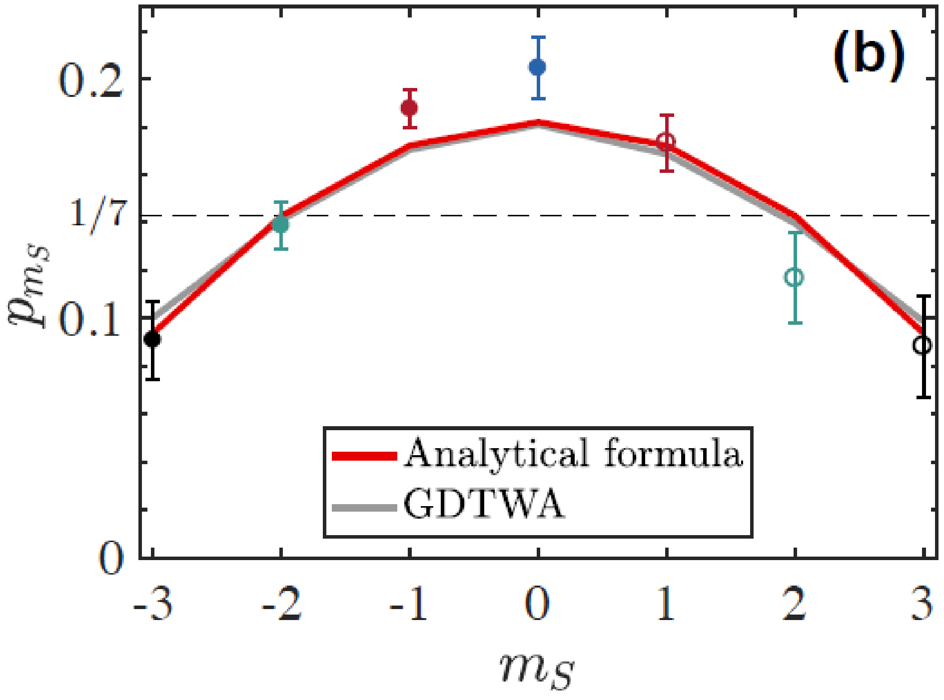
Use $\langle A \rangle = \dfrac{Tr[\rho.A]}{Tr[\rho]}$

$$\frac{1}{k_B T} = \frac{5B_Q + 9\overline{V}}{24B_Q^2 + 24V_{eff}^2}$$

$$P_{m_s} = \frac{1}{7}\left(1 + \beta B_Q (4 - m_s^{\,2})\right)$$

where: $V_{eff}^2 = \sum V_{(i,j)}^2$ $\qquad \overline{V} = \sum V_{(i,j)}$

Results



Consistent with the eigenstate thermalization hypothesis
(thermal-like fluctuations tied to the growth of entanglement)

$$\frac{1}{k_B T} = \frac{5B_Q + 9\overline{V}}{24B_Q^2 + 24V_{eff}^2}$$

where:

$$V_{eff}^2 = \sum V_{(i,j)}^2 \qquad \overline{V} = \sum V_{(i,j)}$$

$$P_{ms} = \frac{1}{7}\left(1 + \beta B_Q(4 - m_s{}^2)\right) \approx exp[-\beta B_Q m_s^2]$$

B_Q quadratic effect

Note that the final probability distribution is that of non-interacting particles!....
An effective temperature (a few nK) for an isolated system

NB: the agreement between experiment and theory shows how little heating there is.

Outline:

1- Theoretical introduction to thermalization

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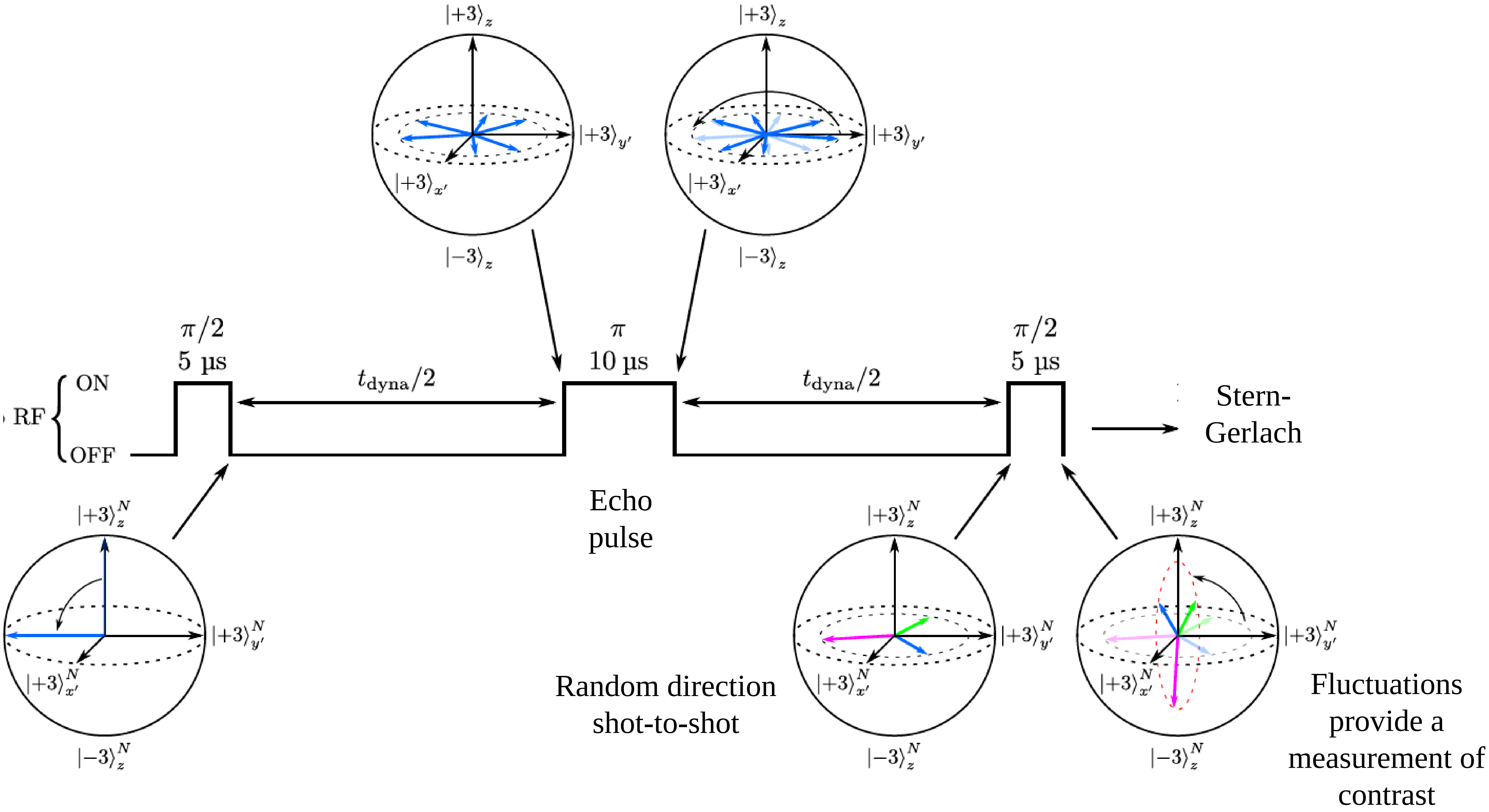
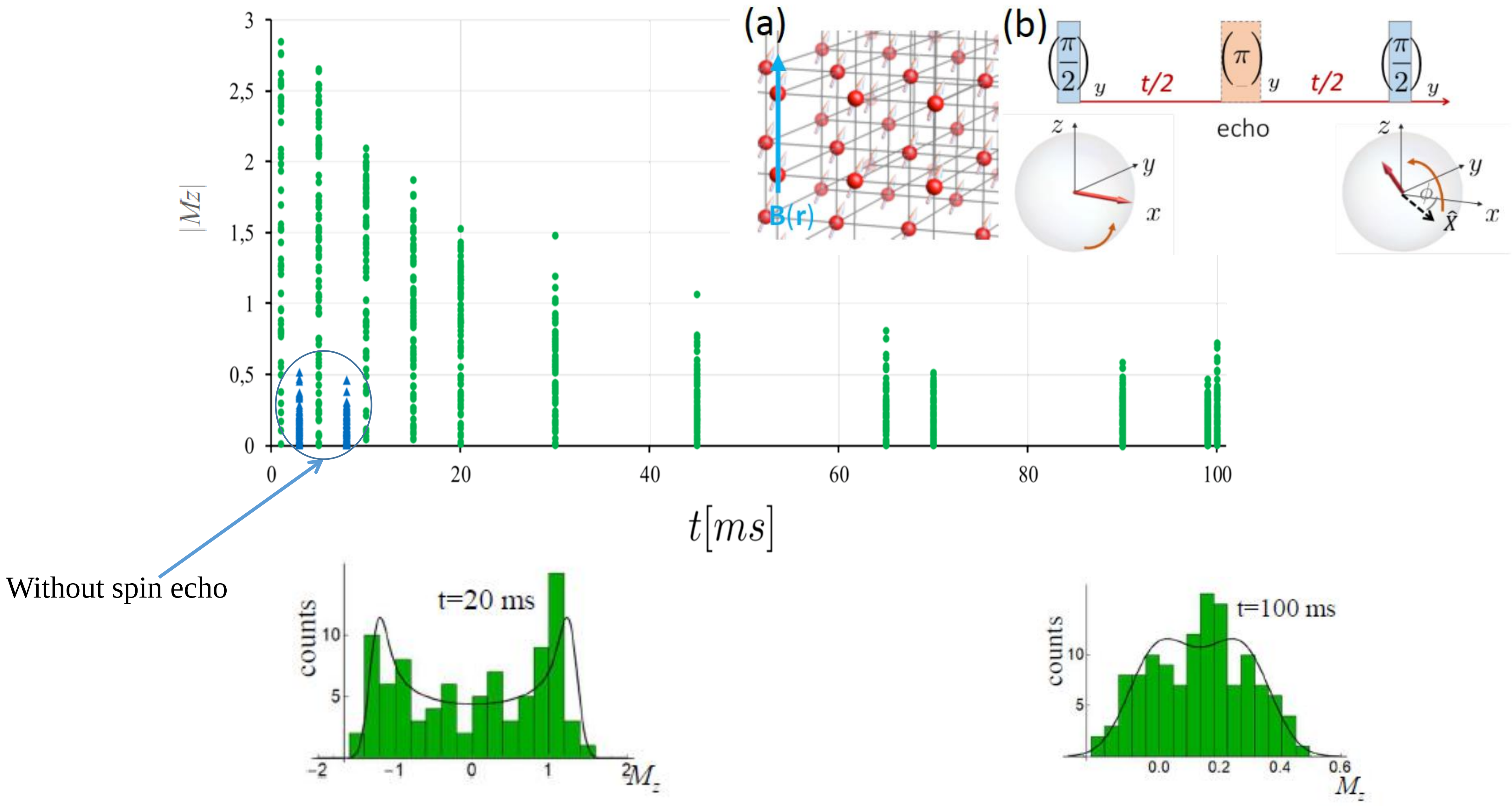
Measuring the collective spin through Ramsey interferometry

$$\ell = \sqrt{\langle S_x \rangle^2 + \langle S_y \rangle^2 + \langle S_z \rangle^2}$$



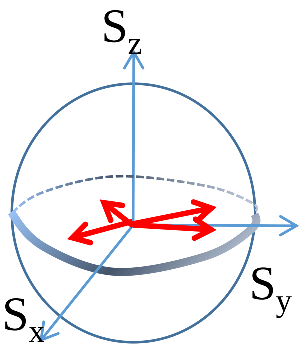
Spin-length data



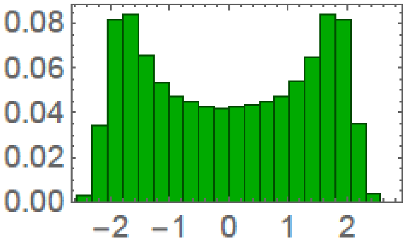
Data analysis: measure collective spins from probability distributions

$$\ell = \sqrt{\langle S_x \rangle^2 + \langle S_y \rangle^2 + \langle S_z \rangle^2}$$

Assume Classical Spin:

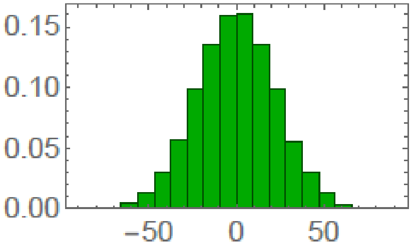


$$\frac{dN}{dM_z} = \frac{1}{\pi \ell} \frac{1}{\sqrt{1 - \frac{M_z^2}{\ell^2}}}$$

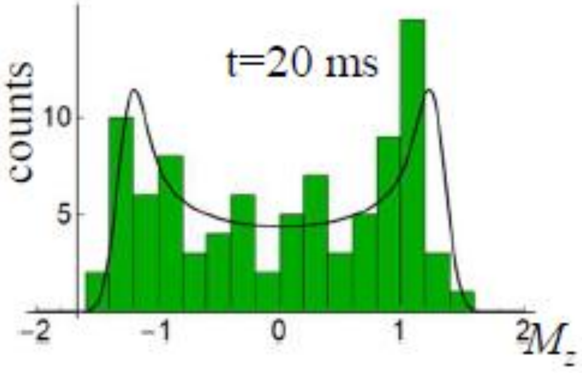


Assume ℓ=0, and Gaussian noise:

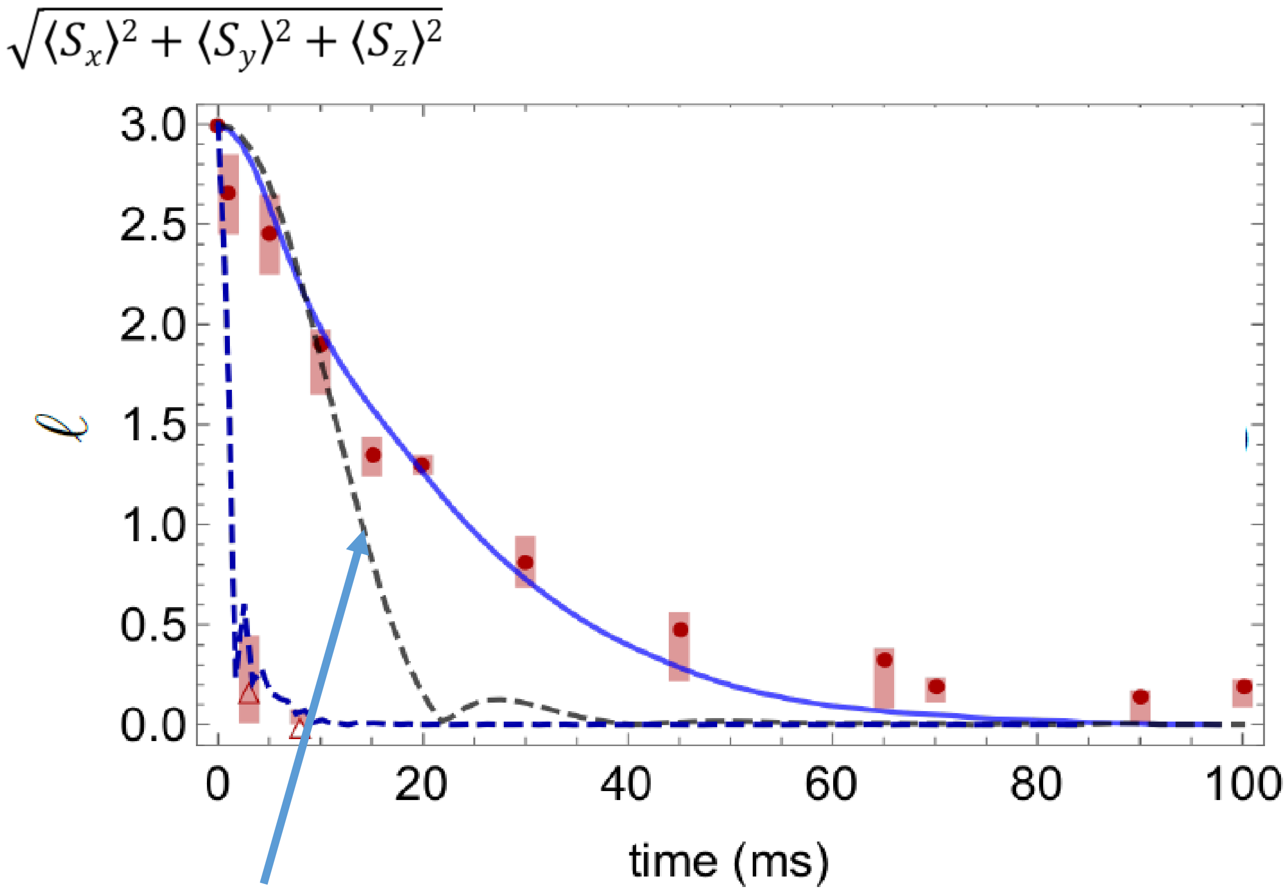
$$\frac{dN}{dM_z} = \frac{1}{\sqrt{\pi}\sigma} \exp\left(-\frac{M_z^2}{\sigma^2}\right)$$



Method to derive ℓ:
fit probability distributions
with a convolution of the
two distributions



Damping of the collective spin due to dipolar interactions



GDTWA

$$|S_\perp|(t) \underset{t\to 0}{\approx} |S_\perp|\left(1 - t^2\left[\Delta B^2 + \frac{1}{N}\sum V_{i,j}^2\right]\right)$$

Classical inhomogeneous precession (↔variance of mean-field)

Beyond mean-field

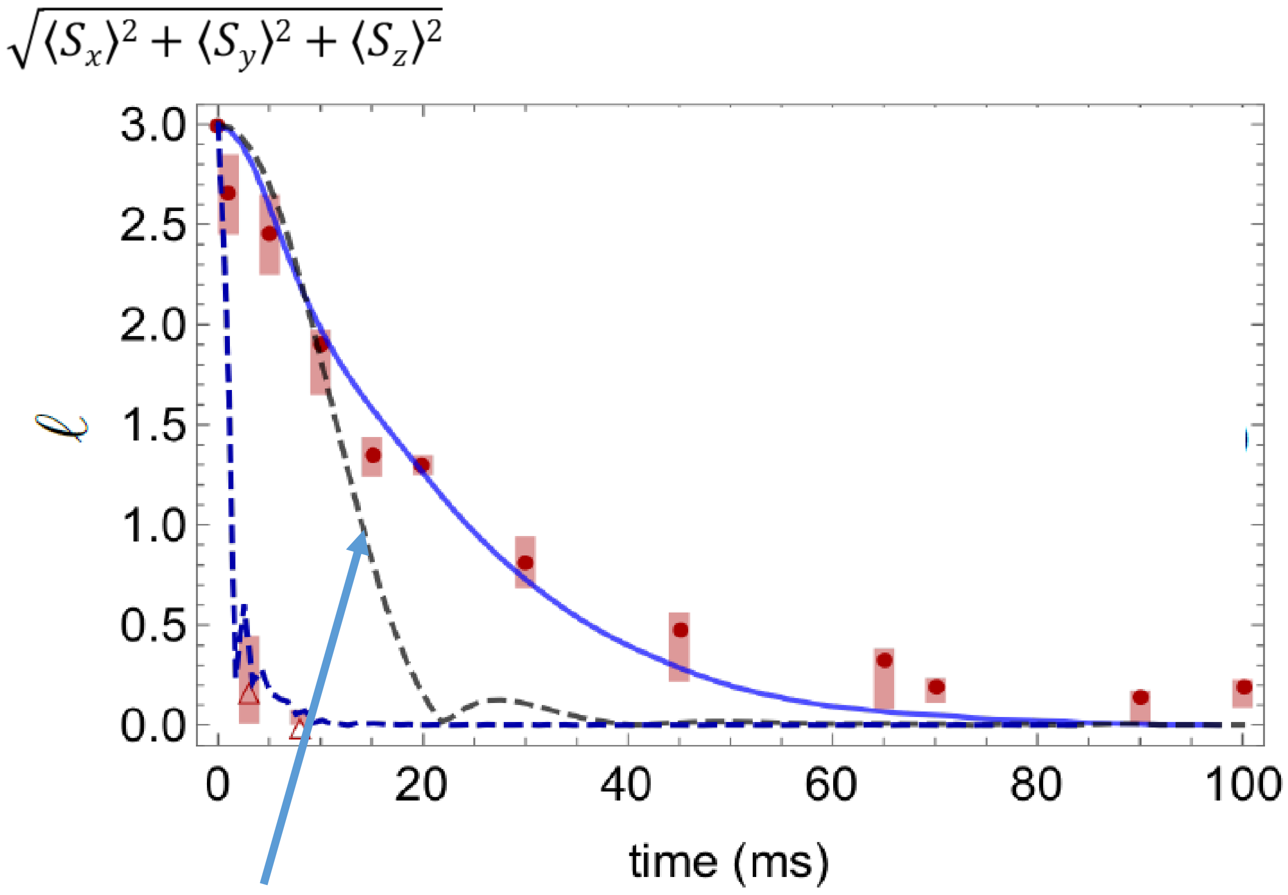
Hazzard et al., PRL **110**, 075301 (2013)

Note that the damping of the spin is a purely dipolar beyond mean-field effect for a homogeneous system, associated with the growth of entanglement

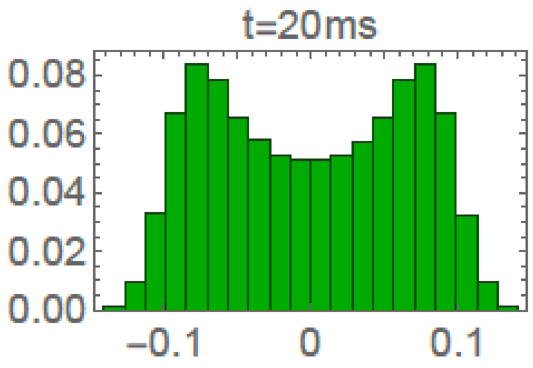
Good agreement at short times

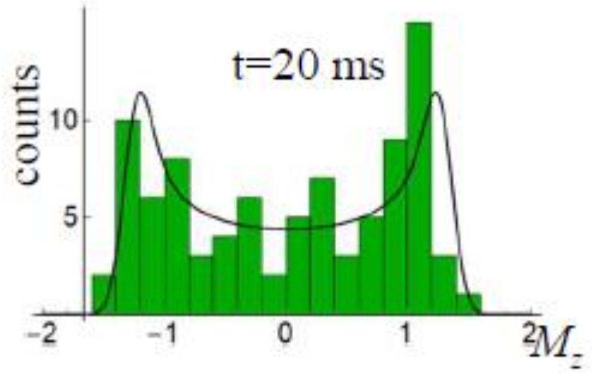
Good agreement with second-order perturbation theory too

Damping of the collective spin due to dipolar interactions



GDTWA





Good agreement at short times
Good agreement with second-order perturbation theory too

At long times, the collective spin decays **SLOWER** than it should !!

Partial conclusions on the collective spin measurements

Strong decay of collective spin, associated with dipole-dipole interactions

The decays is « too » slow.

→ heating in the lattice ?
→ Are there more holes than we thought ?
→ effect of losses ?
→ more subtle effect associated with possibly disorder ?

(see glassy dynamics observed with Rydberg atoms arXiv:1909.11959)

Technical noise leads to* slower *decay of the contrast !

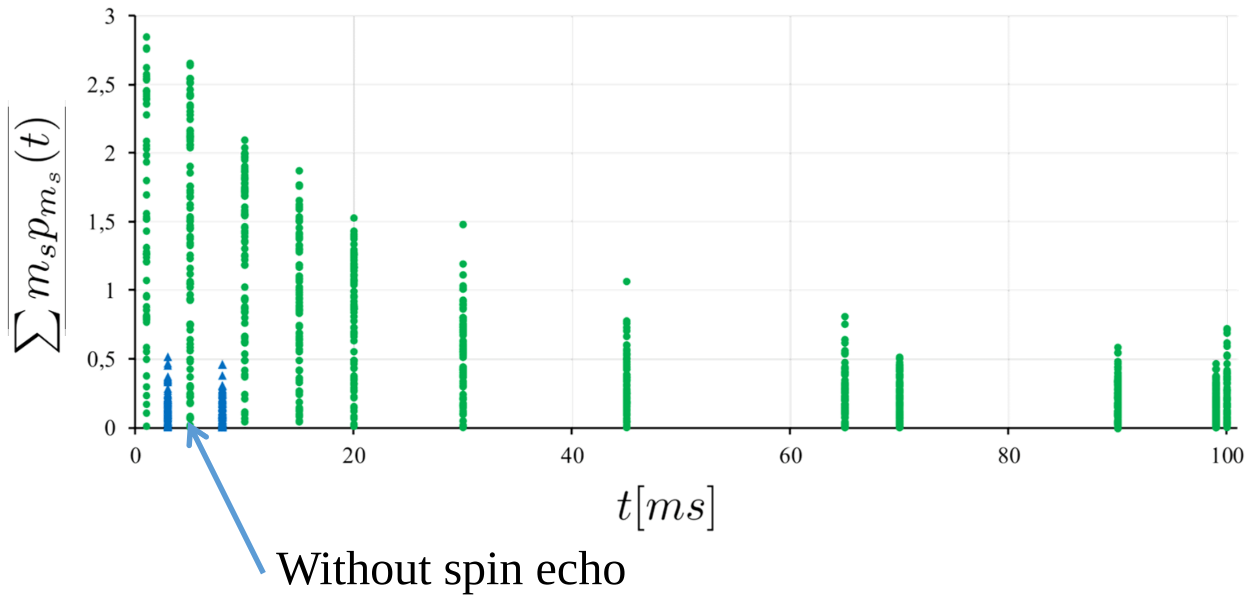
The measurement of coherences (the contrast of the interferometer) gives access to information we could not reach by simply measuring populations.

NB: $\sqrt{\langle S_x\rangle^2 + \langle S_y\rangle^2 + \langle S_z\rangle^2}\Big|_{t\to\infty} \approx 0$ can be related to $P_{ms} = \frac{1}{7}\left(1 + \beta B_Q(4 - m_s{}^2)\right) \approx exp[-\beta B_Q m_s^2]$

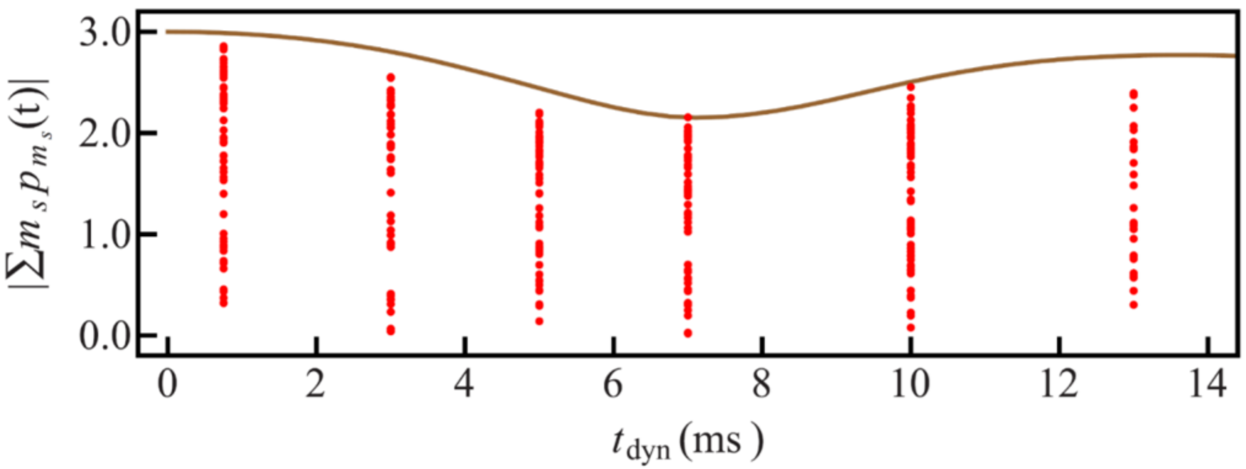
At equilibrium, the strongly interacting many-body system looks like a non-interacting one !

Spin-length data with and without lattice

Collective spin length $\sqrt{\langle S_x \rangle^2 + \langle S_y \rangle^2 + \langle S_z \rangle^2}$ from Ramsey interferometer



Lattice case :
decrease of spin length
due to dipolar interactions



Bulk case :
Spins remain almost locked despite magnetic field gradient

preservation of ferromagnetism

With and without lattice: the main difference (Lanczos approach)

$$H_{dd} = -\frac{1}{2}S'_x.S'_x - \frac{3}{8}(S'^+S'^+ + S'^-S'^-) + \frac{1}{8}[\,(S'^+S'^- + S'^-S'^+)]$$

Lattice case :

$$\Psi_0 = |3_x, 3_x, 3_x, 3_x, \dots, 3_x\rangle \xrightarrow{\;T\;} \Psi_1 = \sum_{(i,j)} |3_x, \underset{\mathrm{i}}{2_x}, 3_x, 3_x, \dots 3_x, \underset{\mathrm{j}}{2_x}, 3_x\rangle \xrightarrow{\;T\;} (\dots)$$

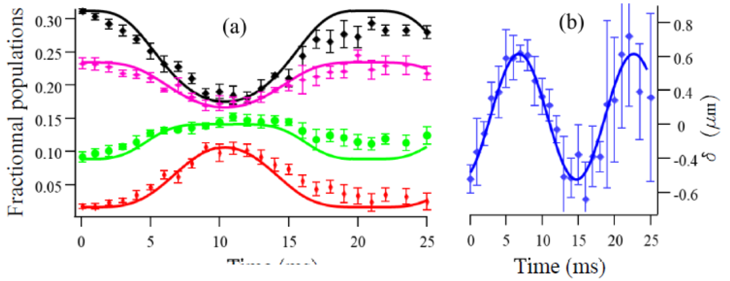
BEC case :

$$\Psi_0 = |3_x, 3_x, 3_x, 3_x, \dots, 3_x\rangle \xrightarrow{\;T\;} \Psi_1 = |2_x, 2_x, 3_x, 3_x, \dots 3_x, 3_x, 3_x\rangle \xrightarrow{\;T\;} (\dots)$$

Spin gap $\propto \dfrac{(a_6 - a_4)\hbar^2 n}{m}$

In the BEC case, protection of ferromagnetism after the quench due to a spin gap

Quench results in the excitation of trapped of magnon mode
(and the retardation of thermalization)

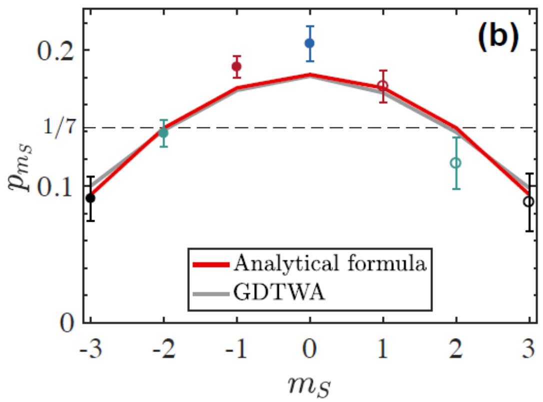


PRL 121, 013201 (2018)

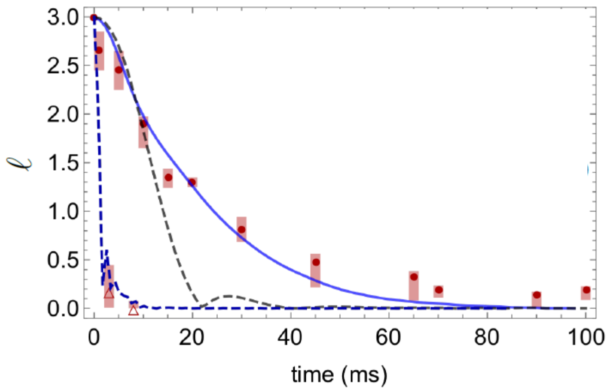
Conclusions

In a lattice

Many-body dynamics
Quantum Thermalization



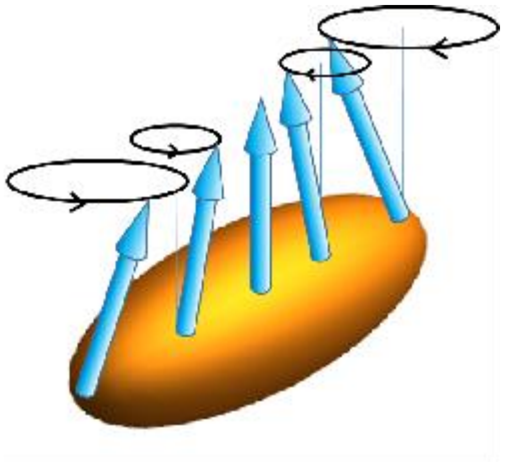
Nature Comm. **10**, 1714 (2019)



arXiv:2005.13487 (2020)

In the BEC phase

BEC is trapped near an energy maximum.
It behaves like a ferrofluid
Collective modes are observed



Phys. Rev. Lett. 121, 013201 (2018)

The routes and timescales to quantum thermalization can be vastly different.
The Lanczos approach and perturbation theory may help building an intuition.

S. Lepoutre, L. Gabardos (PhD)
Youssef Aziz Alaoui
B. Laburthe-Tolra, O. Gorceix, E. Maréchal, L. Vernac,
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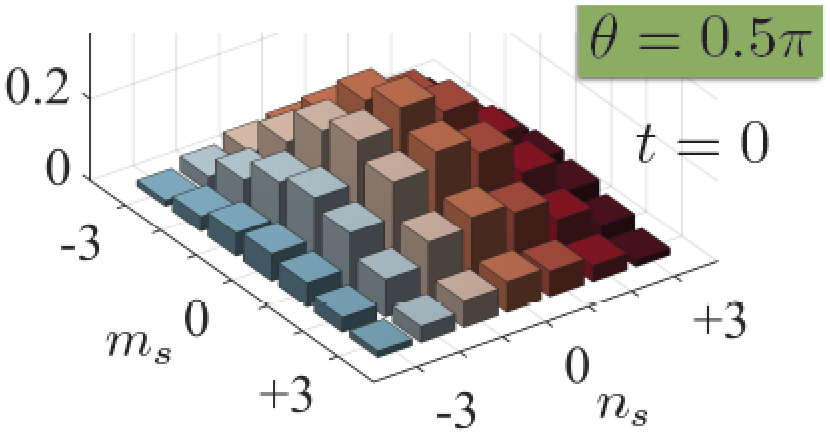
A. M. Rey, J. Schachenmayer, B. Zhu,
B. Blakie, Petra Fersterer, Arghavan Safavi-Naini,
T. Roscilde

!! Thank you !!

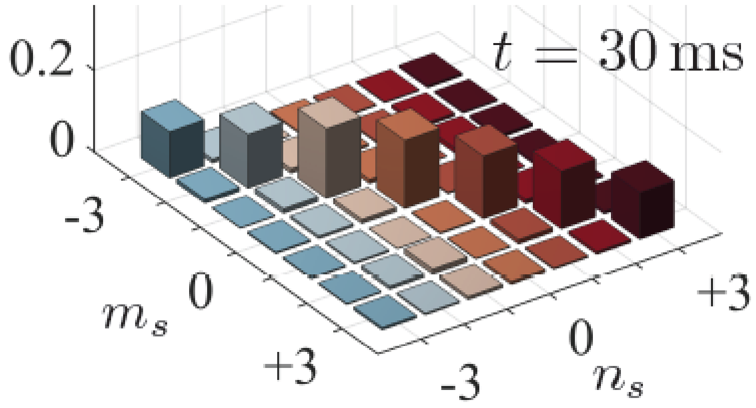
!! Open post-doc position !!

Outlook : Quantum thermalization, entanglement

Reduced density matrix (isolate one spin and trace over the rest of the system)



« coherent »



« mixed »

!! However still a pure state !!

Future: measure bi-partite fluctuations (double-well lattice)
entanglement witness ? (T. Roscilde)