

Non-equilibrium physics of many interacting large spins



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**I am against any nationalism, even in the
guise of mere patriotism. - Albert Einstein**

Atoms are composite objects, whose spin can be larger than 1/2

$$\vec{F} = \vec{S} + \vec{I}$$

Large spin magnetism

Spinor condensates

Stamper-Kurn, Lett, Klempt,
Chapman, Sengstock, Shin, Gerbier

e.g. Na, Rb

$F=0,2$

Alkali: spin arises both from
nuclear and electronic spins

Spin-dependent interactions

SU(N) magnetism

Bloch, Fallani, Ye, Takahashi,...

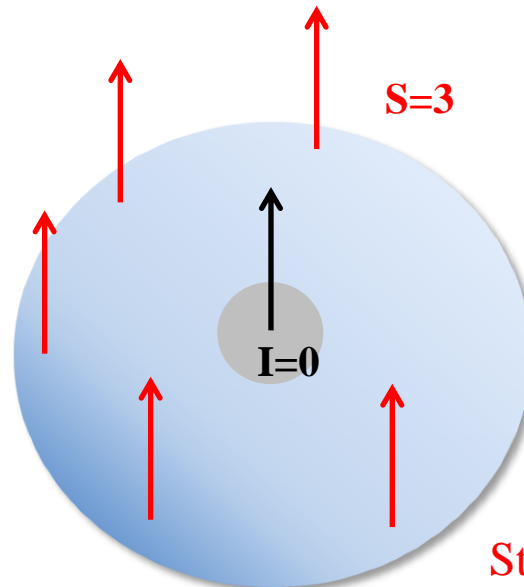
e.g. Sr, Yb

Alkaline-earth: spin is purely
nuclear

Spin-independent interactions

« magnetic atoms »: spin is
purely electronic

e.g. Cr, Er, Dy



**Strong dipole-dipole
interactions**

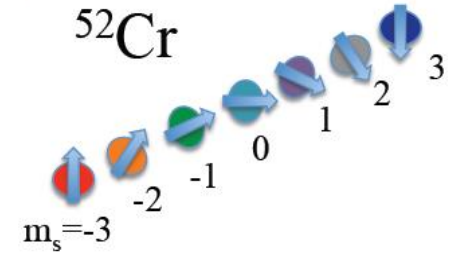
Stuttgart, Paris, Innsbruck, Stanford

This seminar: magnetism with large spin cold atoms (Chromium atoms)

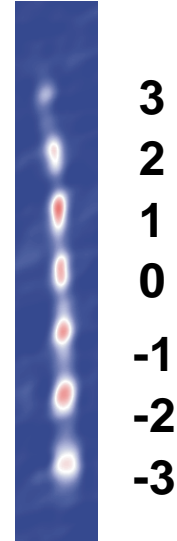
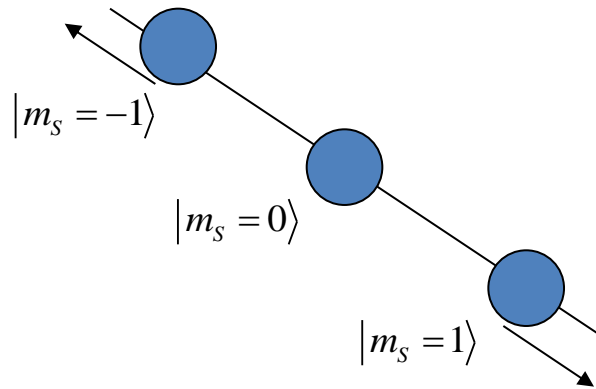
Optical dipole traps equally trap all Zeeman state of a same atom

Linear (+ Quadratic)
Zeeman effect

$$E(m_S) = m_S g \mu_B B (+\alpha B^2)$$

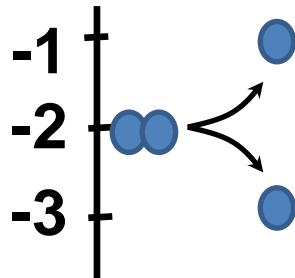


Stern-Gerlach separation:
(magnetic field gradient)

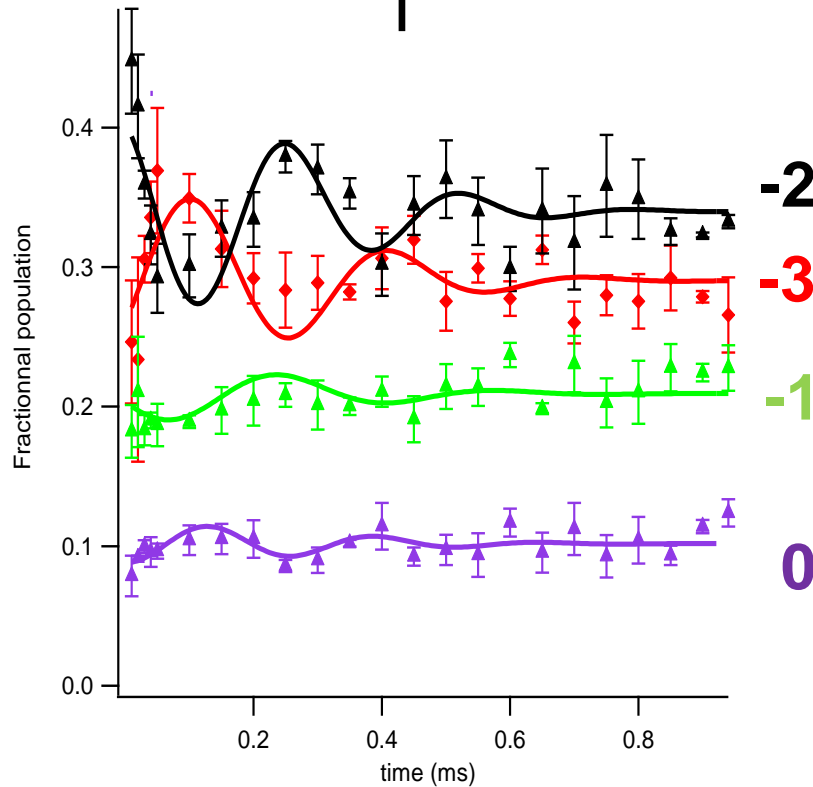


Two main players for magnetism and spin dynamics (1):

Spin-dependent contact interaction:
scattering length depends on molecular channel



Contact exchange



$$\Gamma = \frac{4\pi\hbar^2}{m} n(a_6 - a_4)$$

Second main player:

Dipole-dipole interactions

$$V_{dd} = \frac{\mu_0}{4\pi} S^2 (g_J \mu_B)^2 (1 - 3 \cos^2(\theta)) \frac{1}{R^3}$$

Heisenberg model of magnetism

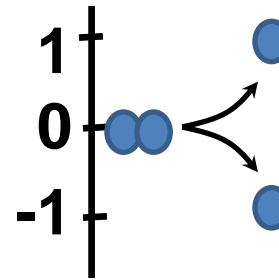
$$\Gamma \propto \frac{t^2}{U} S_{1z} S_{2z} + \frac{1}{2} (S_{1+} S_{2-} + S_{1-} S_{2+})$$

Ising

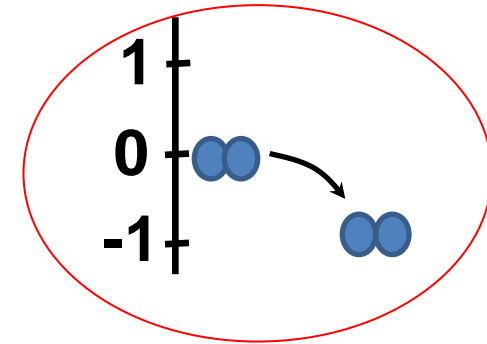
Exchange

Super-Exchange

Dipolar Exchange

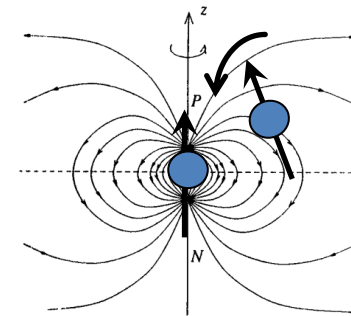


Relaxation



Nuclear Magnetic Resonance

$$S_{1z} S_{2z} - \frac{1}{4} (S_{1+} S_{2-} + S_{1-} S_{2+})$$



!!! Anisotropy !!! Long Range !!! Large Spin !!!

Second main player:

Dipole-dipole interactions

$$V_{dd} = \frac{\mu_0}{4\pi} S^2 (g_J \mu_B)^2 (1 - 3 \cos^2(\theta)) \frac{1}{R^3}$$

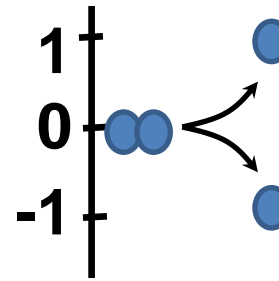
Heisenberg model of magnetism

$$\Gamma \propto \frac{t^2}{U} \left[S_{1z} S_{2z} + \frac{1}{2} (S_{1+} S_{2-} + S_{1-} S_{2+}) \right]$$

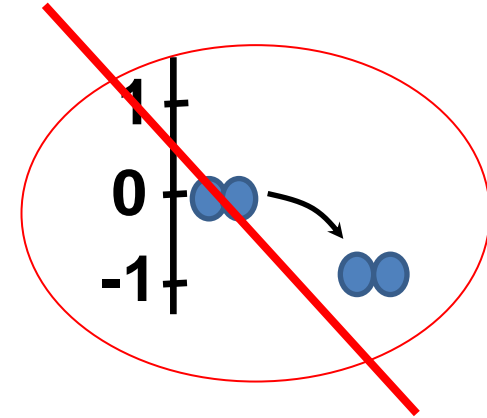
Ising

Exchange

Dipolar Exchange



Relaxation



Nuclear Magnetic Resonance

$$S_{1z} S_{2z} - \frac{1}{4} (S_{1+} S_{2-} + S_{1-} S_{2+})$$

Ising

Exchange

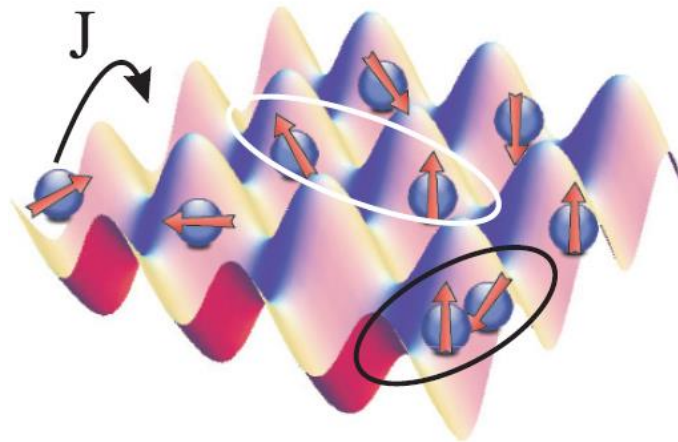
!!! Non-Heisenberg !!! Anisotropy !!! Long Range !!! Large Spin !!!

This Experiment

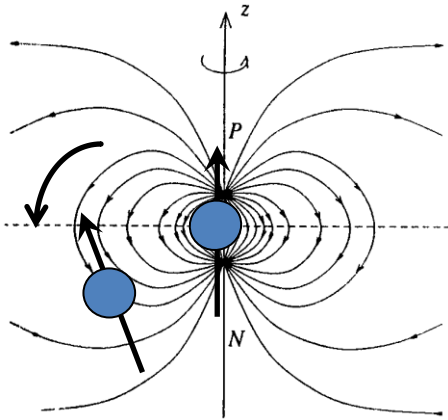
I – Excite the spins

II – Free evolution under the effect of interactions

**Questions: Under which conditions is there spin dynamics ?
Do quantum correlations develop?**



Under which conditions correlations develop? – Classical vs Quantum magnetism

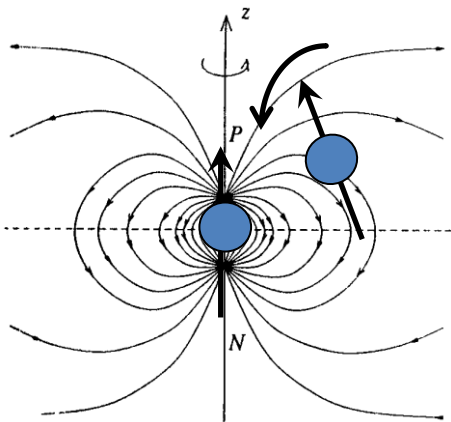
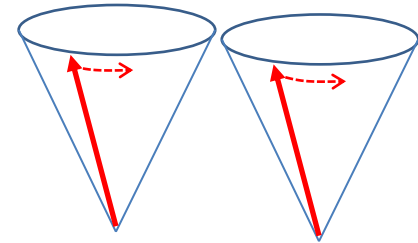


$$\vec{B}_A(B) = \vec{B}_B(A)$$

Classically:

These two atoms undergo identical precession

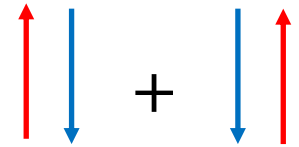
Total spin conserved



Quantum-mechanically:

Possibility for entanglement

Total spin is NOT conserved

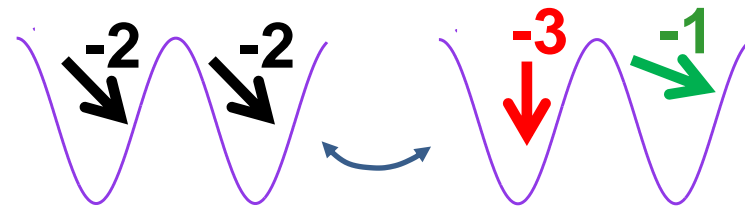
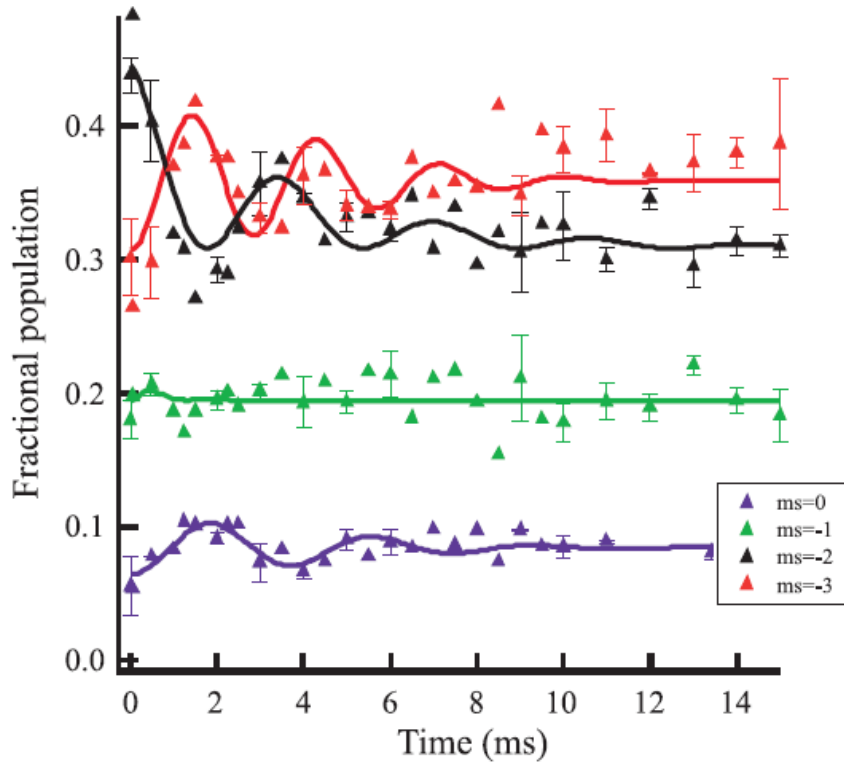


From classical to quantum: dipolar interactions may create correlations

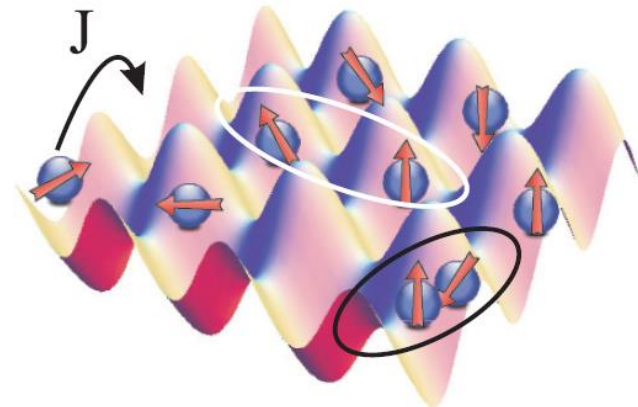
Observation of intersite spin-exchange due to dipolar interactions

$$\Psi(0) = |2,2,\dots,2,2\rangle$$

Start with one atom in each site of a 3D optical lattice in one Zeeman state $m_s=2$

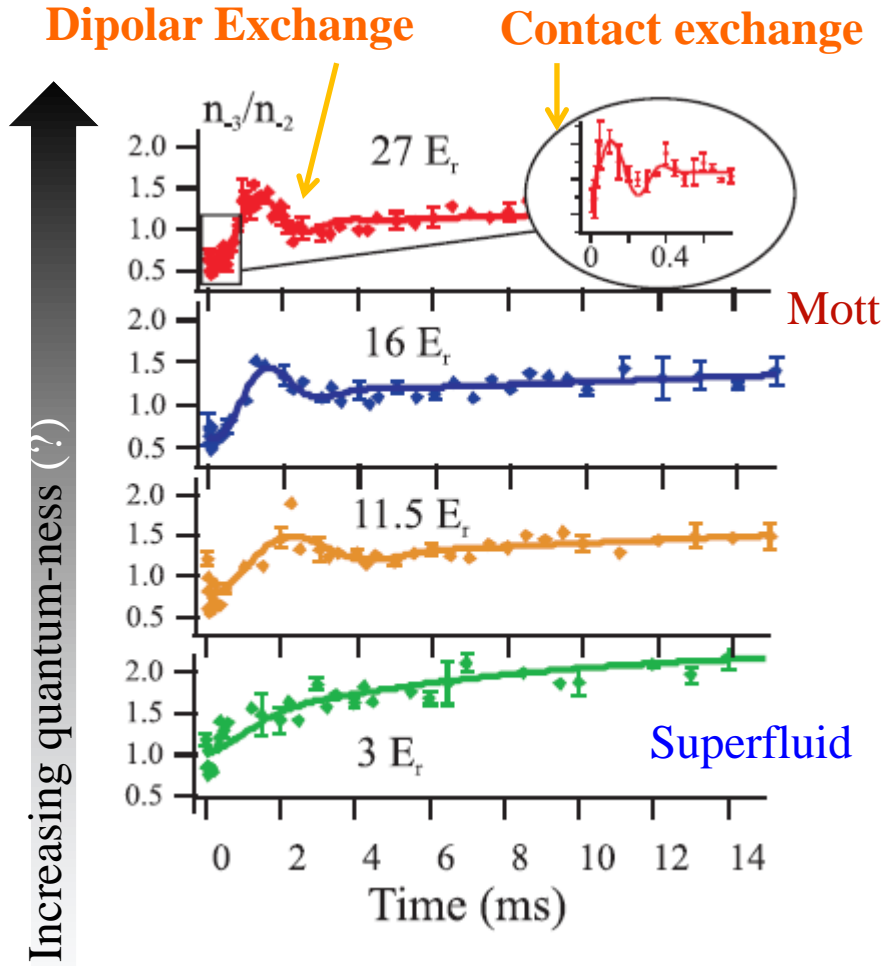


Dynamics inherently many-body
Mean-field theories fails



Exotic quantum magnetism of large spin, from Mott to superfluid

An exotic magnetism driven by the competition between three types of exchange



PRA 93, 021603(R) (2016)

Super- Exchange (I)

(nearest neighbor)

$$\Gamma \propto \frac{t^2}{U}$$

Dipolar exchange (II)

(true long range)

$$V_{dd}$$

Contact exchange (III)

(short range)

$$\frac{4\pi\hbar^2}{m} n(a_6 - a_4)$$

Mean-field theories fail; exact diagonalization techniques unrealistic...

But perturbation theory helps (a little)

$$\Psi_0 = |2,2,\dots,2\rangle \rightarrow \Psi_1 \propto \sum_{(i,j)} V_{(i,j)} |2,2,\dots,2,1,2,\dots,2,3,2,2\rangle$$

(i) (j)

$$\Gamma = \sqrt{\sum_{(i,j)} (V_{(i,j)})^2} \quad \left[= \langle \Psi_0 | \sum V_{(i,j)} | \Psi_1 \rangle \right]$$
$$\left[\neq \int dr' V_{dd}(r-r') n(r') \right]$$

Good agreement with data, see: PRA 93, 021603(R) (2016)

Example 2 : tilt atom with an rf pulse

$$|S_{\perp}|(t) \underset{t \rightarrow 0}{\approx} |S_{\perp}| \left(1 - t^2 \left[\Delta B^2 + \frac{1}{N} \sum V_{i,j}^2 \right] \right)$$

PRL 110, 075301 (2013)

Classical inhomogeneous precession
(\leftrightarrow variance of mean-field)

Beyond mean-field

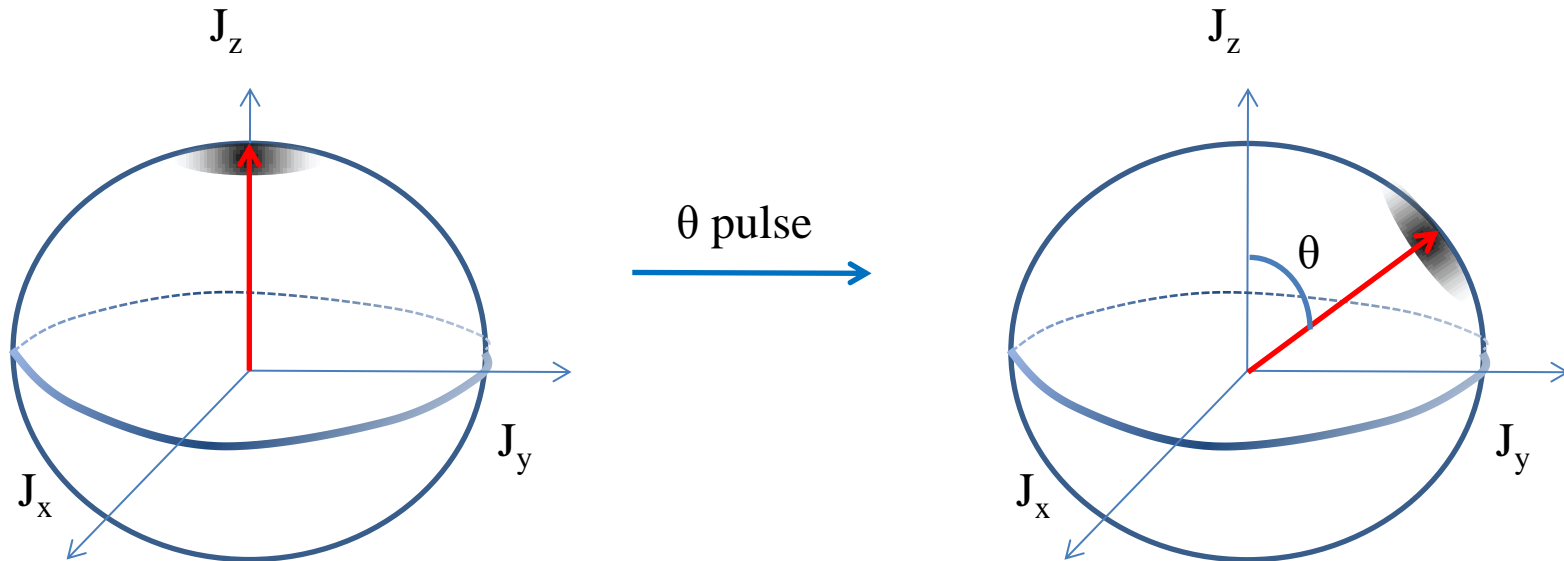
Outline

Spin dynamics after tilting spins
(rf pulse)

I BEC case

II Lattice case

Dynamics after tilting the spins



$$S_{1z} S_{2z} - \frac{1}{4} (S_{1+} S_{2-} + S_{1-} S_{2+})$$

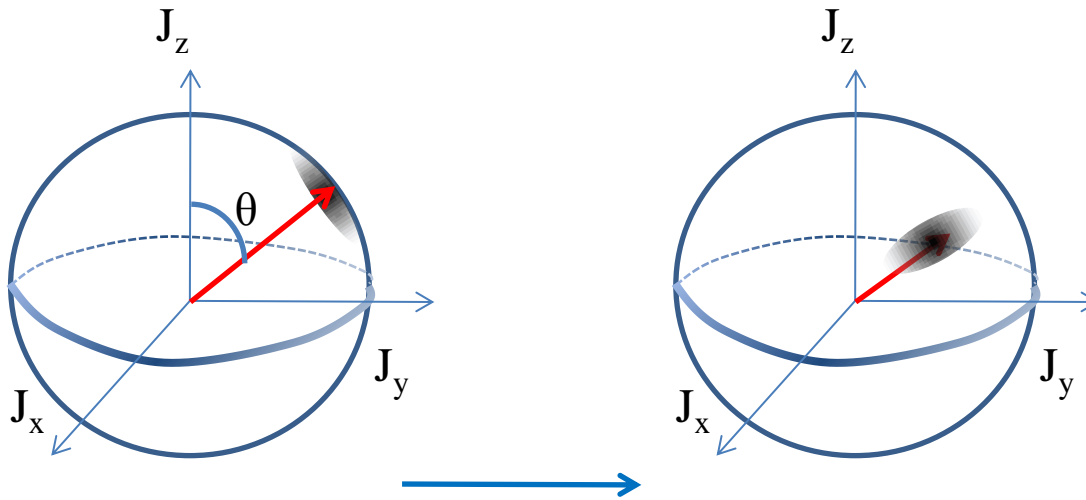
Prediction (Ana Maria Rey):

θ small \rightarrow classical precession
 θ large \rightarrow entanglement grows

See also E. Witkowska,
PRA 93, 023627 (2016)

Note !! Spin-dependent contact interactions trigger NO spin dynamics !!
The initially stretched state remains stretched after any rf pulse

After tilting the spin: from classical to quantum



$$S_{1z}S_{2z} - \frac{1}{4}(S_{1+}S_{2-} + S_{1-}S_{2+})$$

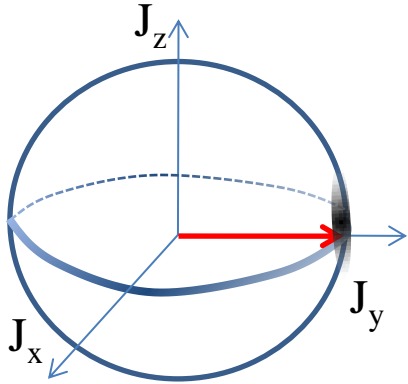
Interpretation: dynamics comes from the difference to the Heisenberg Hamiltonian

$$-\frac{1}{2} \left[S_{1z}S_{2z} + \frac{1}{2}(S_{1+}S_{2-} + S_{1-}S_{2+}) \right] = -\frac{1}{2} \vec{S}_1 \cdot \vec{S}_2$$

$$\delta H \propto S_{1z}S_{2z} \underset{t \rightarrow 0}{\approx} S_z^2$$

Squeezing \leftrightarrow Variance (S_z)

The specific case of $\pi/2$ pulse: mean-field dynamics vanishes



$$\vec{B}_{dd}(\vec{r}) = \gamma \int d\vec{r}' \frac{1 - 3r_z'^2}{|\vec{r} - \vec{r}'|^3} \left(3S_z(\vec{r}') \vec{z} - \vec{S}(\vec{r}') \right)$$

$$\Gamma \propto \vec{B} \times \vec{S}$$

At $\pi/2$ $S_z=0$

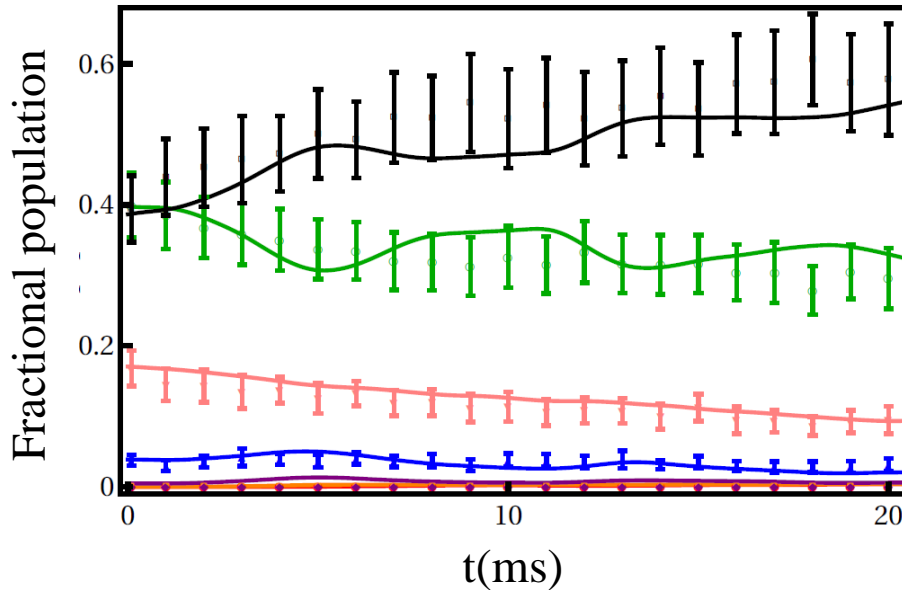


Assuming a BEC initially polarized in $m_s=3$, mean-field theory predicts **no spin dynamics!**

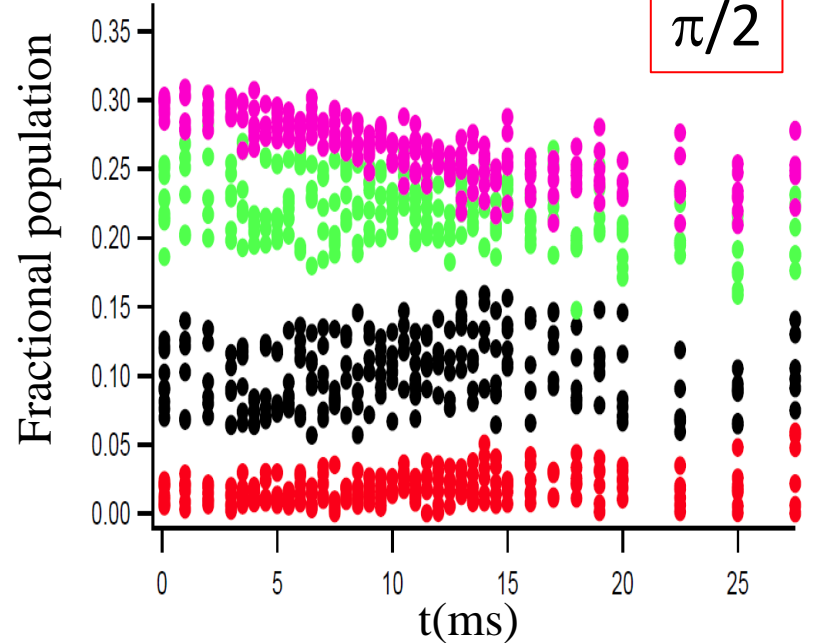
In principle, any dynamics seen is a beyond-mean-field effect (or is it ?)

Experimental results, BEC case...

$\pi/4$



$\pi/2$



**Dynamics
entirely
triggered by
dipolar
interactions!**

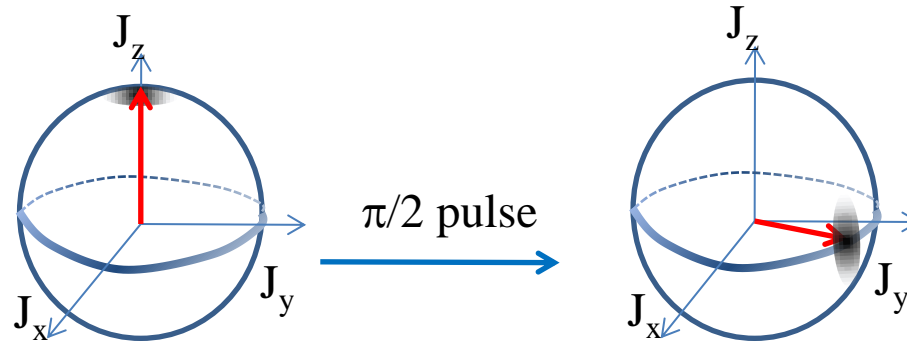
Theory
Pedri/Kechadi
Zhu/Rey

Dynamics vanishes for large angles close to $\pi/2$

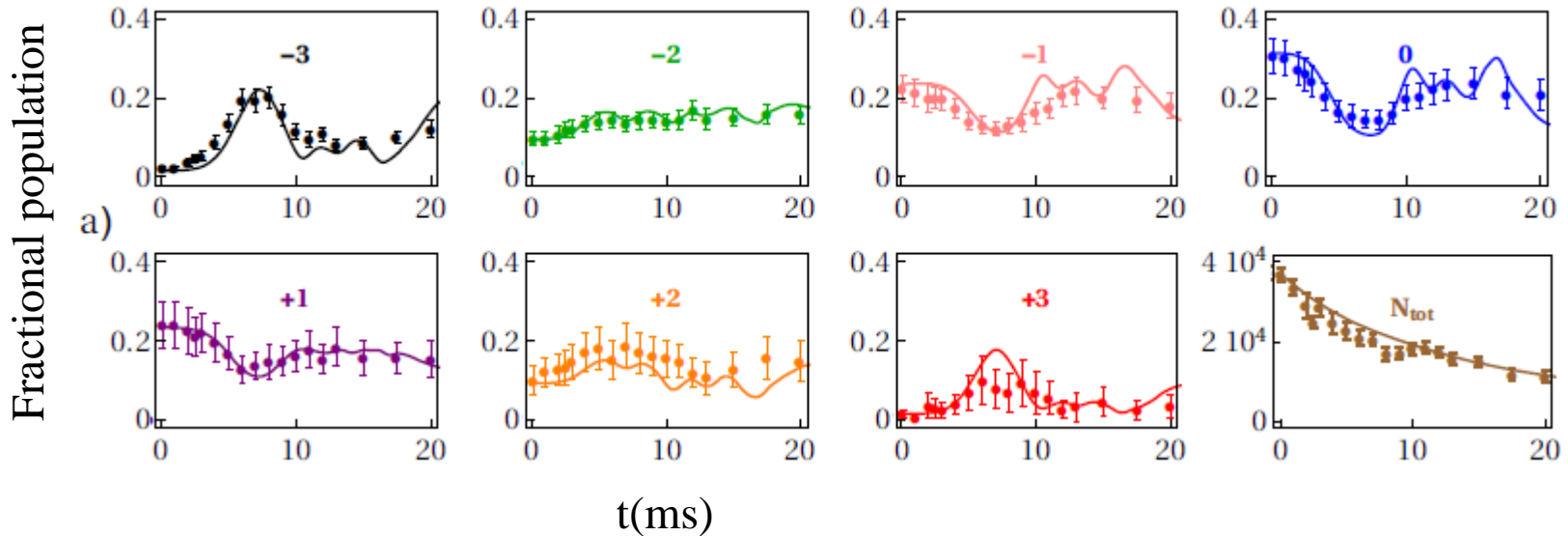
Beyond mean-field effects too small to be observed ??

Which would be the conditions ? $\left[\sqrt{\sum_{(i,j)} (V_{(i,j)})^2} \Leftrightarrow 10ms \right]$

Trigger spin dynamics using magnetic field gradients



GP- Theory :
Pedri/Kechadi
Zhu/Rey

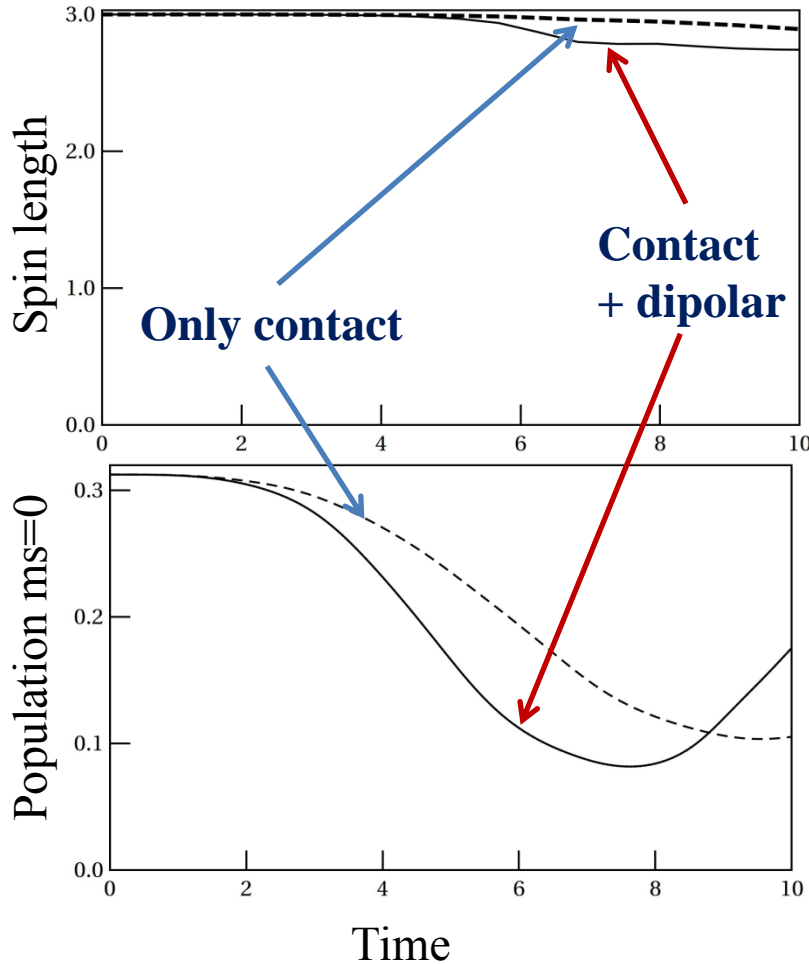


Dynamics is triggered by the existence of a B-field gradient

Dynamics reproduced by mean-field theory!

The unexpected ferromagnetic behavior of an anti-ferromagnetic gas

The spinor remains almost locally locked to total spin 3

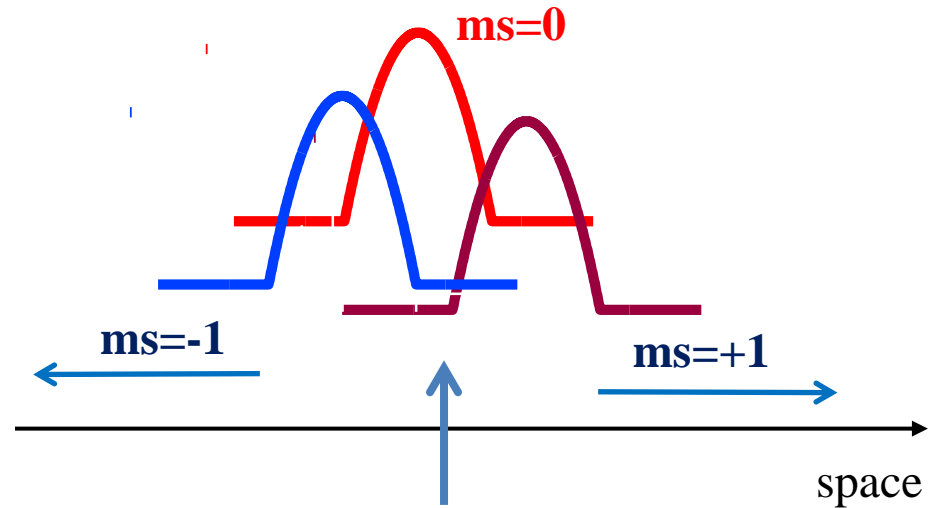
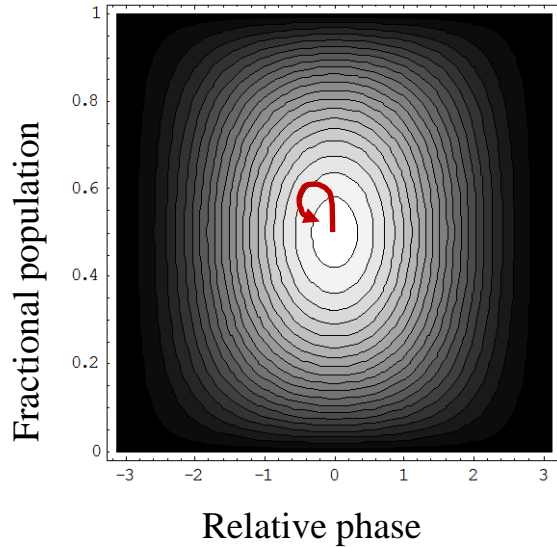


Atoms remain locally polarized in a stretched state (and therefore interact through $S=6$)

This is a surprise because $a_6 > a_4 \rightarrow$ equilibrium favors depolarization

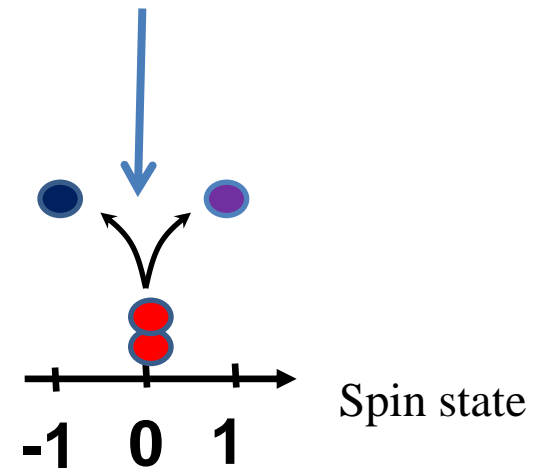
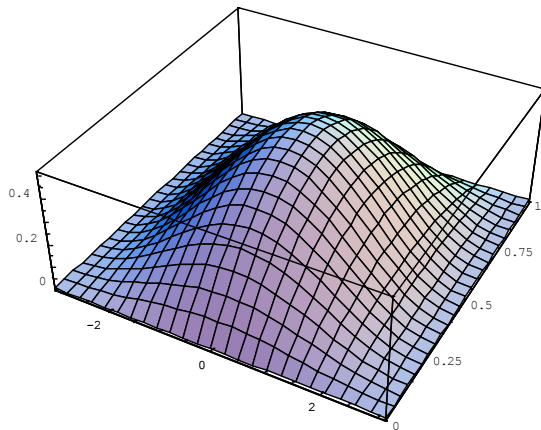
(Spin length characterizes the local polarized character)

**Interpretation: locally, spinor is at a maximum of the interaction energy.
Magnetic field gradients cannot change the spinor structure without violating energy conservation**



Local structure of spinor

- modified by gradient
- Restored by spin-exchange



A simplistic and general equation describes spin-dynamics

$$\frac{p_{m_s}(t)}{p_{m_s}(0)} = 1 + \left(\frac{g\mu_B b}{2Mw} \right)^2 \left(m_s^2 - \sum_{m_{s'}} m_{s'}^2 p_{m_{s'}}(0) \right) t^{\frac{1}{2}}$$

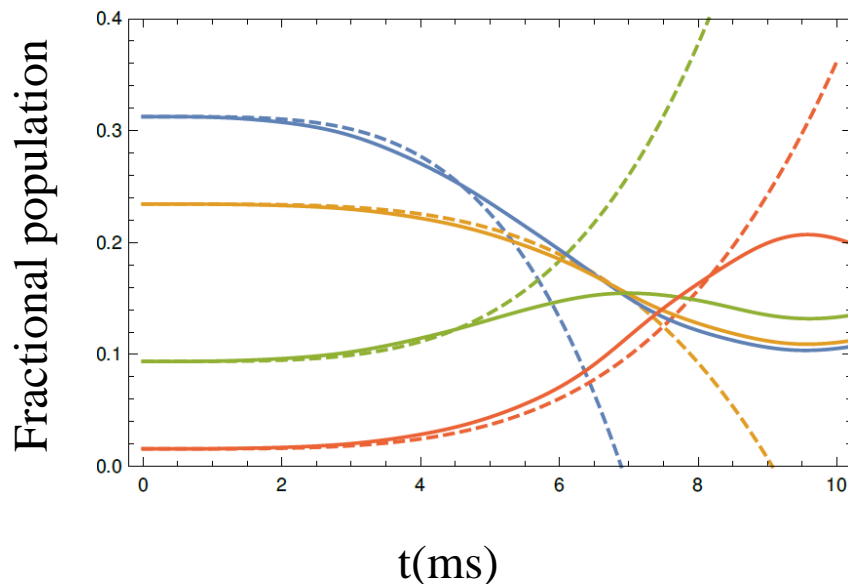
gradient
Radius BEC

(in practice independent of spin-dependent interactions)

All the spin-dependent interactions do is undo whatever population imbalance the magnetic field gradient creates !

Natural timescale

$$\tau = \left(\frac{2Mw}{g\mu_B b'} \right)^{1/2}$$



Speculation on why we do not observe beyond mean-field effects

??? Can beyond mean-field effect be observed if strong spin-dependent interactions favor ferromagnetism ???

Outline

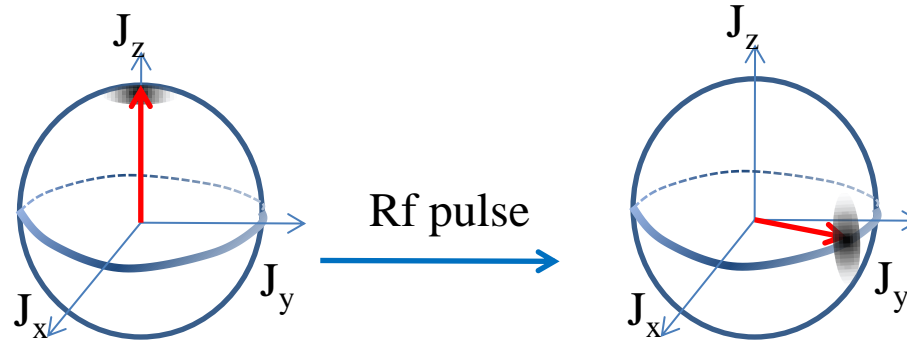
Spin dynamics after tilting spins
(rf pulse)

I BEC case

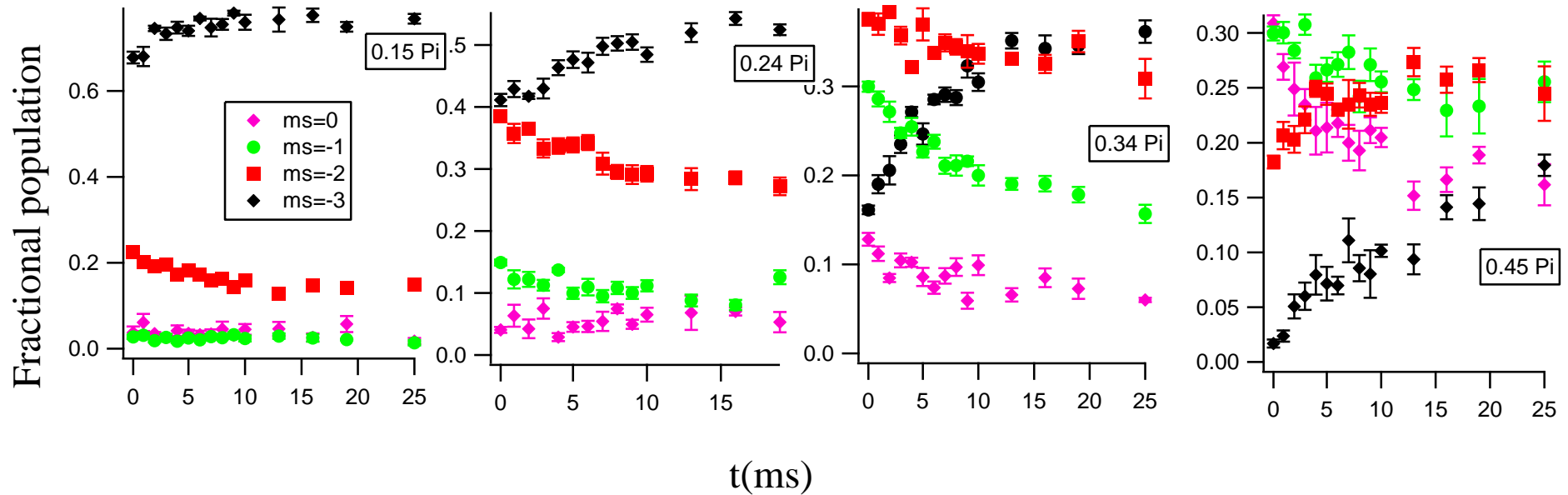
II Lattice case

Mott state, one atom per lattice site

Experimental results, lattice case...



Preliminary



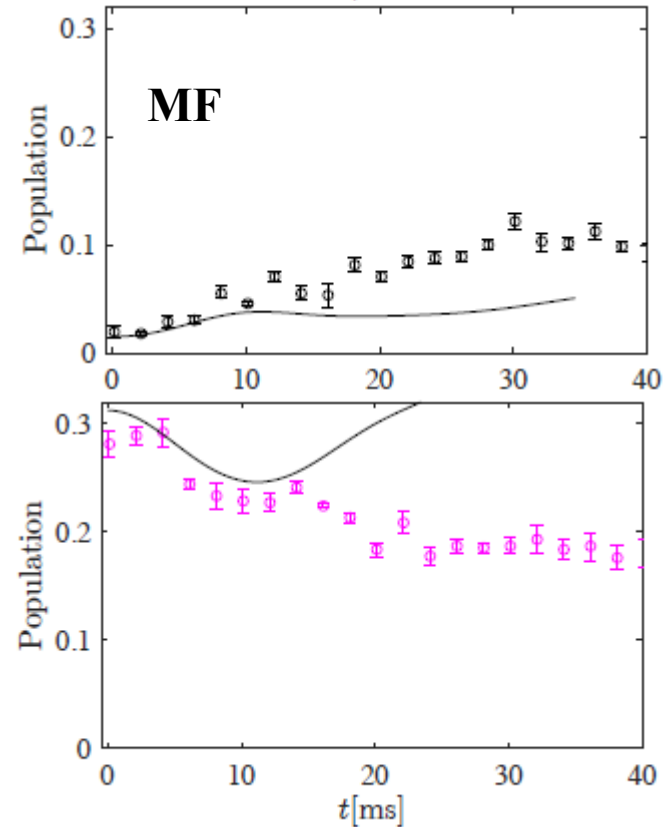
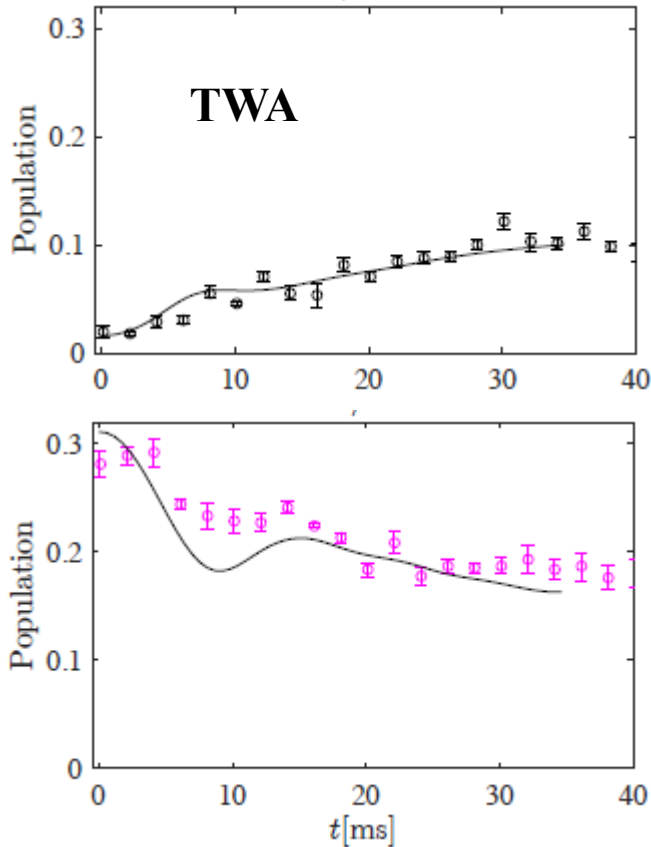
Increasing quantum-ness is expected



How to probe for beyond mean-field effects?

new numerical models being developed in A. M. Rey's group (TWA)
(J. Schachenmayer, B. Zhu).

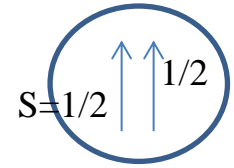
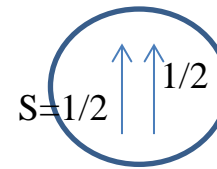
Preliminary



Beyond mean-field theory agrees significantly better
A very good test of the theory for large atom numbers
(no plaquette simulation available)

How to probe for beyond mean-field effects? (when theory not available)

Measure spin fluctuations?



Associated to correlations ??

How to reveal entanglement ?

(Entanglement witness quest - collaboration Perola
Milman; Paris 7 University)

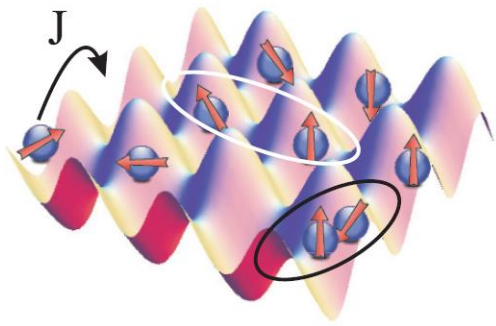
Large Spins are tricky for EWs!!

**!! For example: squeezing is NOT
an entanglement witness for large
spins !!**

See papers by Toth...

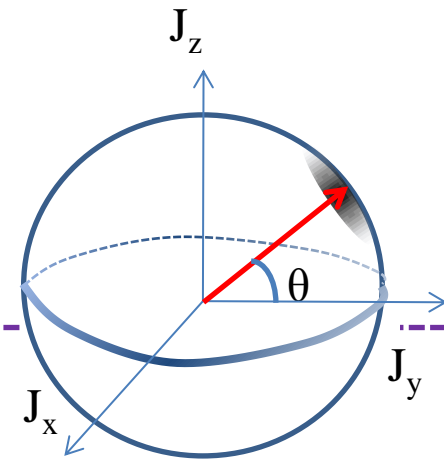
Local probes?

Bi-partite entanglement? (discussions Tommaso Roscilde)



Summing up

Spin dynamics after tilting the spin by rf



- In the BEC phase

- Spin dynamics triggered by dipolar interactions for $\theta \neq \pi/2$
- When spin dynamics is triggered by magnetic field gradients, BEC remains locally almost ferromagnetic
 - Correlations could arise without a lattice but are not seen

- In the lattice

- Correlations develop – How to prove this directly?
 - Correlations increase with $\Delta S_z|_0$
(specific to non-Heisenberg Hamiltonian)
- What happens when super-exchange and dipolar interactions compete?

Thank you

S. Lepoutre, B. Naylor (PhD), L. Gabardos,
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M. Robert-de-St-Vincent,
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Paris 13 University
LPL

L. Santos

- Hannover University

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J. Schachenmayer
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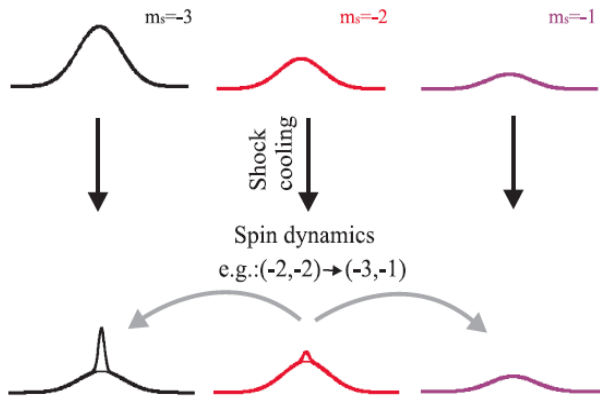
- JILA,
- University Boulder-Colorado

M. Brewczyk
M. Gajda

- Bialymstoku University
- Polish Academy of sciences

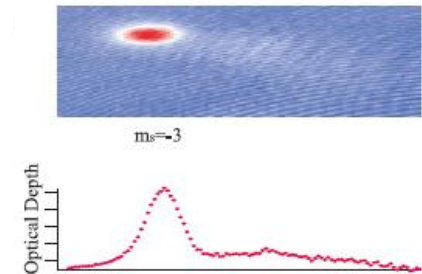
Other things which are being studied in the lab, but were not discussed in this talk

- Dynamics of BEC in presence of the spin degrees of freedom



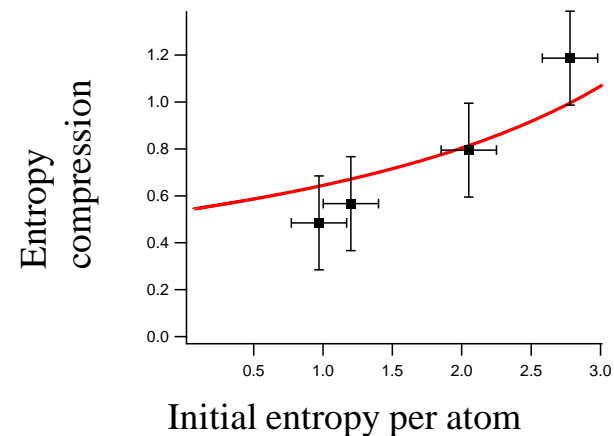
Phys. Rev. Lett. **117**, 185302 (2016)

We observe a partially thermalized spin degrees of freedom, and interplay between spin-dynamics and BEC, which results in a difficulty to produce depolarized BECs.



- Using the spin degrees of freedom to remove entropy in a BEC

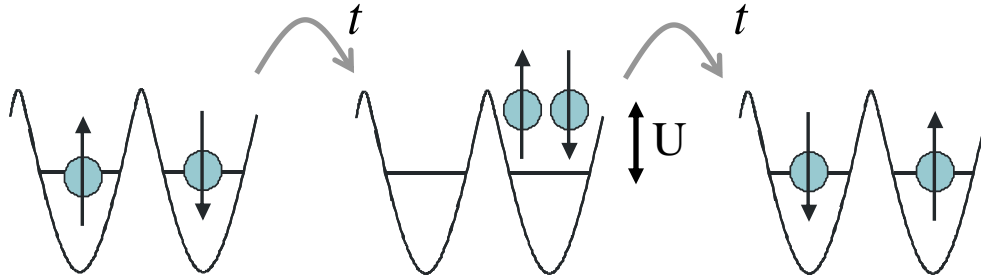
At thermodynamic equilibrium, a BEC is polarized in the lowest energy spin state. Filtering out spin excited states is therefore a very good way to remove entropy from the gas.



Phys. Rev. Lett. **115**, 243002 (2015)

Exotic quantum magnetism of large spin, from Mott to superfluid

An exotic magnetism driven by the competition between three types of exchange



$$\Gamma \propto \frac{t^2}{U}$$

Super-Exchange (I)

(nearest neighbor)

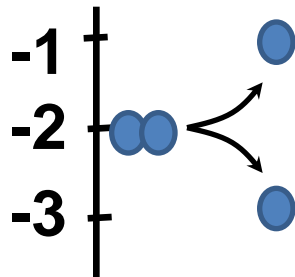
decreases with lattice depth

$$\left(S_{1z} \cdot S_{2z} - \frac{1}{4} (S_{1+} S_{2-} + S_{1-} S_{2+}) \right) \frac{(1 - 3z^2)}{r^3}$$

Dipolar exchange (II)

(true long range)

independent from lattice depth



$$\Gamma = \frac{4\pi\hbar^2}{m} n (a_6 - a_4)$$

Contact exchange (III)

(short range)

Increases with lattice depth

One speculative slide - Which criterion for beyond-mean-field effects?

$$|S_{\perp}|(t) \underset{t \rightarrow 0}{\approx} |S_{\perp}| \left(1 - t^2 \left[\Delta B^2 + \frac{1}{N} \sum V_{i,j}^2 \right] \right)$$

PRL 110, 075301 (2013)

$$\left\langle \vec{S}(t) \right\rangle$$

Measure size of total spin: a good indication for beyond-mean-field effects ... at least for homogeneous systems!

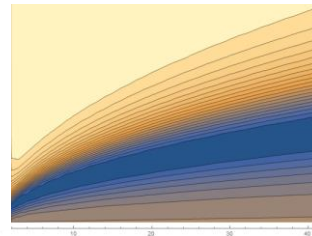
Squeezing by



$$\left\langle S^2(t) \right\rangle$$

In a lattice, energy of two nearby atoms is: $U = -\frac{J^2}{U_S}$ S : molecular potential

$$V_{dd} > \left| \frac{J^2}{U_{S+2}} - \frac{J^2}{U_S} \right|$$



Remember: Classically:
atoms undergo identical precession;
total spin is conserved in time

Speculation: something interesting might happen due to competition between dipolar interaction and super-exchange

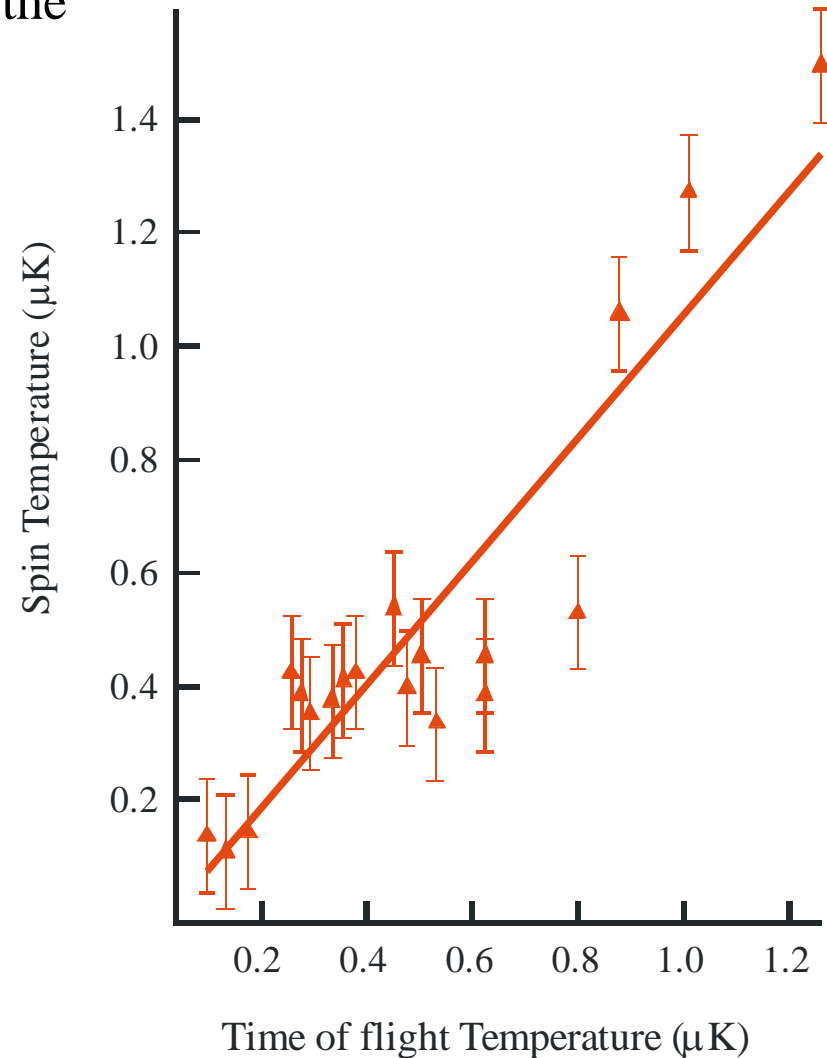
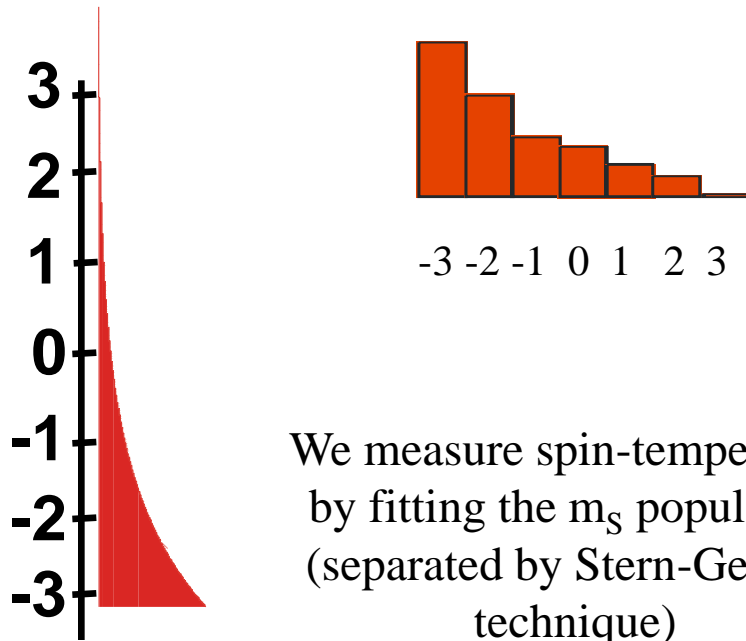
Spin temperature equilibrates with mechanical degrees of freedom

(due to magnetization changing collisions)

At low magnetic field: spin thermally activated

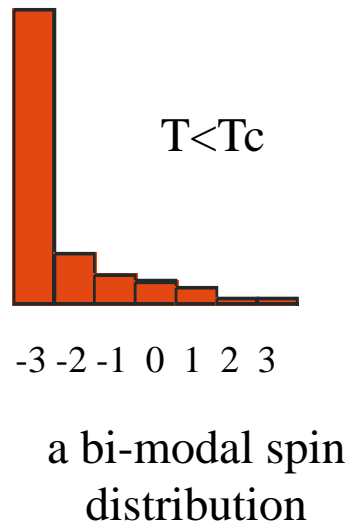
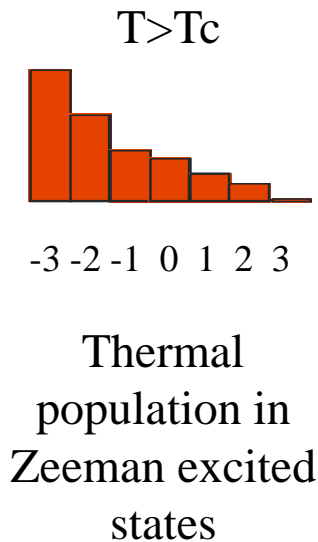
Magnetization adapts to temperature due to the presence of dipolar interactions

$$g\mu_B B \approx k_B T$$

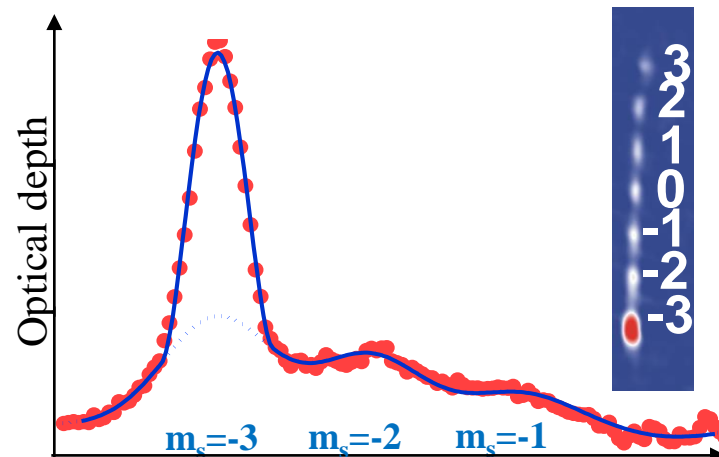


Related to Demagnetization Cooling expts,
T. Pfau, *Nature Physics* **2**, 765 (2006)

The BEC always forms in the $m_s=-3$



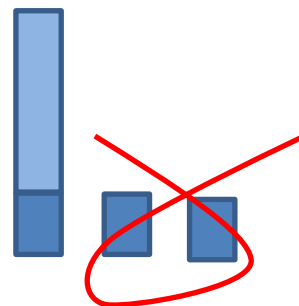
**BEC only in $m_s=-3$
(lowest energy state)**



PRL 108, 045307 (2012)

One idea: Kill spin-excited states ?

**Provides a loss
specific for thermal
fraction**



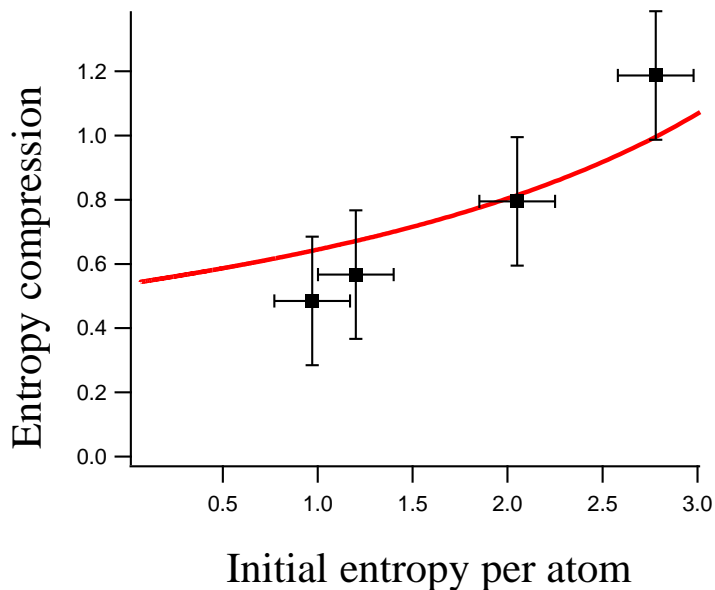
**Should lead to purification of the BEC, thus cooling
(and this process can be repeated after waiting for more depolarization)**

Cooling efficiency

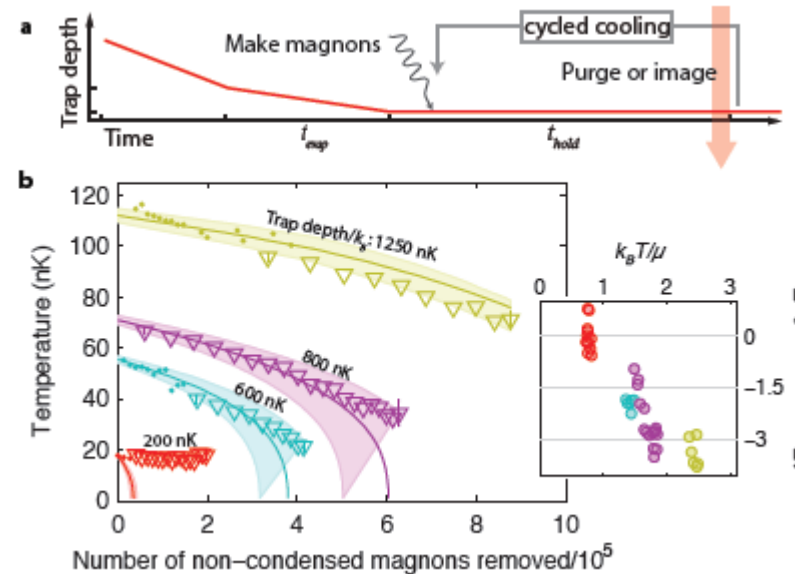
All the entropy lies in the thermal cloud

Thus spin filtering is extremely efficient!

In principle, cooling efficiency has no limitation



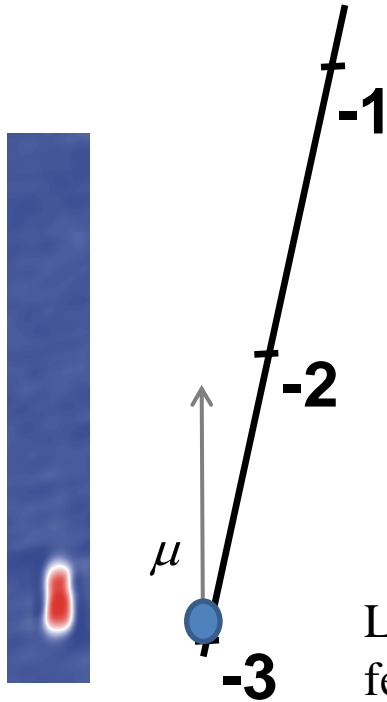
Chromium, LPL, Phys. Rev Lett. (2015)



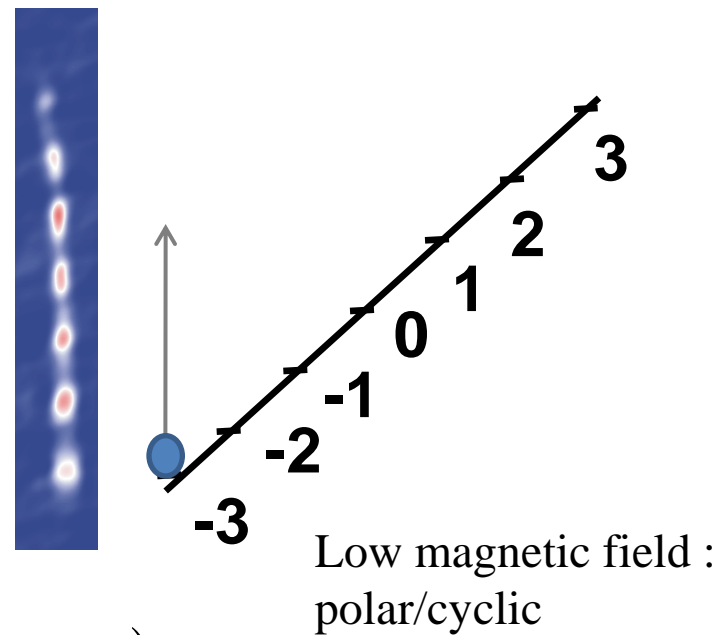
Rb, Stamper Kurn, Nature Physics (2015)

Use spin to store and remove entropy

New magnetic phases at low field

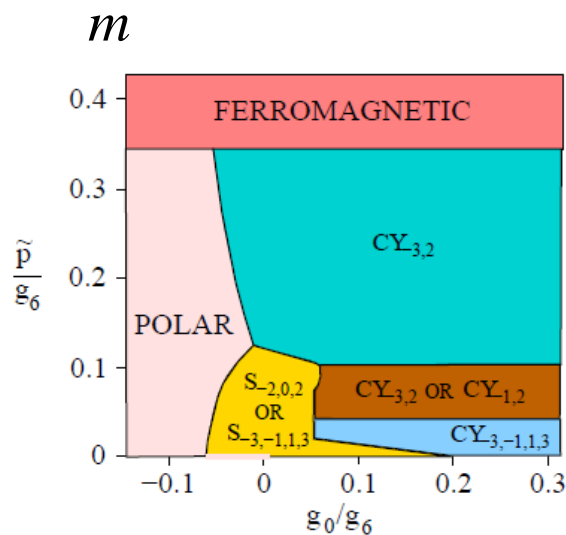
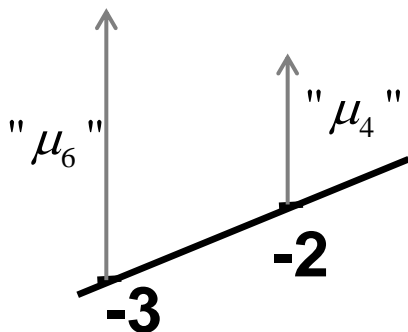


Large magnetic field :
ferromagnetic



Low magnetic field :
polar/cyclic

$$g_J \mu_B B_c \approx \frac{2\pi \hbar^2 n_0 (a_6 - a_4)}{m}$$



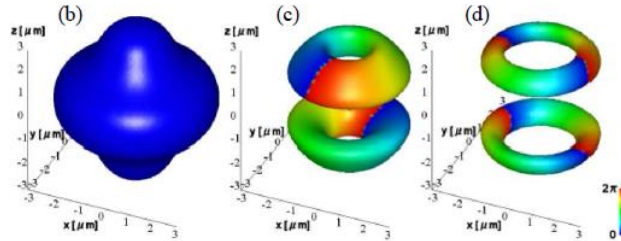
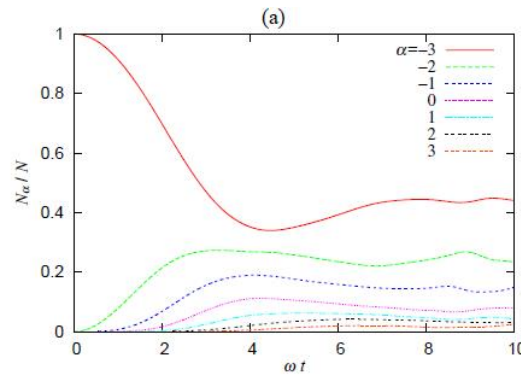
Santos PRL **96**,
190404 (2006)

Ho PRL. **96**,
190405 (2006)

Depolarization observed (Phys. Rev. Lett. **106**, 255303 (2011)) ; phases remain to be studied

Two interesting proposals:

Einstein-de Haas effect



Santos PRL **96**,
190404 (2006)

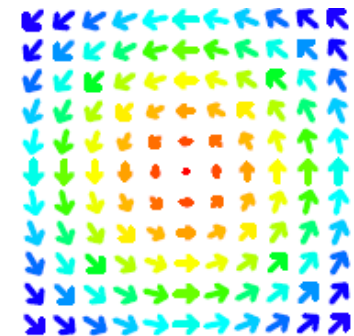
Ho PRL. **96**,
190405 (2006)

Spontaneous circulation in the ground state

$$\tilde{V}_{dd} \propto \int d^3\vec{k} \left[3|\vec{F}(\vec{k}) \cdot \vec{k} / k|^2 - |\vec{F}(\vec{k})|^2 \right]$$

$\vec{F}(\vec{k})$ Fourier transform of magnetization vector

Maximize $|\vec{F}(\vec{k})|$ and $\vec{F} \perp \vec{k}$



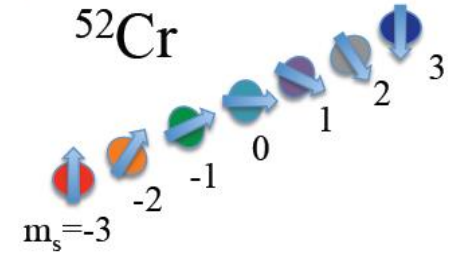
Ueda PRL **97**, 130404 (2006)
S. Yi and H. Pu,
PRL **97**, 020401 (2006)

This seminar: magnetism with large spin cold dipolar atoms

Optical dipole traps equally trap all Zeeman state of a same atom

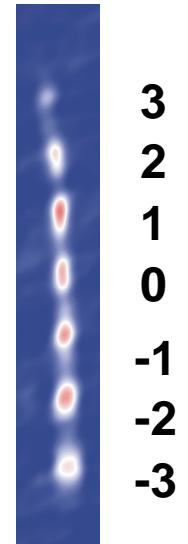
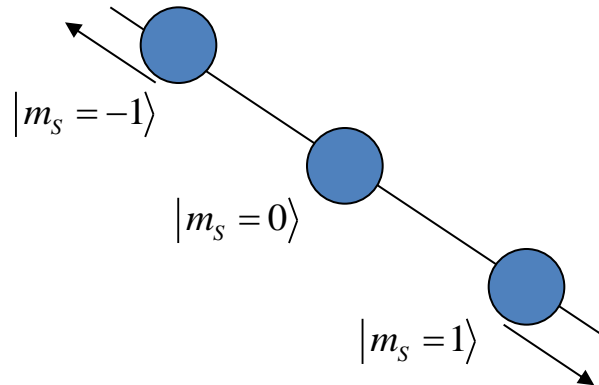
Linear (+ Quadratic)
Zeeman effect

$$E(m_S) = m_S g \mu_B B (+\alpha B^2)$$



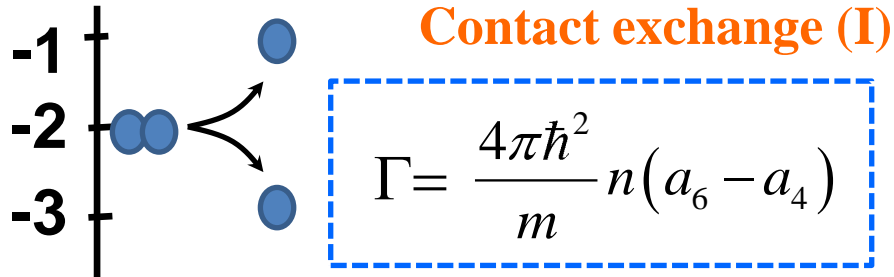
Stern-Gerlach separation:
(magnetic field gradient)

Simultaneously measure spin and
momentum distribution



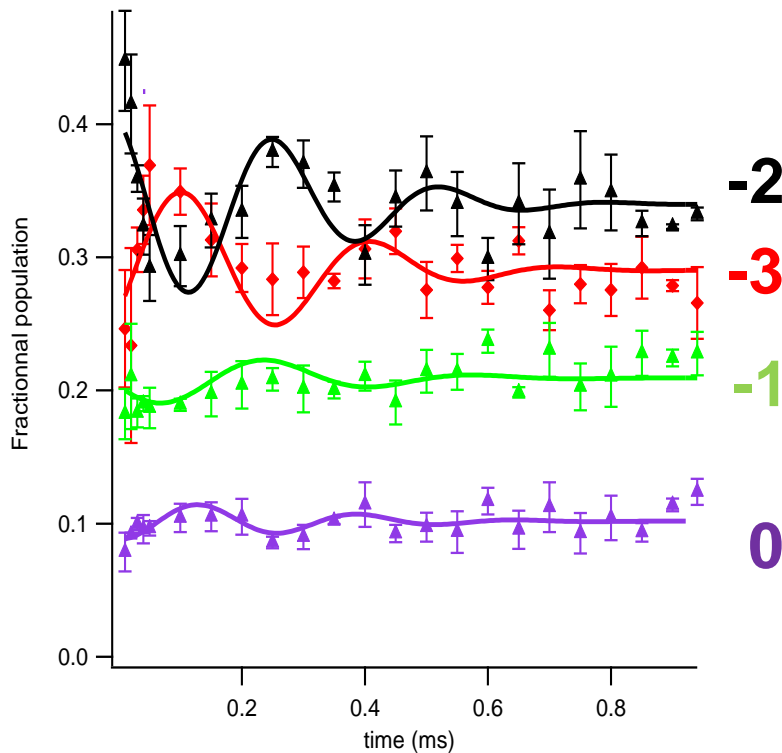
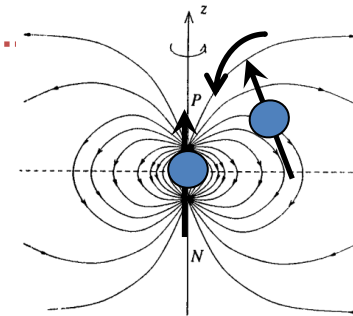
Two main players for magnetism and spin dynamics:

Spin-dependent contact interaction

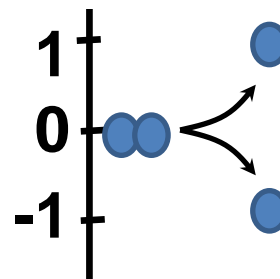


Dipole-dipole interactions

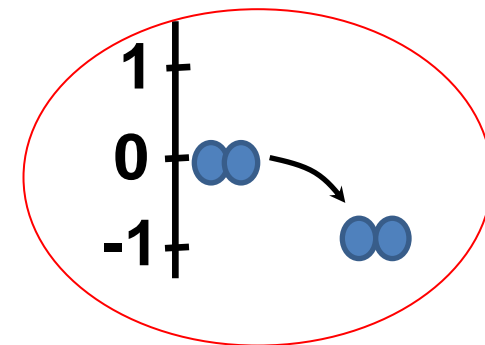
$$V_{dd} = \frac{\mu_0}{4\pi} S^2 (g_J \mu_B)^2 (1 - 3\cos^2(\theta)) \frac{1}{R^3}$$



Dipolar Exchange (II)



Relaxation

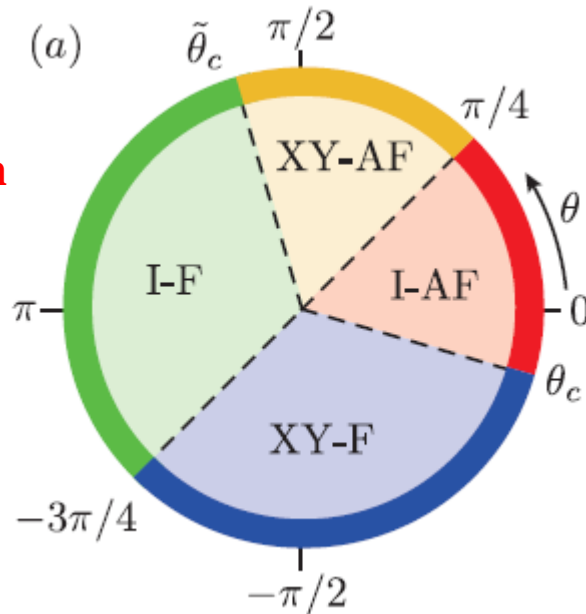


Unique!

Other consequences, due to long range character **and** anisotropy

Anomalous spin behavior

long range character



$$H = \frac{Ja^3}{\hbar^2} \sum_{i \neq j} \frac{\cos \theta S_i^z S_j^z + \sin \theta (S_i^x S_j^x + S_i^y S_j^y)}{|\mathbf{R}_i - \mathbf{R}_j|^3}$$

Possibility of long-range ferromagnetic order in 2D (in contrast to Mermin Wagner theorem for short-range interactions)

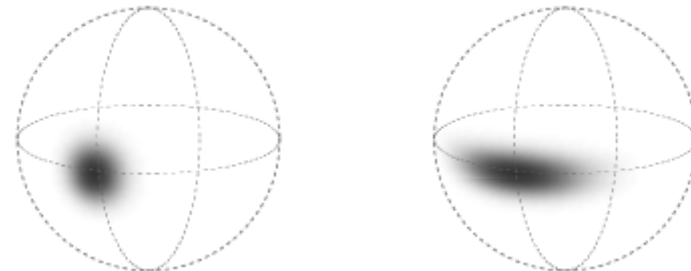
Buchler PRL **109**, 025303 (2012)

**XYZ Hamiltonian
Spin-orbit coupling
when magnetization is
free (anisotropy)**

(Rey, Buchler, Zoller, Karr, Lev...)

Needs to engineer two degenerate states of different magnetization

**How correlations develop and spread
(Non-Heisenberg physics)**

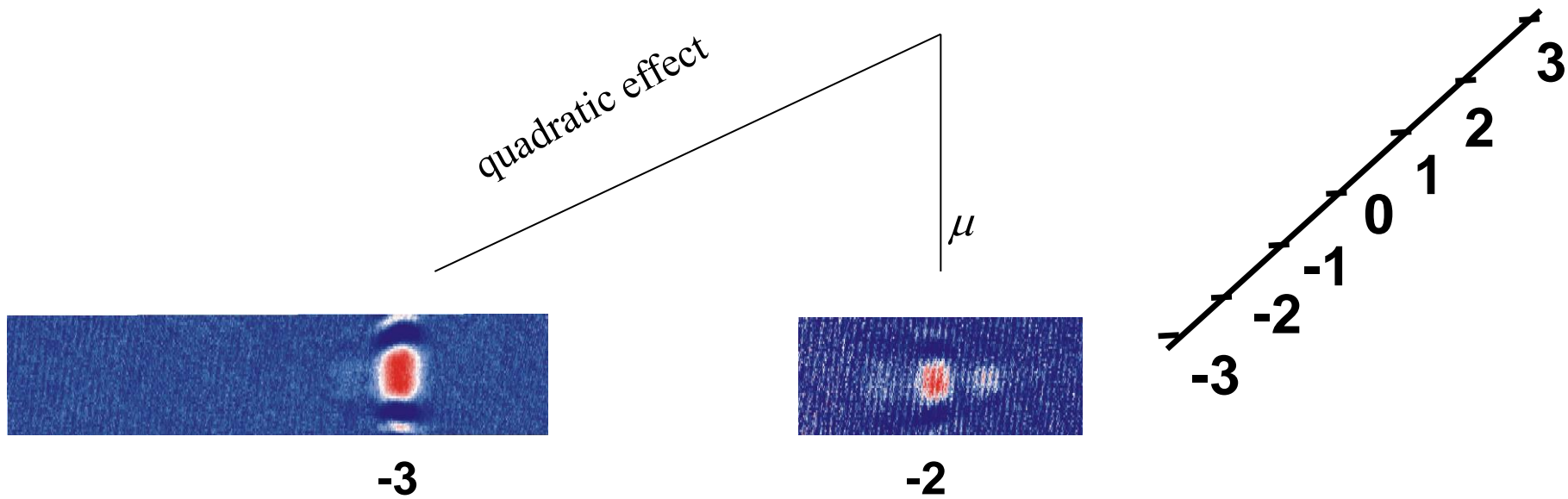


(on-going collaboration with A. M. Rey)

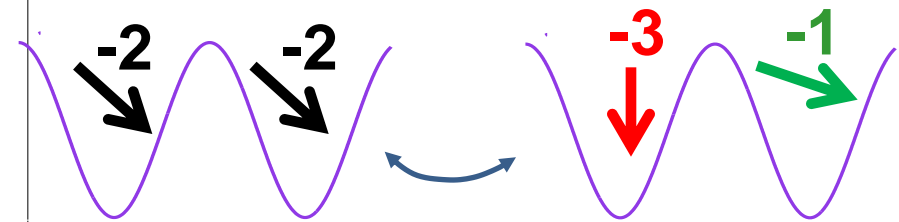
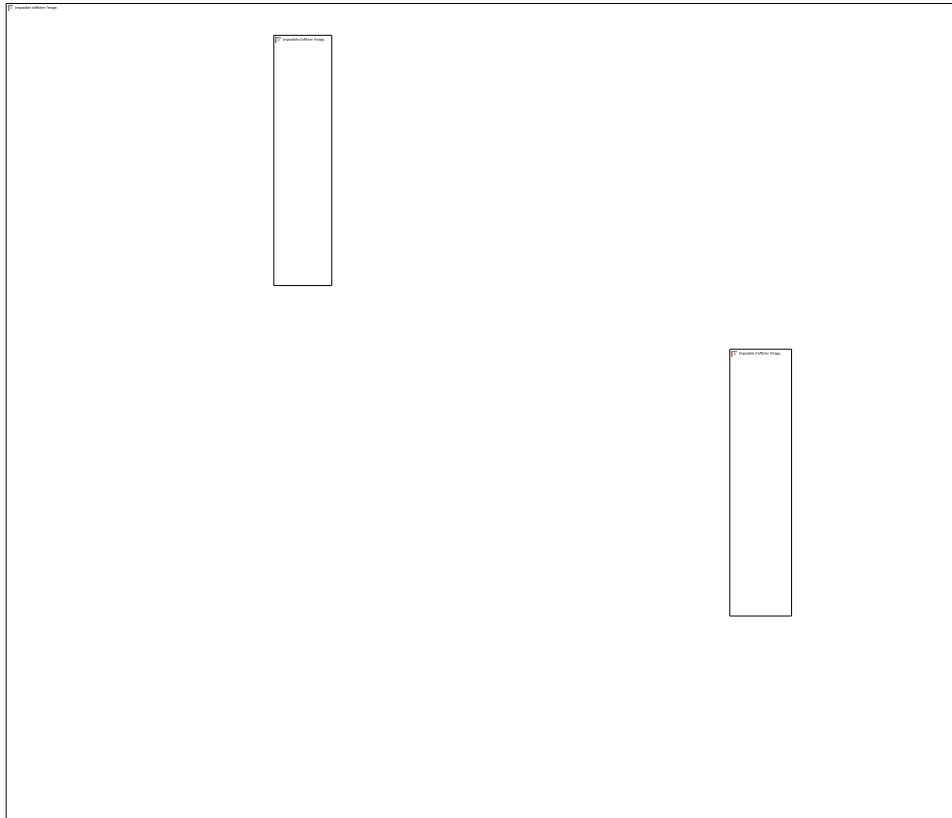
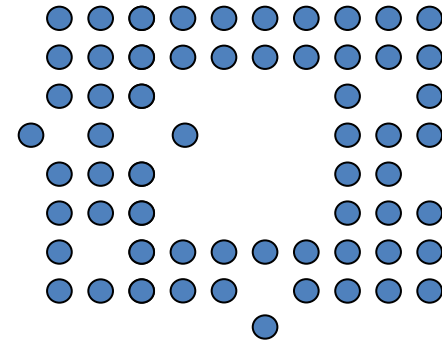
First case: prepare all atoms in a well-defined Zeeman state

$$\Psi(0) = |2, 2, \dots, 2, 2\rangle$$

(each site contains one atom in state $m_s = -2$)



Spin dynamics after emptying doubly-occupied sites: A proof of inter-site dipole-dipole interaction



Magnetization is constant

Timescale for spin dynamics = 20 ms

Tunneling time = 100 ms

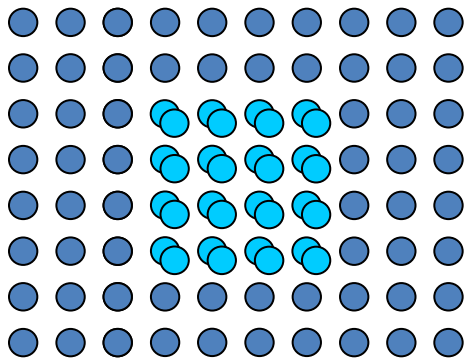
Super-exchange > 10s

Experiment: spin dynamics after the atoms are promoted to $m_s = -2$

Theory: exact diagonalization of the t-J model on a 3*3 plaquette (P. Pedri, L. Santos)

!! Many-body dynamics !!
(each atom coupled to many neighbours)
Mean-field theories fail

Phys. Rev. Lett., 111, 185305 (2013)

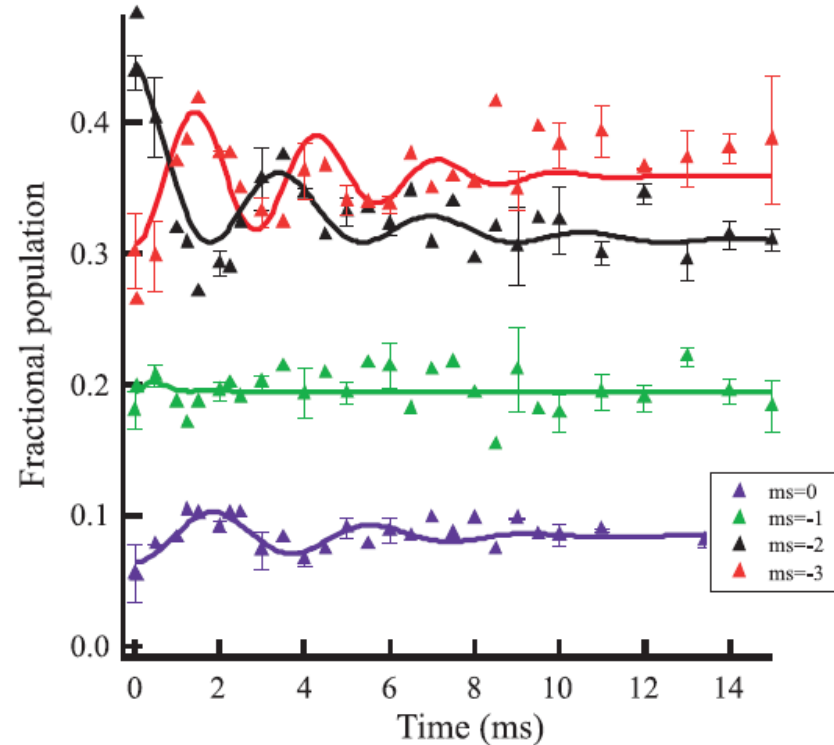


1- At large lattice depths (Mott regime)

In presence of doubly-occupied sites:

A complex oscillatory behavior displaying two distinct frequencies

Phys. Rev. Lett., 111, 185305 (2013)



Contact exchange (III)

$$\Gamma = \frac{4\pi\hbar^2}{m} n(a_6 - a_4)$$

Dipolar Exchange (II)

Exact diagonalization is excluded with two atoms per site
(too many configurations for even a few sites)

A toy many-body model for the dynamics at large lattice depth

Exact diagonalization is excluded with two atoms per site
(too many configurations for even a few sites)

Perturbative theory for singlons

$$\left(S_{1z} \cdot S_{2z} - \frac{1}{4} (S_{1+} S_{2-} + S_{1-} S_{2+}) \right) (1 - 3z^2)$$

$$|2,2,\dots,22\rangle \rightarrow \infty \sum_{(i,j)} V_{(i,j)} |2,2,\dots,2,1,2,\dots,2,3,2,2\rangle$$

(i) (j)

$$\Gamma = 2\pi \sqrt{\sum_{(i,j)} (V_{(i,j)})^2}$$

Toy model for doublons: replace S=3 by S=4 or S=6

Measured frequency: 300 Hz

Calculated frequency: S=4: 220 Hz
 S=6: 320 Hz

4th order correction
included

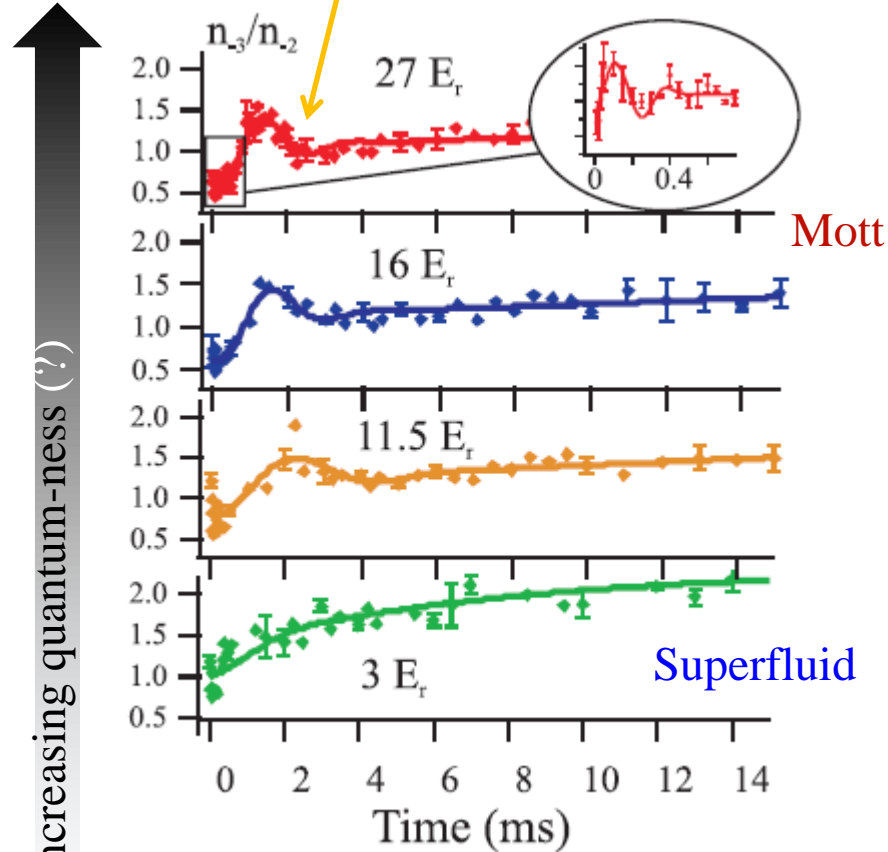
Toy models seems to qualitatively reproduce oscillation;

see related analysis in Porto, Science, (2015)

2- Spin dynamics as a function of lattice depth

Dipolar Exchange (II)

Contact exchange (I)



Phys. Rev. A 93, 021603(R) (2016)

Large lattice depth:
dipolar exchange and contact
exchange contribute on different
timescales (~qualitatively well
understood)

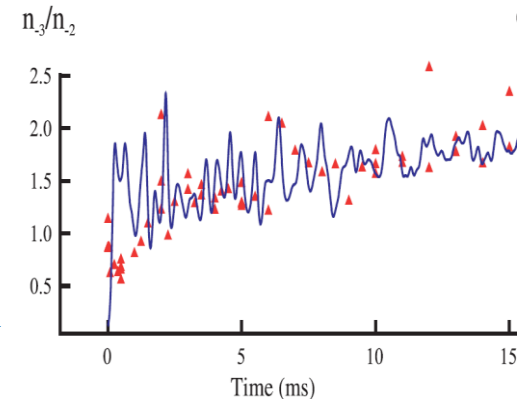
Intermediate lattice depth:
super-exchange may occur and
compete

No theoretical model yet
All three exchange mechanisms
contribute

A unique and exotic situation!!

Low lattice depth:
An interplay
between contact and
dipolar interactions

« Good » agreement with
GP (P. Pedri)



Similarities and differences

Our Hamiltonian =
NMR Secular Hamiltonian
+ magnetization-changing collisions

$$S_{1z}S_{2z} + \frac{1}{2}(S_{1+}S_{2-} + S_{1-}S_{2+})$$
$$- \frac{3}{4}(2zS_{1z} + r_-S_{1+} + r_+S_{1-})$$
$$(2zS_{2z} + r_-S_{2+} + r_+S_{2-})$$

Heisenberg hamiltonian

$$S_{1z}S_{2z} + \frac{1}{2}(S_{1+}S_{2-} + S_{1-}S_{2+})$$

NMR Secular hamiltonian

$$S_{1z}S_{2z} - \frac{1}{4}(S_{1+}S_{2-} + S_{1-}S_{2+})$$

Magnetization-changing collisions

$$S_{1-}S_{2-}$$

$$- \frac{1}{4}(S_{1+}S_{2-} + S_{1-}S_{2+})$$

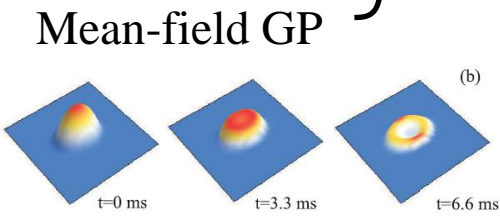
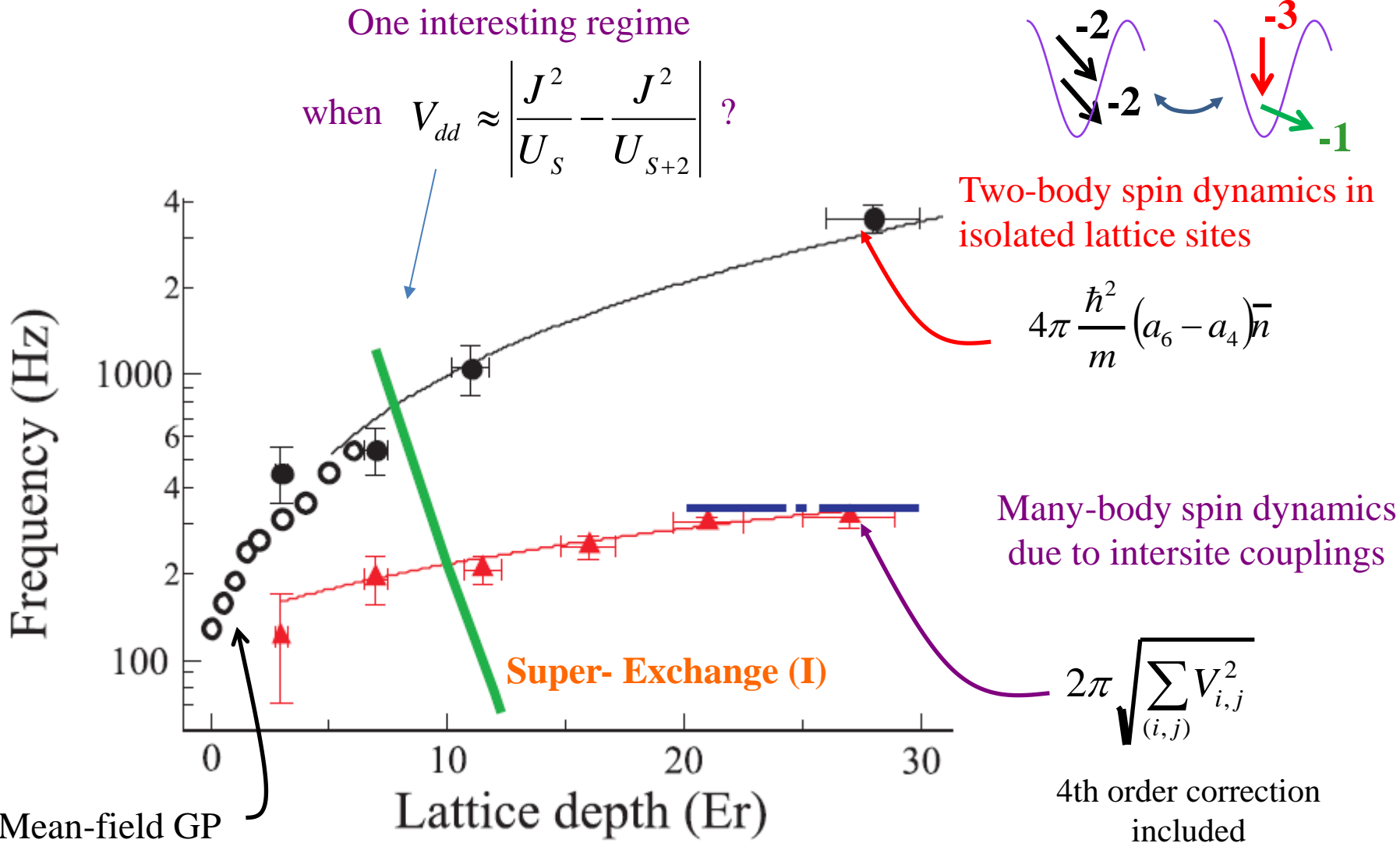
Anisotropy $(1 - 3\cos^2 \theta)$

Long Range

Tunneling

Large Spin

Dipolar exchange (II)



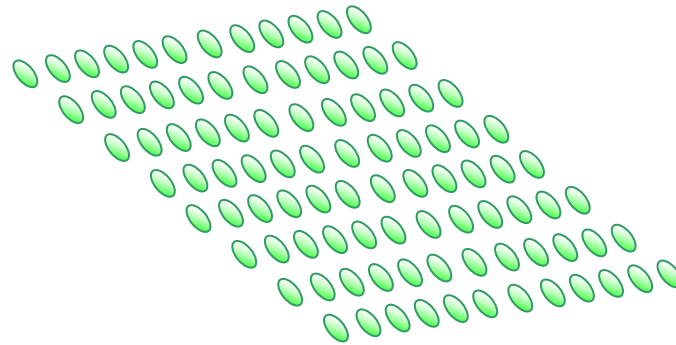
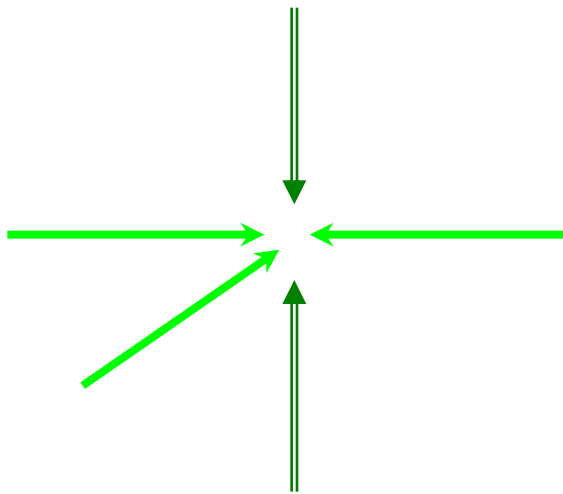
Increasing quantum-ness (?)

A ^{52}Cr BEC in a 3D optical lattice

Optical lattice: Periodic potential made by a standing wave

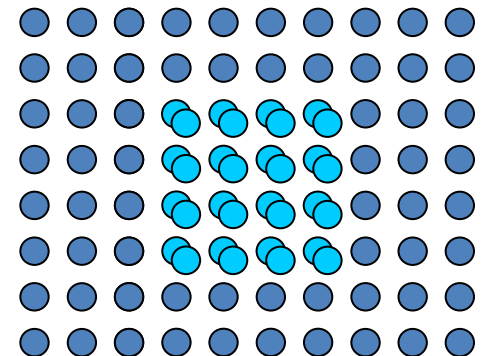
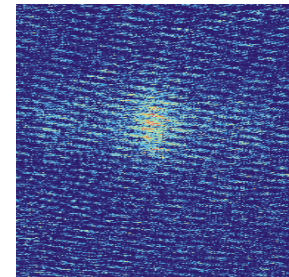
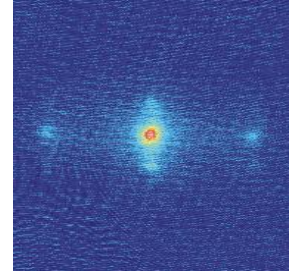
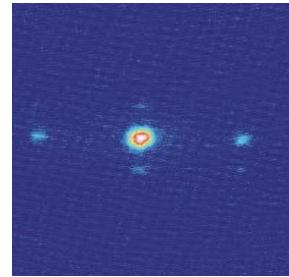
Our lattice architecture:

(Horizontal 3-beam lattice) \times (Vertical retro-reflected lattice)



Rectangular lattice of anisotropic sites

3D lattice \rightarrow Strong correlations, Mott transition...



From 2 to N atoms (interacting through dipolar interactions)

$$\frac{dS}{dt} = \gamma S \times (B + \sum_j B_j)$$

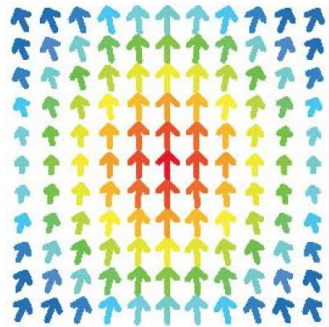
$$\vec{B}_j$$

Field created by atom j

$$\begin{aligned} \frac{d\langle S \rangle}{dt} &= \gamma \left\langle S \times (B + \sum_j B_j) \right\rangle \\ &= \gamma \langle S \rangle \times (B + \sum_j \langle B_j \rangle) \end{aligned}$$



Neglect correlations



$$\Gamma(r) = \int dr' V_{dd}(r - r') n(r')$$

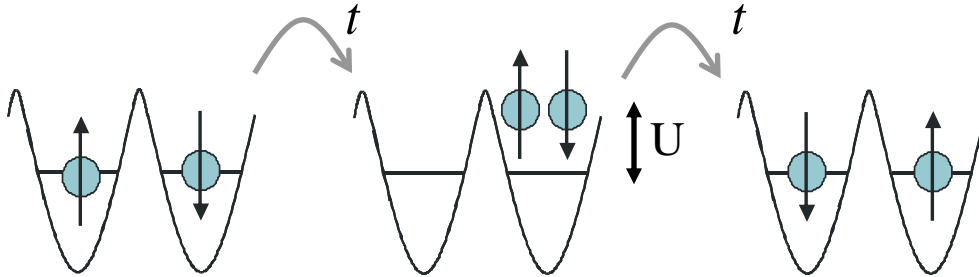
Mean-field interaction ↔
Gross-Pitaevskii equation

In the **mean-field** approximation, atoms undergo (**classical**) precession around dipolar field

Mean-field may be inhomogeneous, and total spin may not be conserved

Exotic quantum magnetism of large spin, from Mott to superfluid

An exotic magnetism driven by the competition between three types of exchange

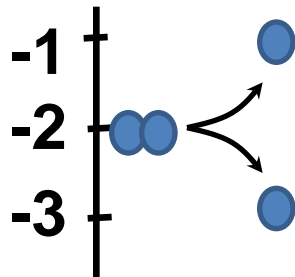


$$\Gamma \propto \frac{t^2}{U}$$

Super-Exchange (I)
(nearest neighbor)
decreases with lattice depth

$$\left(S_{1z} \cdot S_{2z} - \frac{1}{4} (S_{1+} S_{2-} + S_{1-} S_{2+}) \right) \frac{(1 - 3z^2)}{r^3}$$

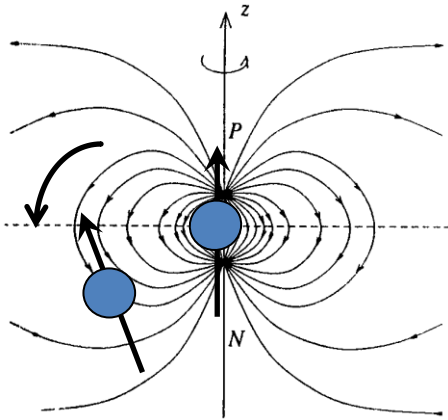
Dipolar exchange (II)
(true long range)
independent from lattice depth



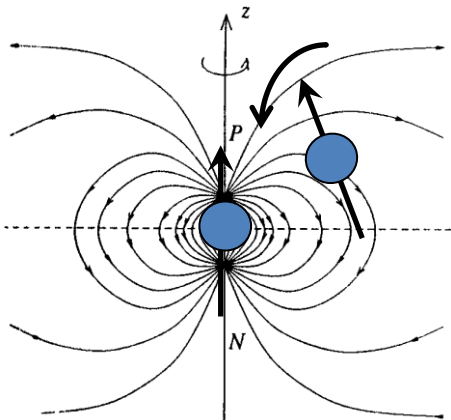
$$\Gamma = \frac{4\pi\hbar^2}{m} n (a_6 - a_4)$$

Contact exchange (III)
(short range)
Increases with lattice depth

Under which conditions correlations develop? – Classical vs Quantum magnetism



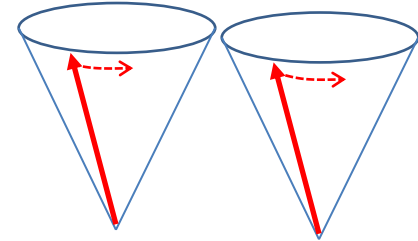
$$\vec{B}_A(B) = \vec{B}_B(A)$$



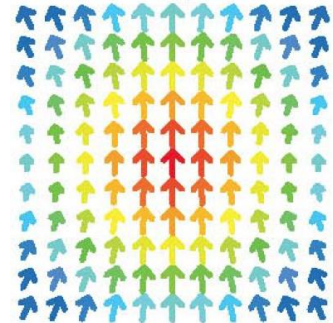
Classically:

These two atoms undergo identical precession

Total spin conserved



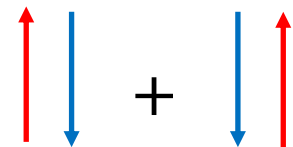
In the **mean-field** approximation, atoms undergo (**classical**) precession around dipolar field (may be inhomogeneous) ↔ GP equation



Quantum-mechanically:

Possibility for entanglement

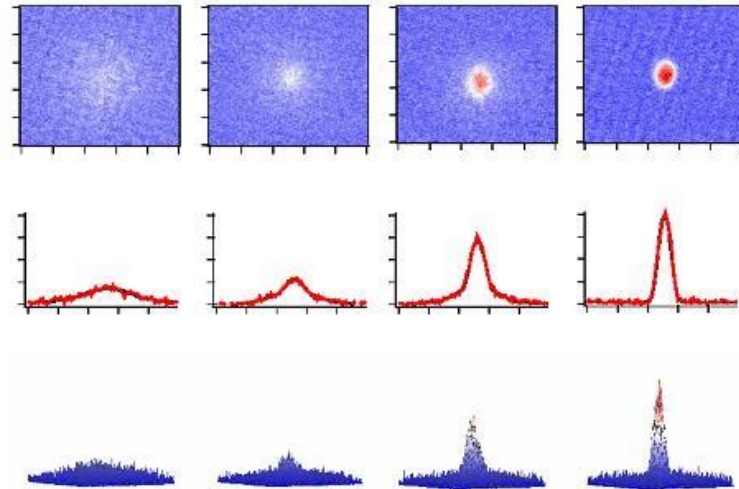
Total spin is NOT conserved



This experiment: chromium quantum gases

Nov 2007 : Chromium BEC

$4 \cdot 10^4$ atoms

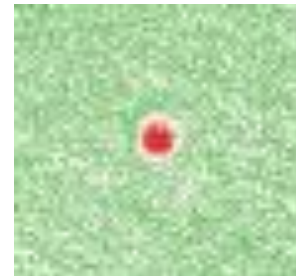


$S=3$

April 2014 : Chromium Fermi sea

10^3 atoms

(from only $3 \cdot 10^4$ atoms in dipole trap !)
Phys. Rev. A 91, 011603(R) (2015)



$F=9/2$

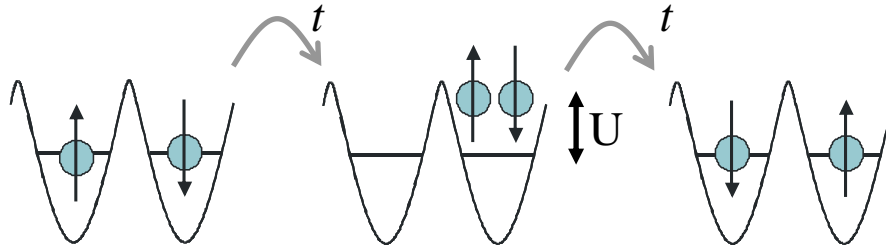
Chromium: unusually large dipolar interactions

(only few experiments worldwide with non-negligible dipolar interactions
-, Boulder, Boston, Hong-Kong,...)

Quantum magnetism, some paradigms from solid-state physics

Strongly correlated (s=1/2) electrons

Condensed-matter: effective spin-spin interactions due to exchange interactions



Super-Exchange

$$\Gamma \propto \frac{t^2}{U}$$

Heisenberg model of magnetism

(**real spins, effective spin-spin interaction**)

Magnetism is driven by super-exchange

$$S_{1z}S_{2z} + \frac{1}{2}(S_{1+}S_{2-} + S_{1-}S_{2+})$$

Ising

Exchange

Cold atoms offer to revisit paradigms from solid-state physics experimentally

... and go beyond ?...