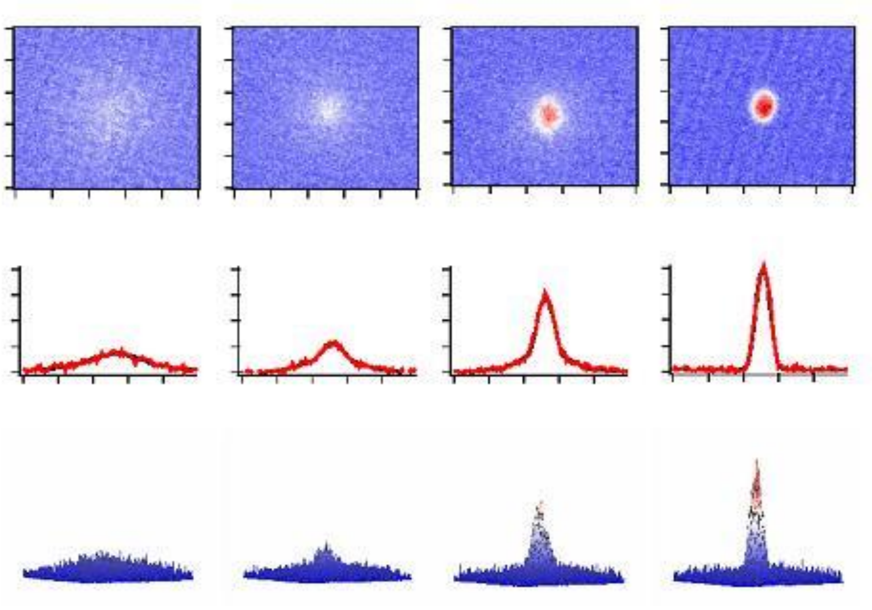
Dipolar Quantum Gases



-Quantum many-body physics with particles interacting through dipole-dipole interactions

-Magnetism

Bruno Laburthe-Tolra (Université Paris 13)

Cold atom activity at Paris North University

Rubidium BEC in rf-dressed magnetic traps
Hélène Perrin
2D Physics ; BEC in a ring

Sodium BEC on a chip (project)
Aurélien Perrin
Non-equilibrium dynamics

Metastable atoms flying by nanostructures
Gabriel Dutier
Atom interferometry, atom-surface interactions

Chromium dipolar gases
Laurent Vernac
Dipolar BEC
Magnetism
Fermi gases

Strontium (project)
Martin Robert-de-Saint-Vincent
Magnetism
New cooling mechanisms
New measurement procedures

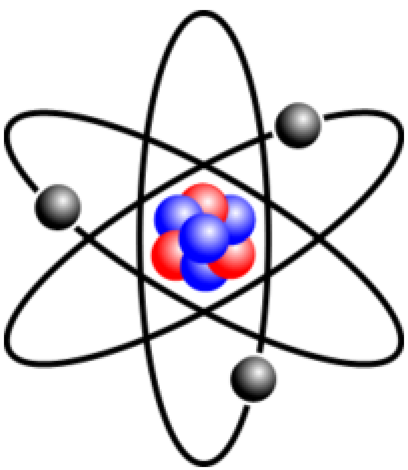
I Quantum Gases to explore many-body physics

From elementary particles…

electrons

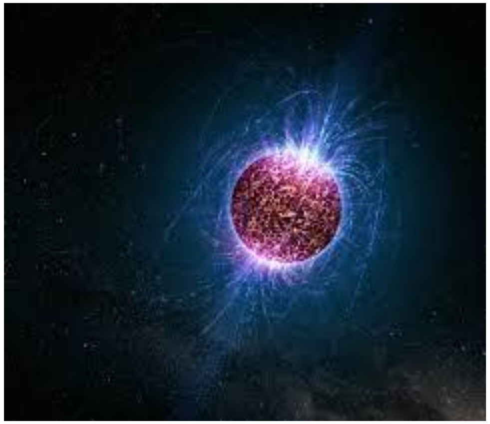
photons …

Quantum Mechanics



…to structured matter

Atoms, liquids, solids, …



Collective states, correlations, Quantum vs Classical physics

From individual particles atoms…

atoms

Quantum Mechanics

…to structured matter

Collective states, correlations, Quantum vs Classical physics

Quantum Gases

(Here, « elementary particles » are atoms)

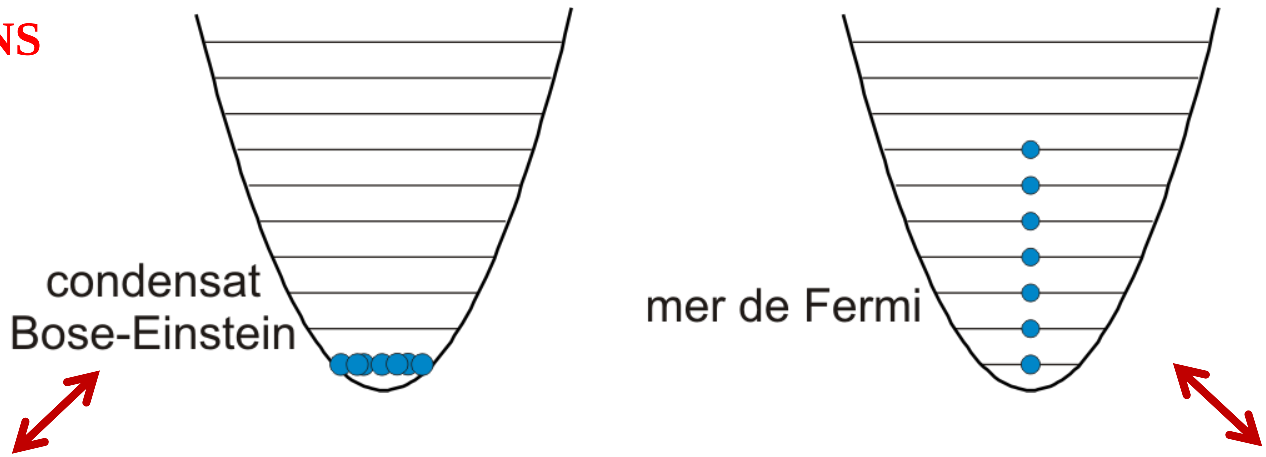
Density : 10^{12} to 10^{15} at/cm^3 ($\leftrightarrow 10^{22}$ at/cm^3 for liquid He)

Temperature : 1 nK to 1µK

De Broglie wavelength > 100 nm $\Delta x . \Delta p > \hbar / 2$

Distance between atoms > 100 nm

BOSONS **FERMIONS**



Lasers

(Bose-stimulation)

Electrons in solids

(Pauli-exclusion principle)

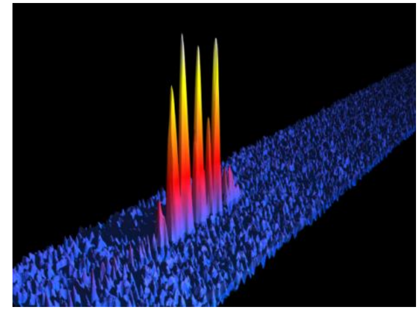
Collective behaviour even without interactions

Effect of interactions on condensates, cold atoms *vs* condensed matter

Attractive interactions

Implosion of BEC for large atom number

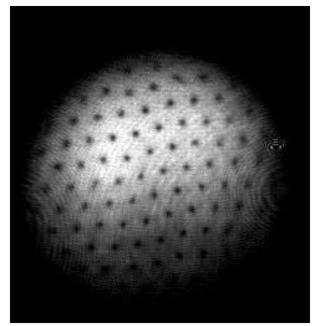
Small solitons



ENS,
Rice…

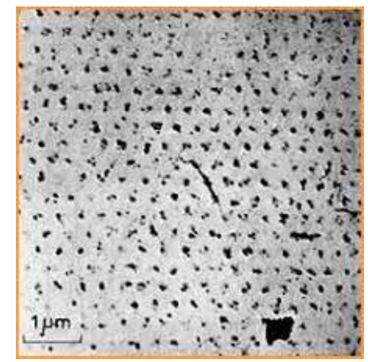
Repulsive interactions

Stable condensate
Phonon spectrum



Superfluidity

ENS, JILA…



Abrikosov lattice in type II superconductors

Spin dependent interactions

Magnetism

ENS, Berkeley…

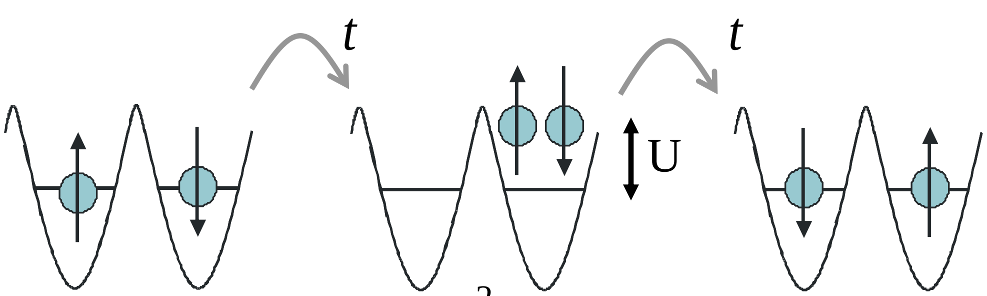
Classical vs Quantum Magnetism

Classical Magnetism
Interaction between magnetic domains through dipolar interactions
(Spintronics)

Bloch precession equation

$$\frac{d\vec{\mu}}{dt} = \gamma\vec{\mu}\times\left(\vec{B} + \sum \vec{B}_j\right)$$

Condensed-matter: effective spin-spin interactions arise due to exchange interactions



$$\Gamma \propto \frac{t^2}{U}$$

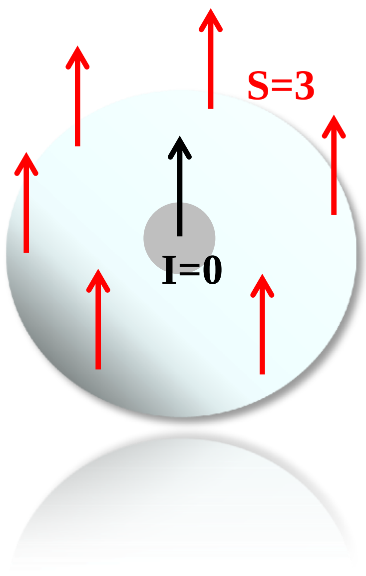
$$S_{1z}S_{2z} + \frac{1}{2}\left(S_{1+}S_{2-} + S_{1-}S_{2+}\right)$$

Ising Exchange

Describes materials with strong correlations
Quantum Magnetism

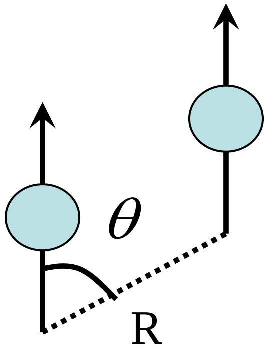
Heisenberg model of magnetism
(real spins, effective spin-spin interaction)

"Our" Magnetism



Dipole-dipole interactions

$$V_{dd} = \frac{\mu_0}{4\pi} S^2 (g_J \mu_B)^2 \left(1 - 3\cos^2(\theta)\right) \frac{1}{R^3}$$



Long range

Anisotropic

Unusually large dipolar interactions due to large electronic spin

Van-der-Waals (contact) interactions

$$V(R) = -\frac{C_6}{R^6} \longrightarrow V(R) = \frac{4\pi\hbar^2}{m} a_S \delta(R)$$

Short range

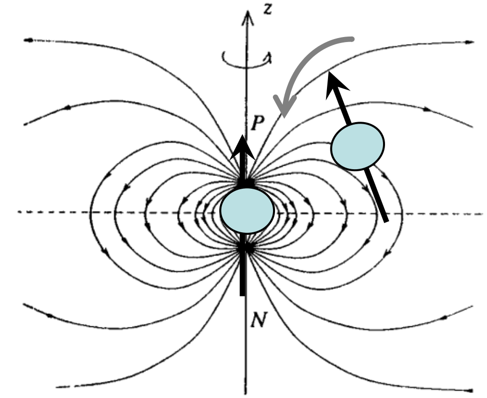
Isotropic

(only few experiments worldwide with non-negligible dipolar interactions - Stuttgart, Villetaneuse, Innsbruck, Stanford, Boulder, Boston, Hong-Kong,…)

“Our” Magnetism

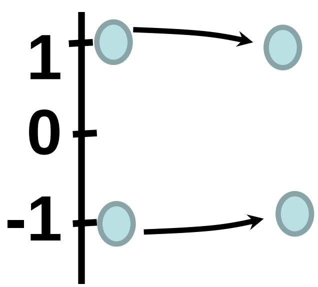
Dipole-dipole interactions

$$V_{dd} = \frac{\mu_0}{4\pi} S^2 \left(g_J \mu_B\right)^2 \left(1 - 3\cos^2(\theta)\right) \frac{1}{R^3}$$

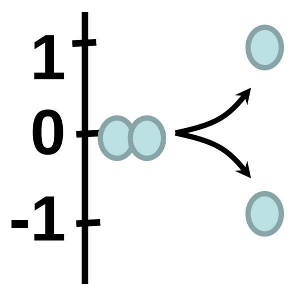


$$S_{1z} S_{2z} - \frac{1}{4}\left(S_{1+} S_{2-} + S_{1-} S_{2+}\right)$$

↔ **NMR Secular hamiltonian**
≠ **Heisenberg**

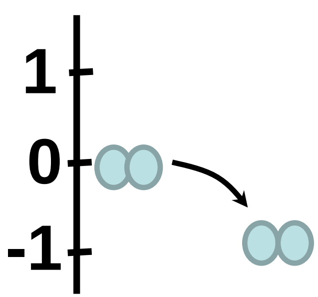


Ising



Exchange

+ Magnetization-changing collisions $S_{1-} S_{2-}$



Anisotropy $\left(1 - 3\cos^2\theta\right)$

Long Range $1/R^3$

Large Spin
(i.e. s>1/2)

I Quantum Gases to explore many-body physics

II How to produce quantum gases

Trapping neutral atoms

No electric charge → More difficult than trapping ions

Permanent magnetic moment

$\mu_B B(r)$ → **Few K using superconducting magnets**

(Larmor shift)

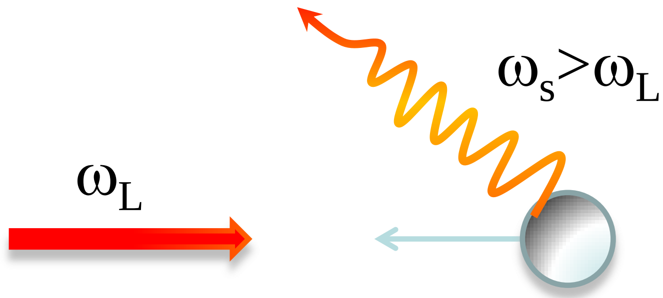
(induced) electric dipole moment

$dE(r)$ → **Few mK using most powerful lasers**

(AC-Stark shift)

Cooling atoms

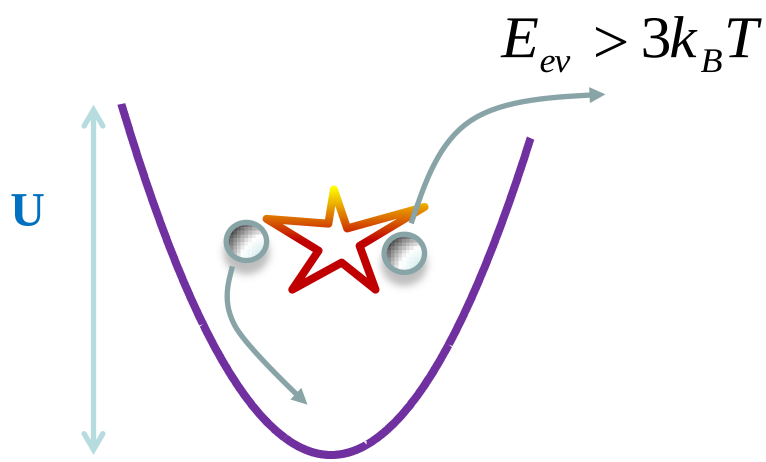
Laser cooling



Temperature limit:

Doppler,
Recoil,
1-100 μK

Evaporative cooling



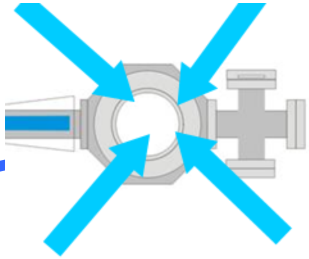
Temperature limit:

T~U/10
1-100 nK

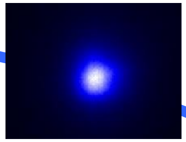
How to produce a quantum degenerate gase (20 s) ?

oven

(100 to 1500 °C)



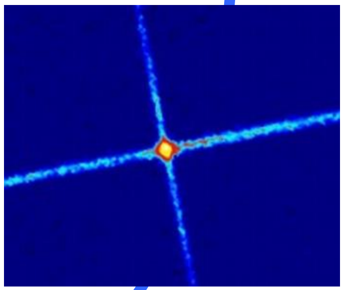
Magneto-optical trap



N = 10^6 to 10^{10}

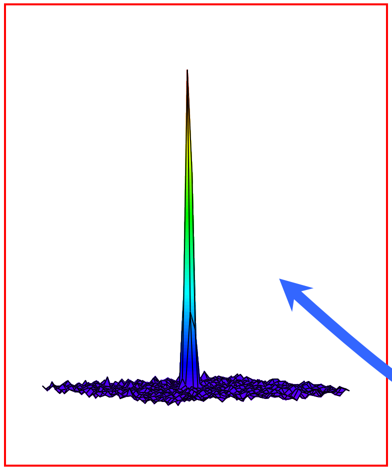
T=120 μK

Many lasers
Ultra-vacuum : 10^{-11} mbar
Building: 2 ans
Life-time: >10 ans
1 M€
Teams~ 5 people

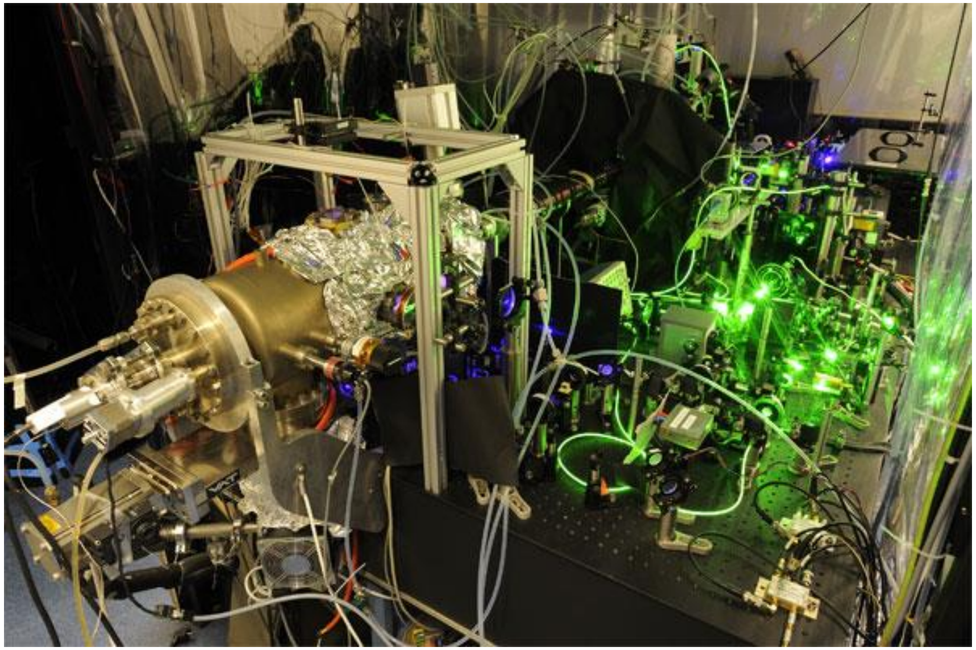
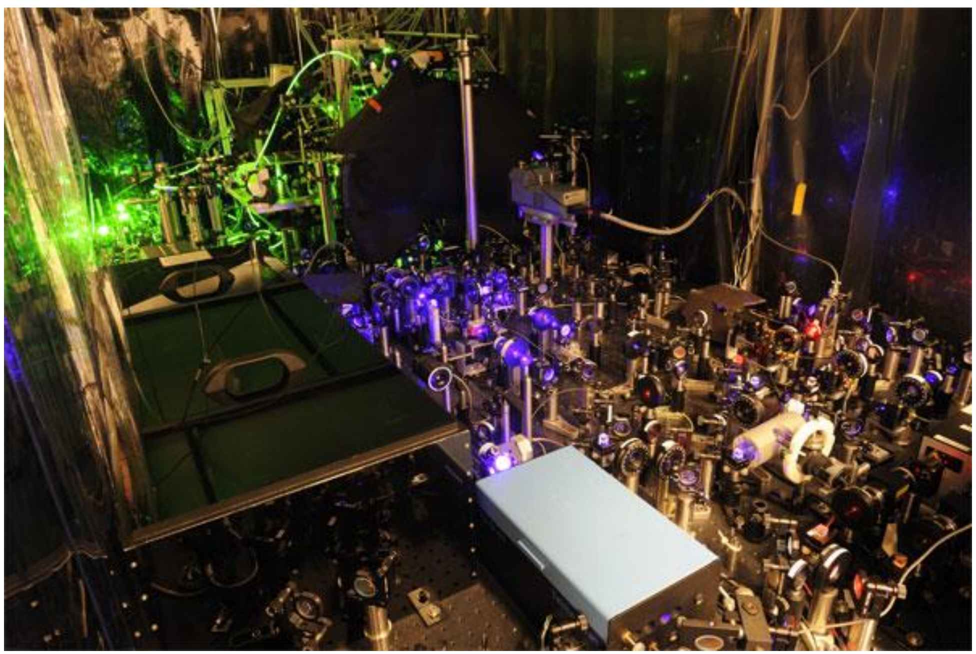
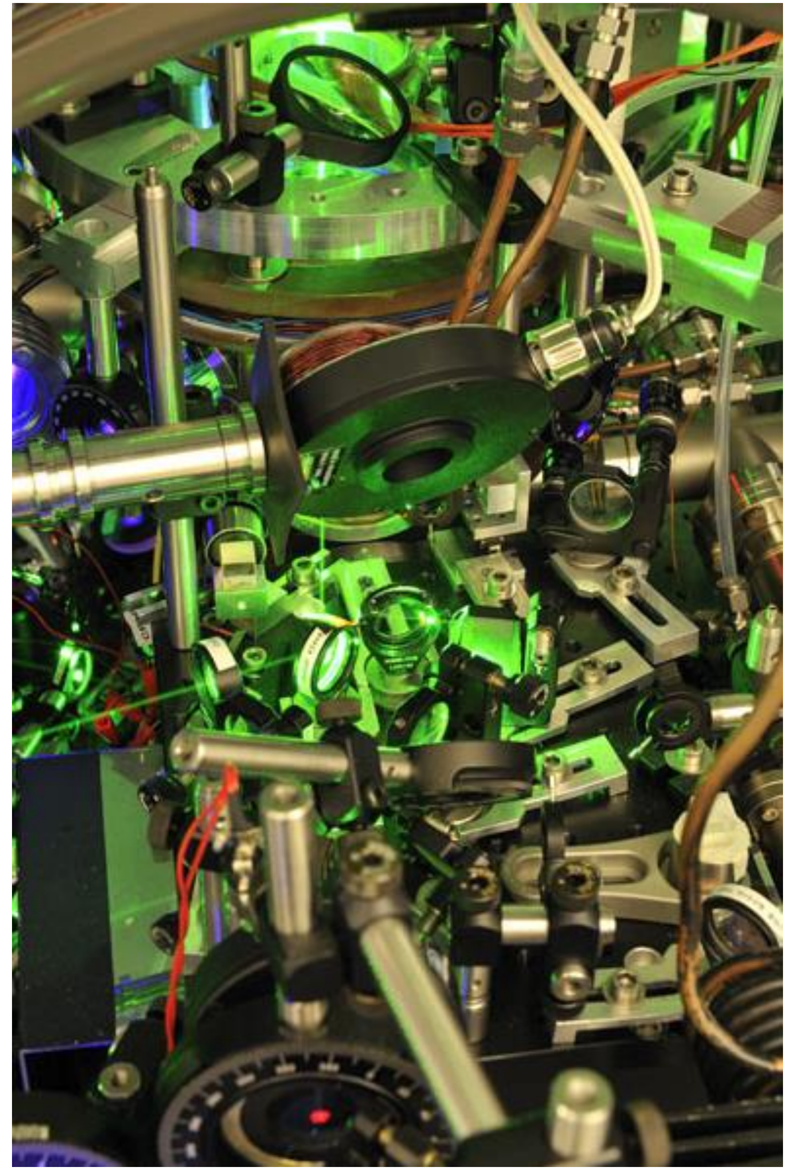


- Crossed dipole (or magnetic) trap

- Evaporative cooling



BEC, Fermi sea

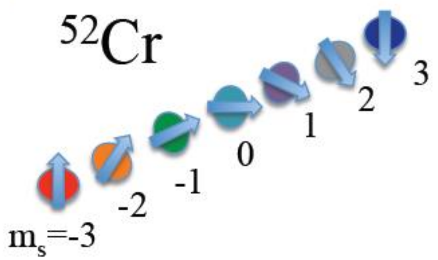
I Quantum Gases to explore many-body physics

II How to produce quantum gases

III Large Spin magnetism with dipolar atoms

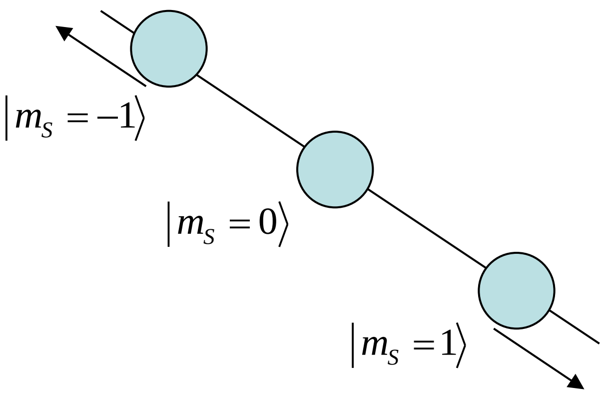
Large spin magnetism

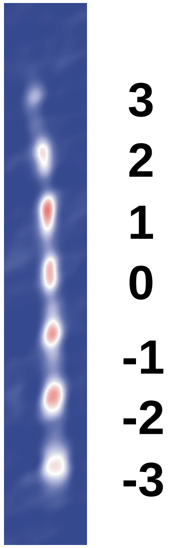
Optical dipole traps equally trap all Zeeman state of a same atom (AC Stark shift)



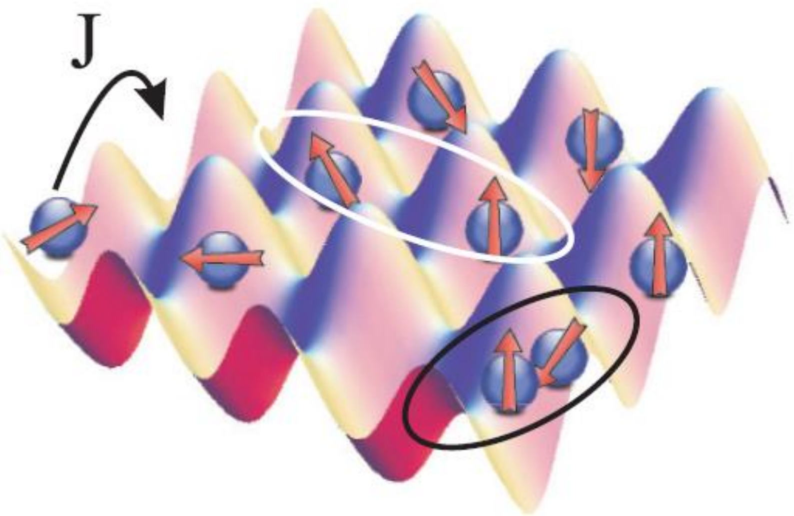
How to measure?

Stern-Gerlach separation:
(magnetic field gradient)





This Experiment

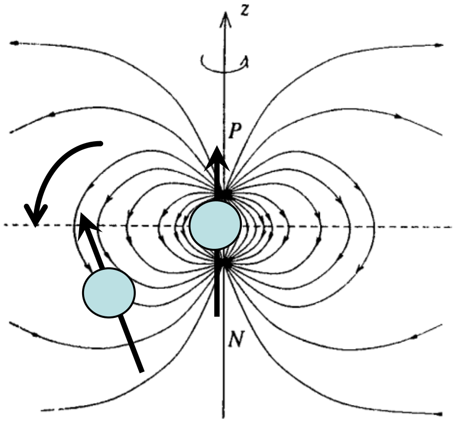


I – Excite the spins

II – Free evolution under the effect of interactions

Question: Under which conditions correlations develop?

Under which conditions correlations develop? – Classical vs Quantum magnetism

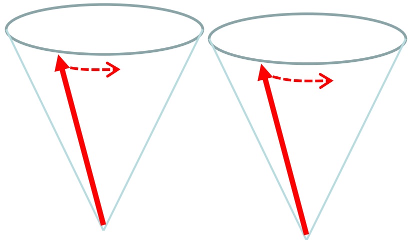


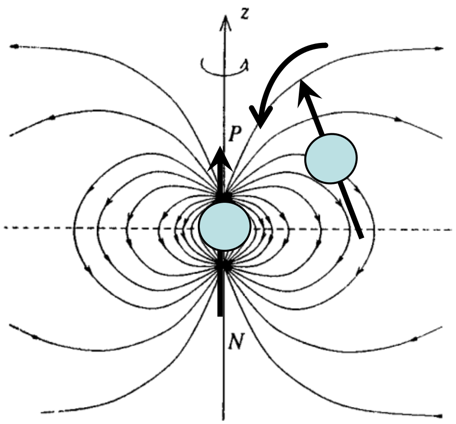
$$\vec{B}_A(B) = \vec{B}_B(A)$$

Classically:

These two atoms undergo identical precession

Total spin conserved

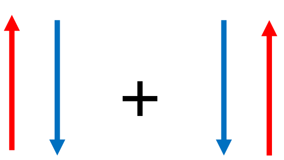




Quantum-mechanically:

Possibility for entanglement

Total spin is NOT conserved

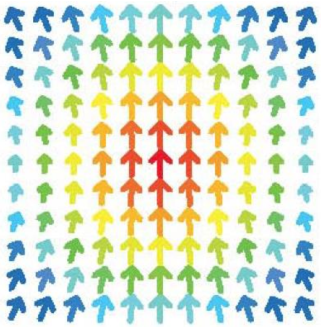


From 2 to N atoms

$$\frac{dS}{dt} = \frac{i}{\hbar}[H,S] \xrightarrow{S \times S = i\hbar S} \frac{dS}{dt} = \gamma S \times B$$

$$\frac{dS}{dt} = \gamma S \times (B + \sum_j B_j)$$

$\vec{B}_j$ **Field created by atom j**



$$\frac{d\langle S\rangle}{dt} = \gamma \left\langle S \times (B + \sum_j B_j) \right\rangle$$
$$= \gamma \langle S\rangle \times (B + \sum_j \langle B_j\rangle)$$

Neglect correlations (HUGE simplification !) ↔ classical

$$\Gamma(r) = \int dr' V_{dd}(r-r')n(r')$$

Mean-field interaction ↔
Bloch equation ↔
Gross-Pitaevskii equation

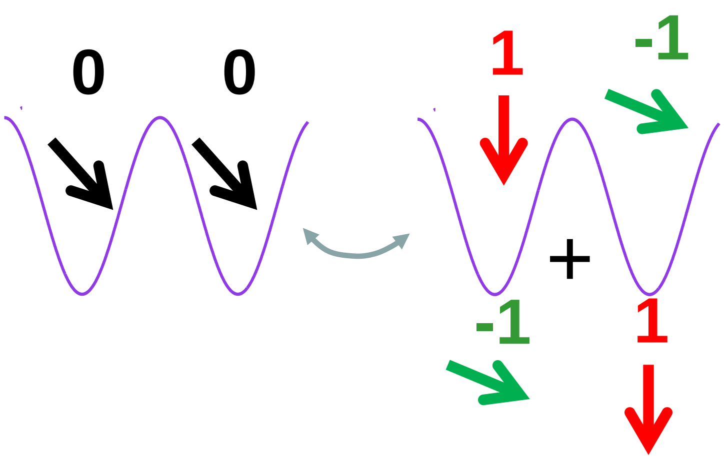
From classical to quantum

$$\left(S_{1z}.S_{2z} - \frac{1}{4}(S_{1+}S_{2-} + S_{1-}S_{2+}) \right)\left(1 - 3z^2\right)$$

Example: one atom in each lattice site in on Zeeman state m_s=0

$$|m_s = 0, m_s = 0\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|m_s = 1, m_s = -1\rangle + |m_s = -1, m_s = 1\rangle\right)$$

Entanglement ! $\Psi_{AB} \neq \Psi_A \otimes \Psi_B$



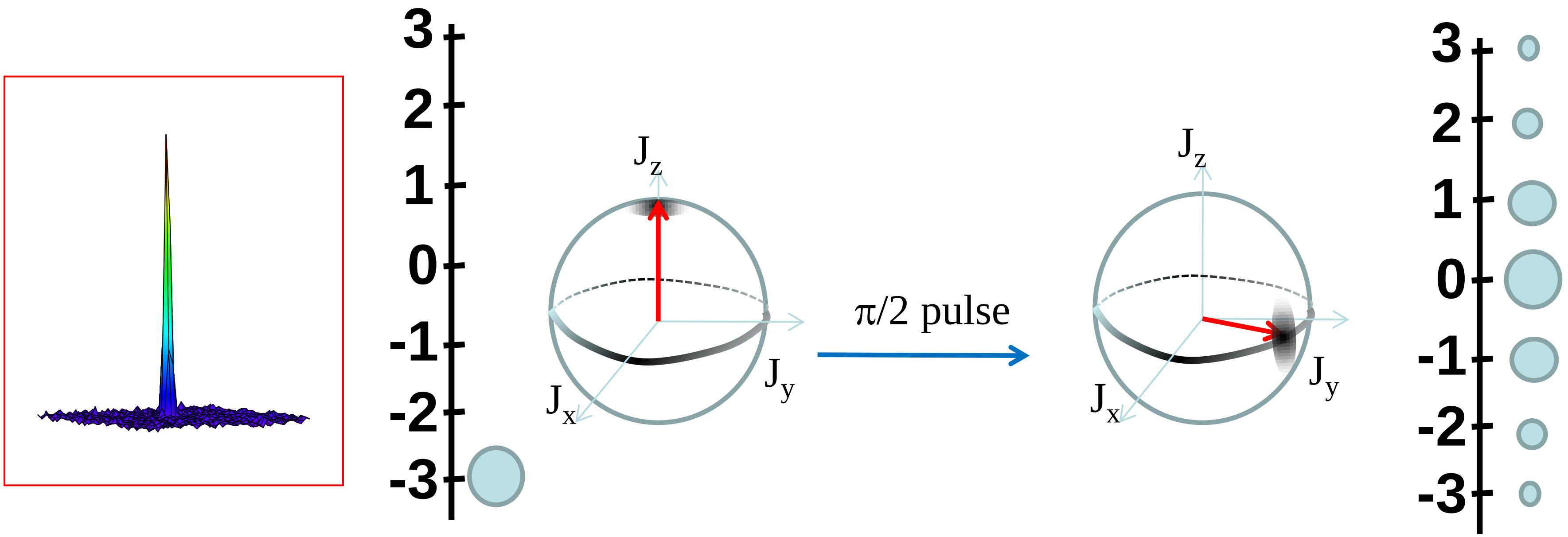
I Quantum Gases to explore many-body physics

II How to produce quantum gases

III Large Spin magnetism with dipolar atoms

IV Experimental results

Experimental protocol

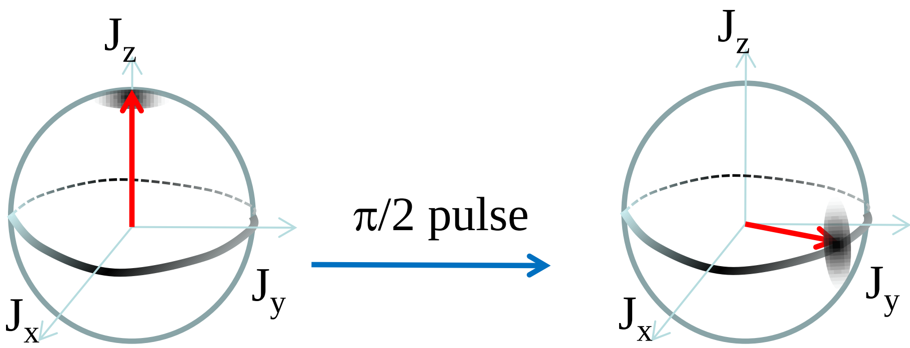


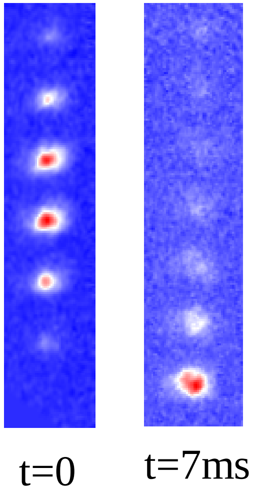
I – Excite the spins

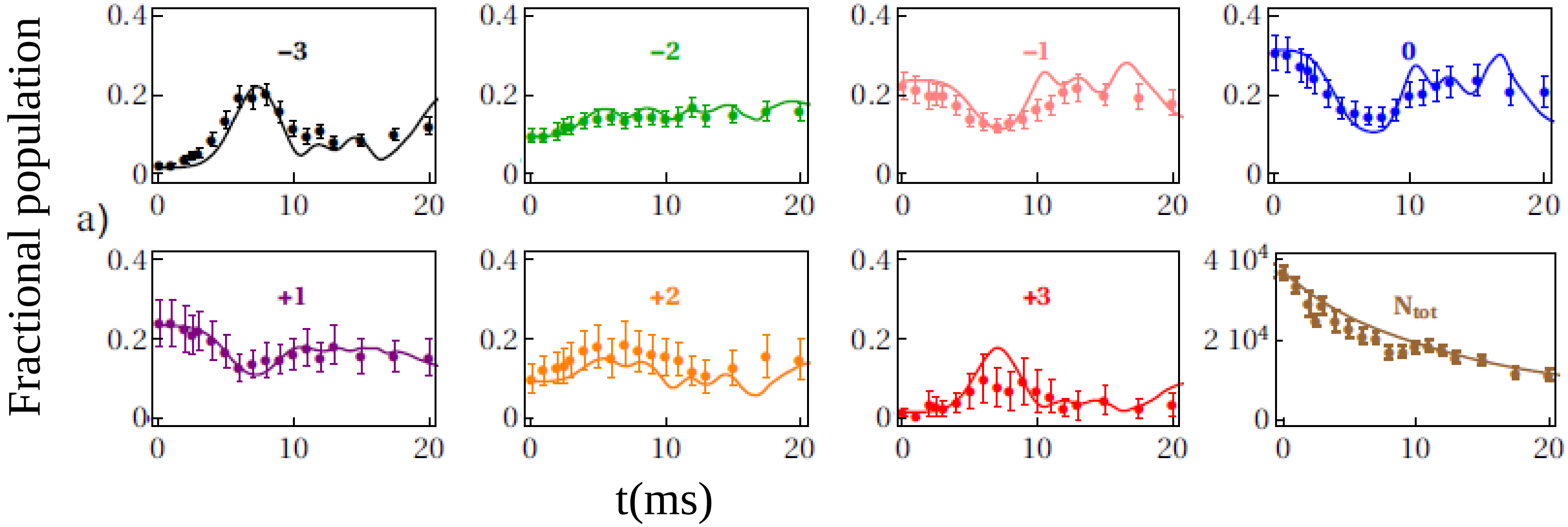
II – Free evolution under the effect of interactions

(here, both contact and dipole-dipole interactions contribute)

BEC case: Trigger spin dynamics using magnetic field gradients







Dynamics is triggered by the existence of a B-field gradient

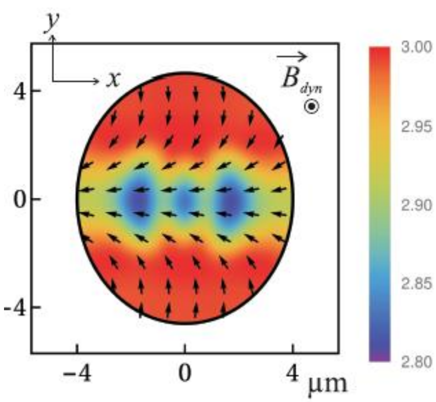
GP- Theory : Pedri/Kechadi Zhu/Rey

Dynamics reproduced by mean-field theory!

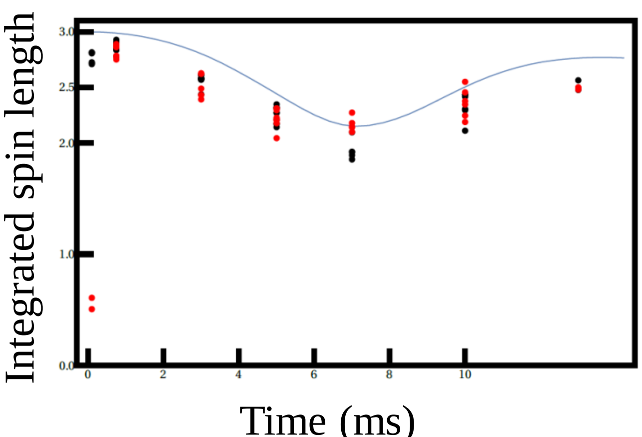
Simplification of the theoretical description

$$
\begin{aligned}
H = & \int d^3\mathbf{r}\left(\mathbf{\Psi}^\dagger \hat{H}_0 \mathbf{\Psi} + \mu_B g \left|\boldsymbol{B}_{\text{dyn}}\right| S^Z(\mathbf{r}) + \frac{c_0}{2}|n(\mathbf{r})|^2\right) \\
& + \int d^3\mathbf{r}\left(\frac{c_1}{2}|\mathbf{S}(\mathbf{r})|^2 + \frac{c_2}{2}|A_{00}(\mathbf{r})|^2 + \frac{c_3}{2}\sum_{M=-2}^{2}|A_{2M}|^2\right) \\
& + \frac{c_{dd}}{2}\int d^3\mathbf{r}d^3\mathbf{r}'\frac{1-3(\hat{e}\cdot\hat{u}_B)^2}{|\mathbf{r}-\mathbf{r}'|^3}\Big[S^Z(\mathbf{r})S^Z(\mathbf{r}') \\
& - \frac{1}{2}\left(S^X(\mathbf{r})S^X(\mathbf{r}') + S^Y(\mathbf{r})S^Y(\mathbf{r}')\right)\Big] - i\Gamma \qquad (1)
\end{aligned}
$$

Gross-Piaevskii equation (already a HUGE simplification due to mean-field approximation)



Spins remain locally polarized



$$
\frac{\partial \vec{f}}{\partial t} = -\vec{f} \times \left[-\frac{\hbar}{2m}\left(\frac{\nabla n}{n}.\nabla\right).\vec{f} - \frac{\hbar}{2m}\nabla^2 \vec{f} + \vec{B}\right]
$$

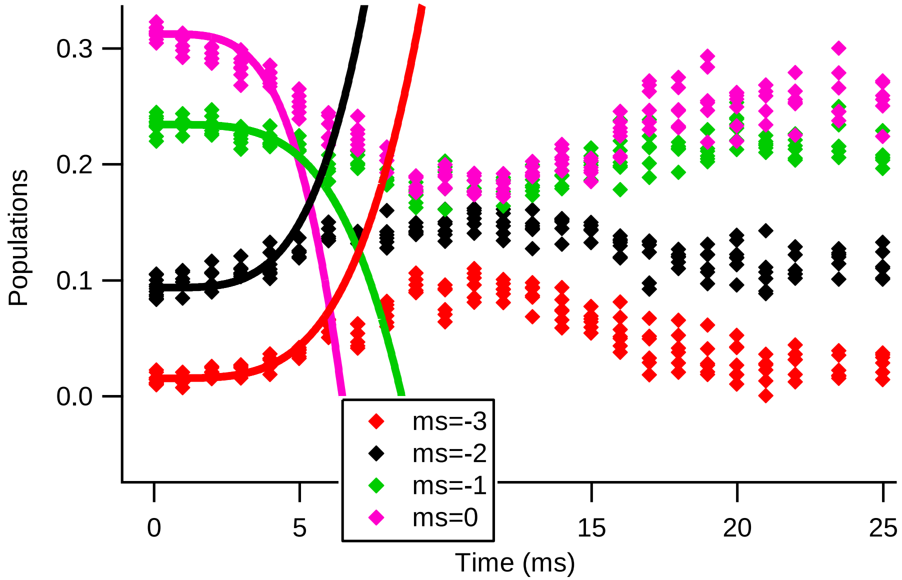
Hydrodynamic equations of a ferrofluid

A very simple description of a complex system

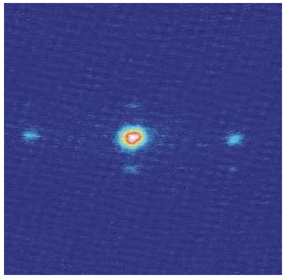
$$\frac{p_{m_s}(t)}{p_{m_s}(0)} = 1 + \left(\frac{g\mu_B b}{2Mw}\right)^2 \left(m_s^2 - \sum_{m_{s'}} m_{s'}^2 p_{m'_s}(0)\right) t^4$$

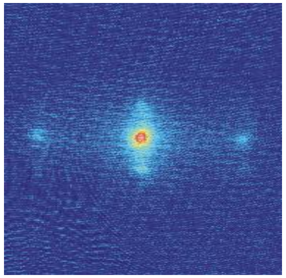
(in practice independent of spin-dependent interactions)

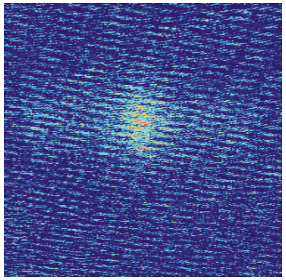
All the spin-dependent interactions do is undo whatever population imbalance the magnetic field gradient creates !



A ^{52}Cr BEC in a 3D optical lattice



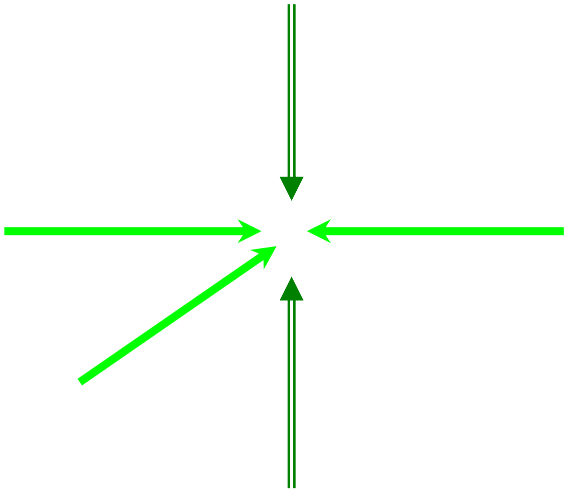


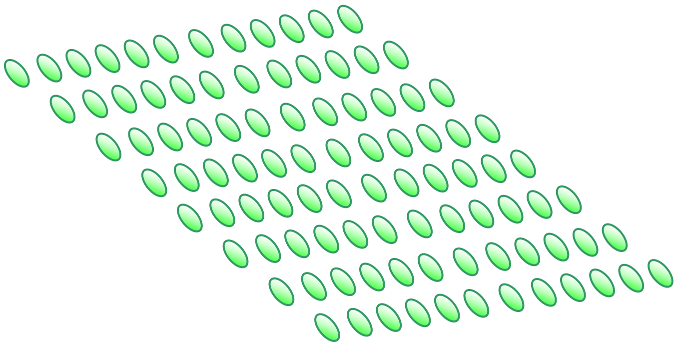


Optical lattice: Perdiodic potential made by a standing wave

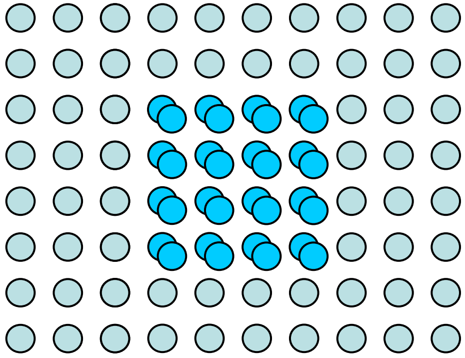
Our lattice architecture:

(Horizontal 3-beam lattice) x (Vertical retro-reflected lattice)



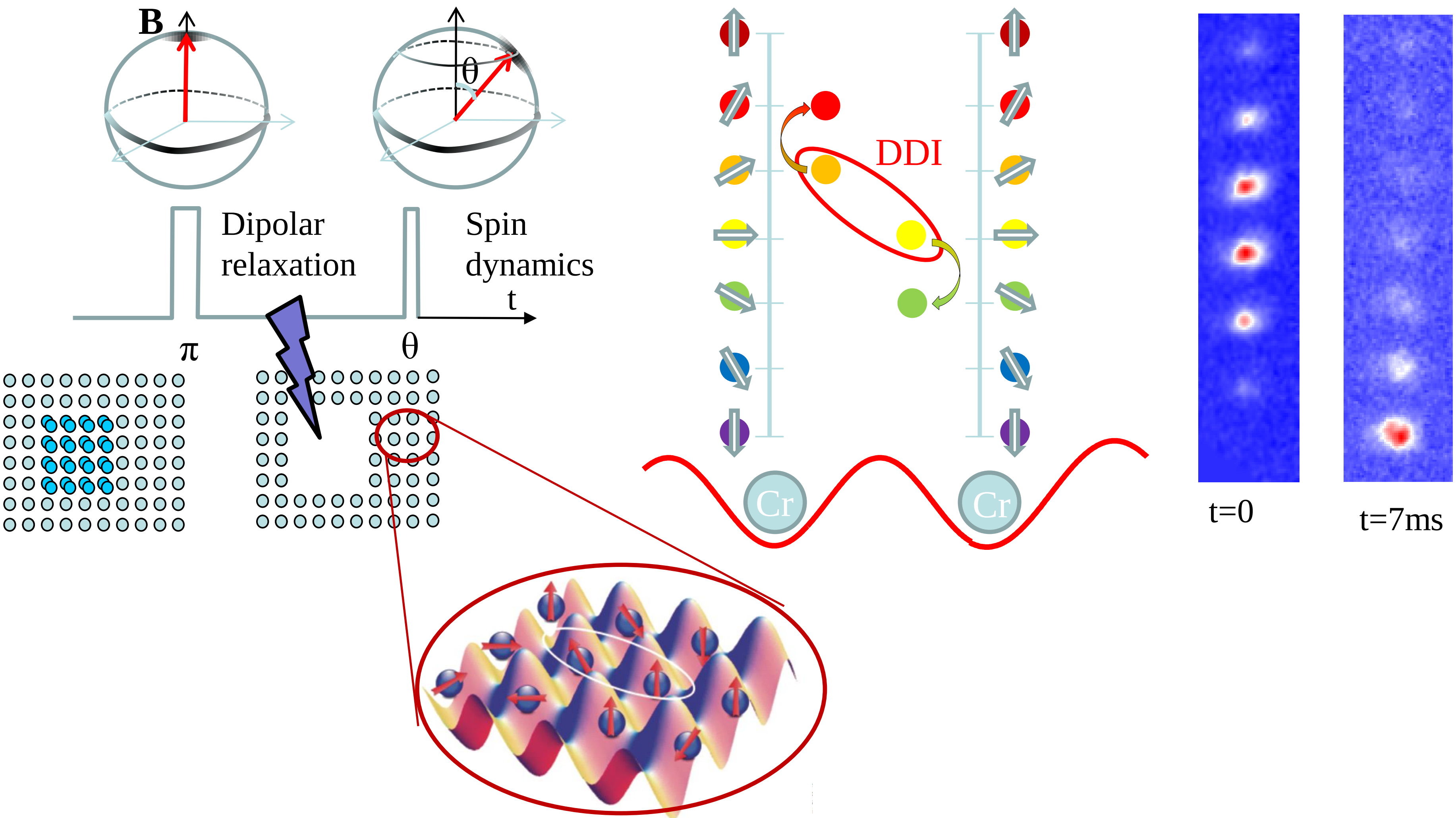


Rectangular lattice of anisotropic sites

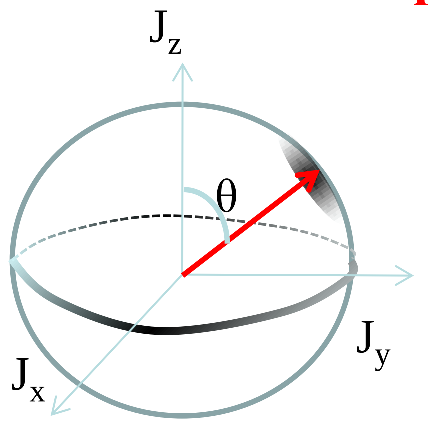


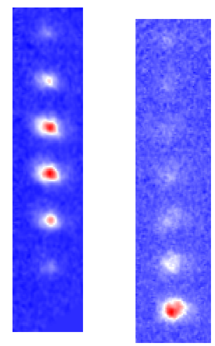
3D lattice → Strong correlations, Mott transition…

Experimental protocol, lattice case

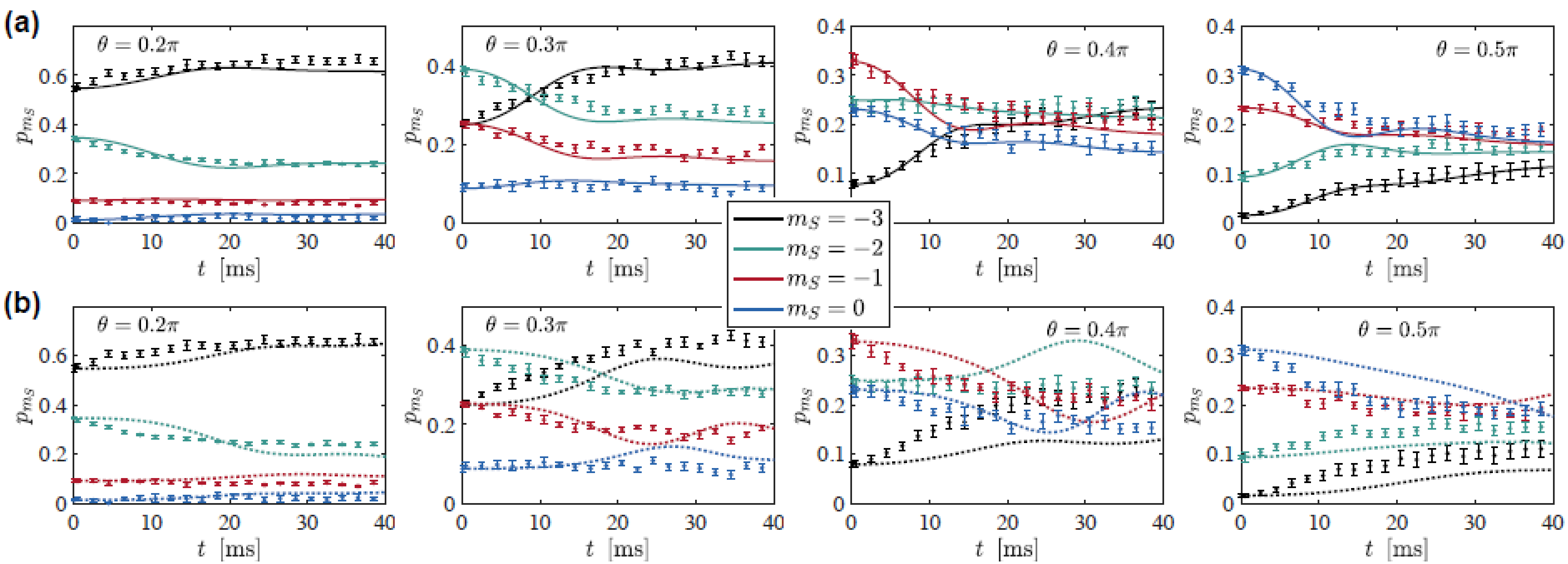


Experimental results, lattice case…





Quantum theory
Schachenmayer/Rey



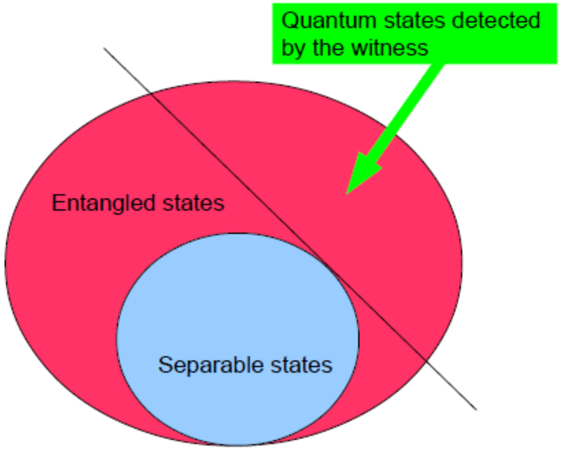
Classical theory

Increasing quantum-ness is seen

How to *experimentally* characterize the growth of entanglement ?

Entanglement witness based on measurements of global spin variables.

(e.g. $(\Delta S_x)^2 + (\Delta S_y)^2 + (\Delta S_z)^2 \geq S$ for any mixture of separable states)



Another possibility: measure the **entropy associated to entanglement**

Basic idea

$$\frac{1}{\sqrt{2}}\left(|m_s = 1, m_s = -1\rangle + |m_s = -1, m_s = 1\rangle\right)$$

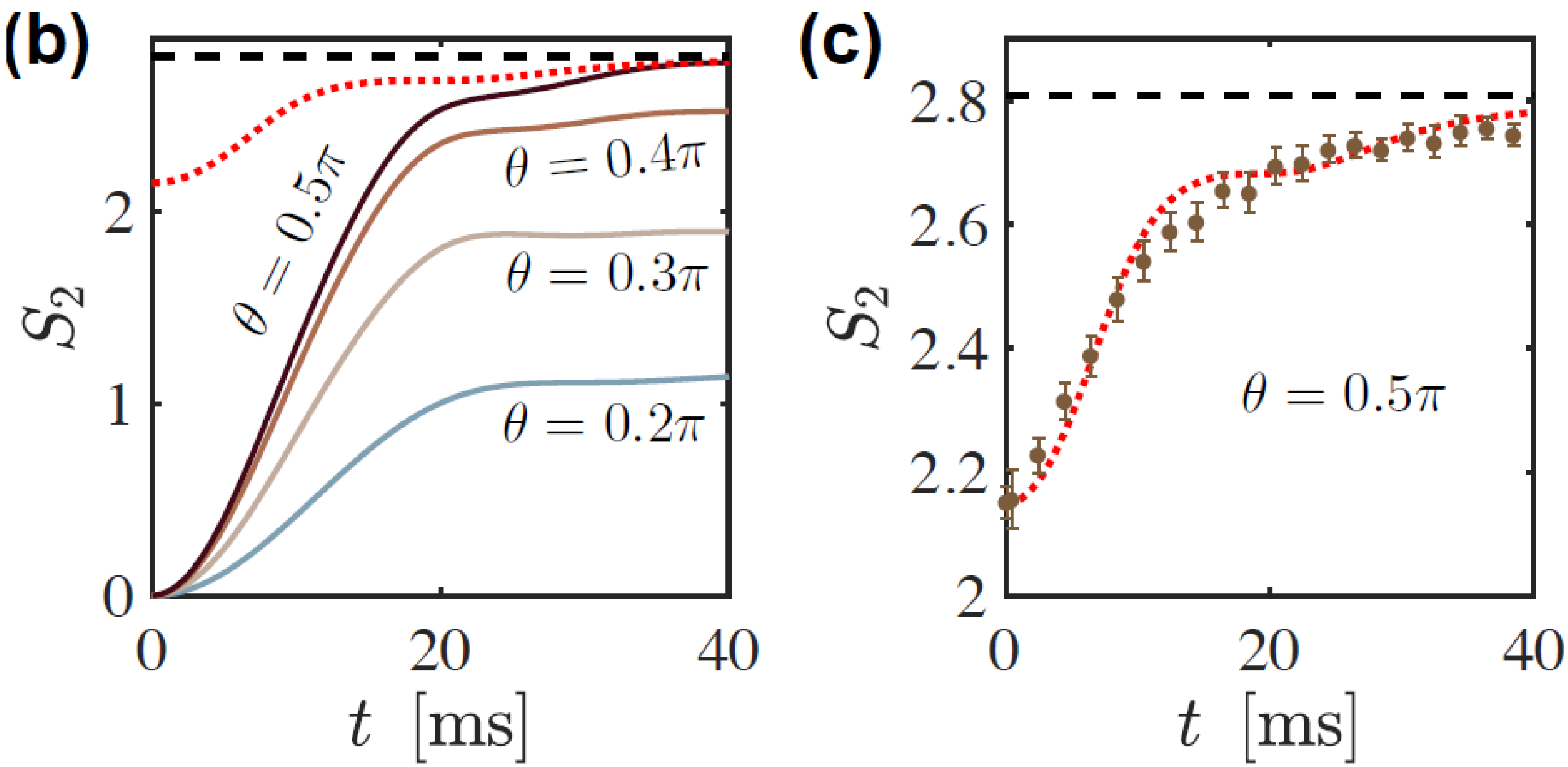
= PURE STATE

But measurement performed in just one lattice site

will show random fluctuations ($|m_s = \pm 1\rangle$)

→ associated entropy

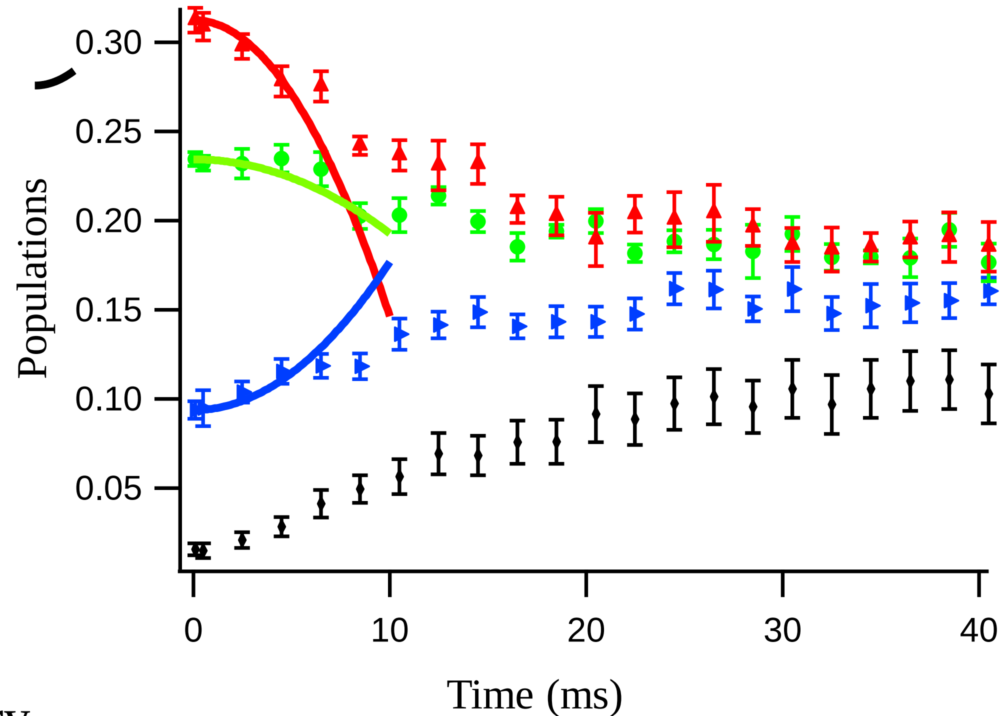
Calculated growth of entanglement

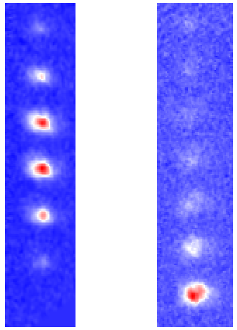


Renyi entanglement entropy (NOT measured here)

Measured diagonal entropy (NOT a proof of entanglement !)

A simple equation for short term dynamics





Perturbation theory

$$\langle A(t)\rangle = \langle A\rangle - it\langle [A,H]\rangle - \frac{t^2}{2}\langle [[A,H],H]\rangle + \dots \longrightarrow \Gamma = \sqrt{\sum_{(i,j)} \left(V_{(i,j)}{}^2\right)}$$

$$p_{m_s}(t) = p_{m_s}(0) + \alpha_m \sum_i V_{dd}^2(\vec{r}_i)t^2$$

$$\alpha_m = 135/512(1,2,-1,-4,-1,2,1)$$

Quantum Gases are well suited to describe from first principles many-body quantum physics

Here: the example of quantum gases with dipolar interactions…

…in the regime where theories still keep up with experiments…

S. Lepoutre, L. Gabardos (PhD), B. Naylor (PhD)
B. Laburthe-Tolra, O. Gorceix, E. Maréchal, **L. Vernac**,
M. Robert-de-St-Vincent,
K. Kechadi (PhD), P. Pedri

A. M. Rey, J. Schachenmayer, B. Zhu,
L. Santos, M. Gajda, M. Brewczyk

LPL Laboratoire de physique des lasers

UNIVERSITÉ PARIS 13 NORD

Have left: A. de Paz, A. Chotia, A. Sharma, B. Pasquiou , G. Bismut, M. Efremov, Q. Beaufils, J. C. Keller, T. Zanon, R. Barbé, A. Pouderous, R. Chicireanu

Collaborators: Johnny Huckans, Perola Milman, Rejish Nath

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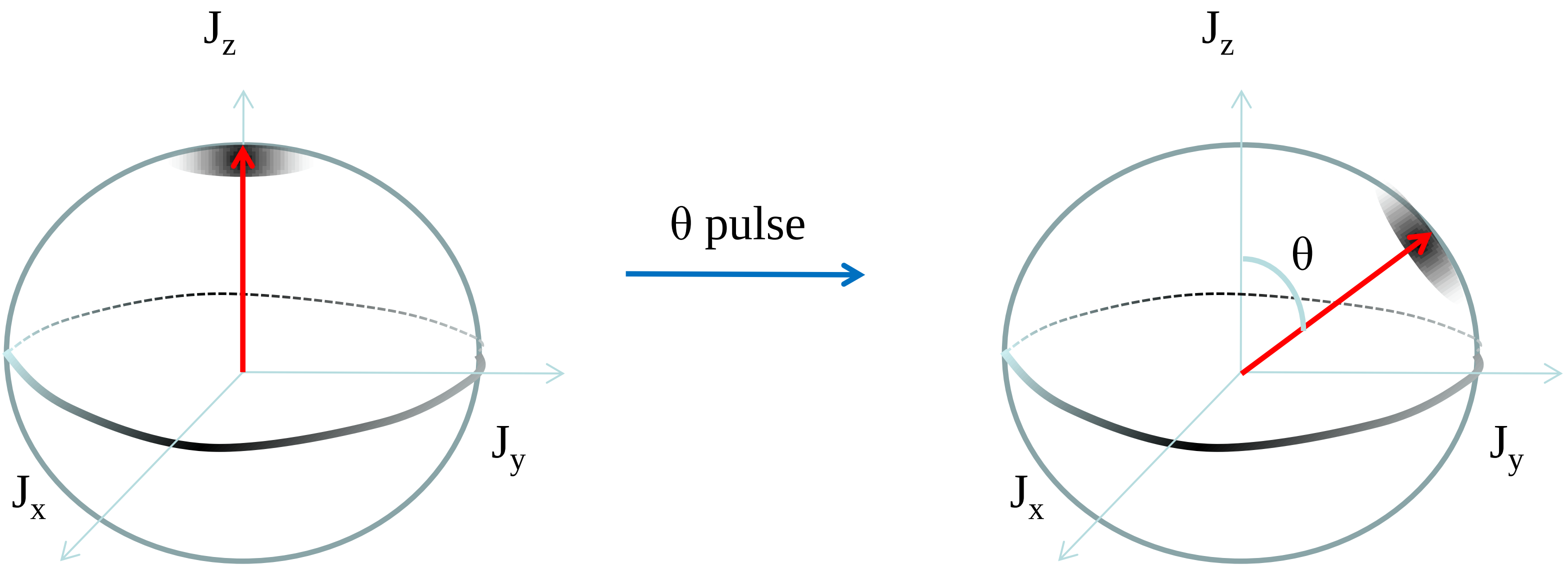
SORBONNE PARIS CITÉ RECHERCHE ENSEIGNEMENT SUPÉRIEUR

cnrs dépasser les frontières

Institut Francilien de Recherche sur les Atomes Froids IFRAF

Dynamics after tilting the spins



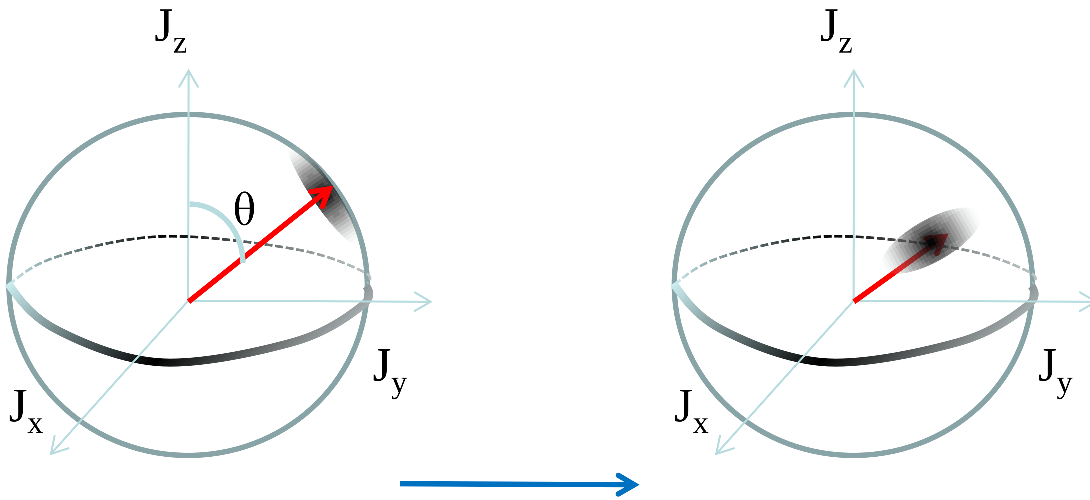
$$S_{1z}S_{2z} - \frac{1}{4}\left(S_{1+}S_{2-} + S_{1-}S_{2+}\right)$$

Prediction (Ana Maria Rey):

θ small → classical precession
θ large → entanglement grows

See also E. Witkowska, PRA 93, 023627 (2016)

After tilting the spin: from classical to quantum



$$S_{1z}S_{2z} - \frac{1}{4}\left(S_{1+}S_{2-} + S_{1-}S_{2+}\right)$$

Interpretation: **dynamics comes from the difference to the Heisenberg Hamiltonian**

$$-\frac{1}{2}\left[S_{1z}S_{2z} + \frac{1}{2}\left(S_{1+}S_{2-} + S_{1-}S_{2+}\right)\right] = -\frac{1}{2}\vec{S}_1.\vec{S}_2$$

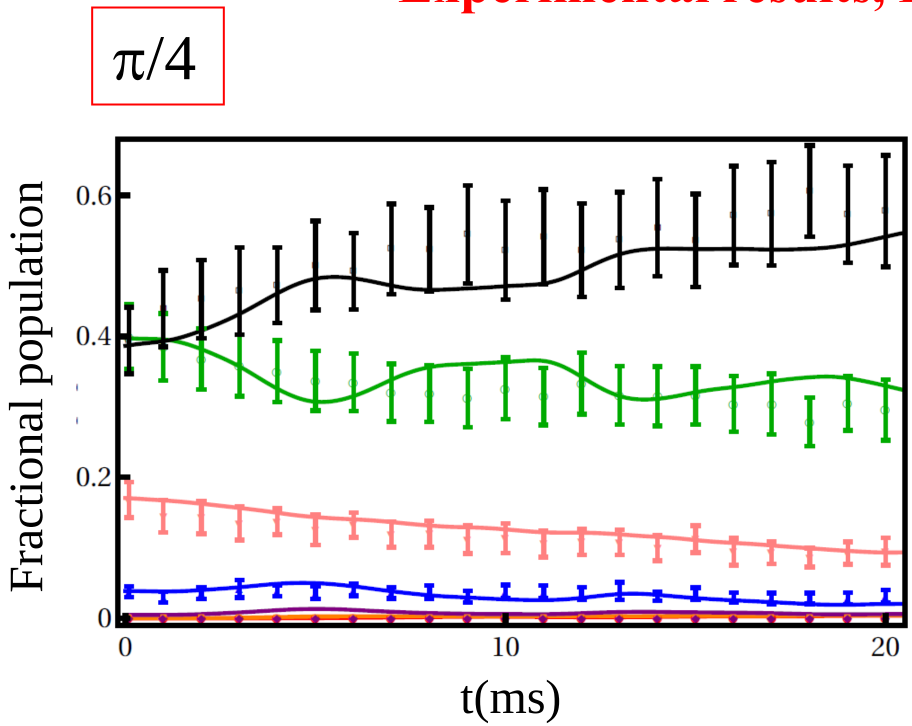
$$\delta H \propto S_{1z}S_{2z} \underset{t\to 0}{\approx} S_z^2$$

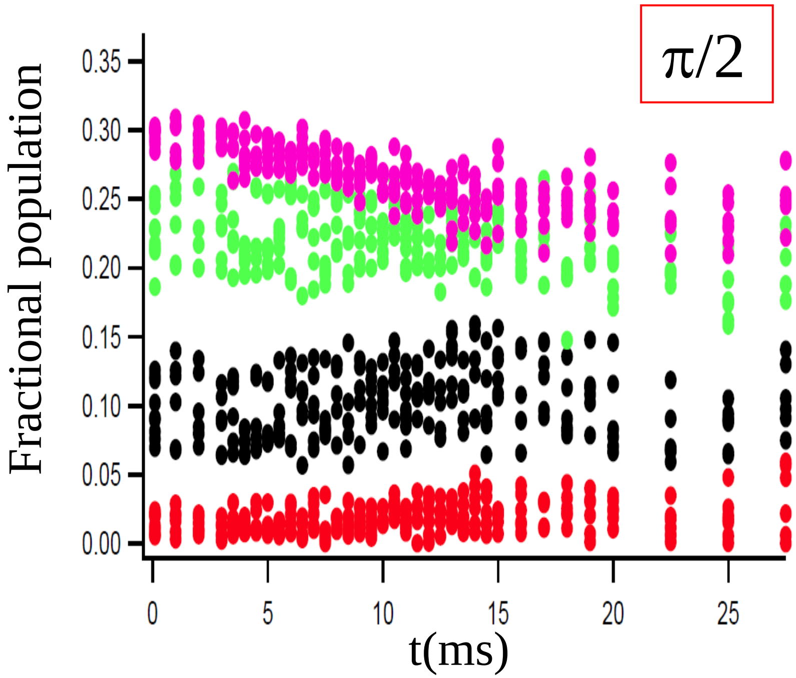
Squeezing ↔ Variance (S_z)

θ small → classical precession

θ large → entanglement grows

Experimental results, BEC case…





Dynamics entirely triggered by dipolar interactions!

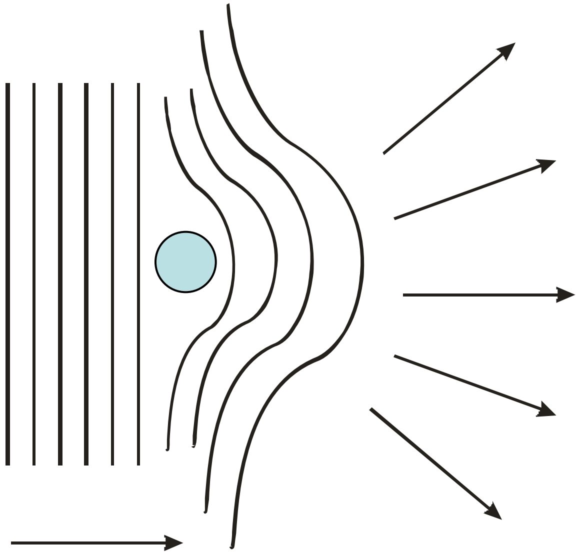
Theory
Pedri/Kechadi
Zhu/Rey

Dynamics vanishes for large angles close to π/2

Beyond mean-field effects too small to be observed ??

Which would be the conditions ? $\left[\sqrt{\sum_{(i,j)}\left(V_{(i,j)}\right)^2} \Leftrightarrow 10ms\right]$

Van-der-Waals interactions between neutral atoms (s orbital): $V(R) = -\frac{C_6}{R^6}$

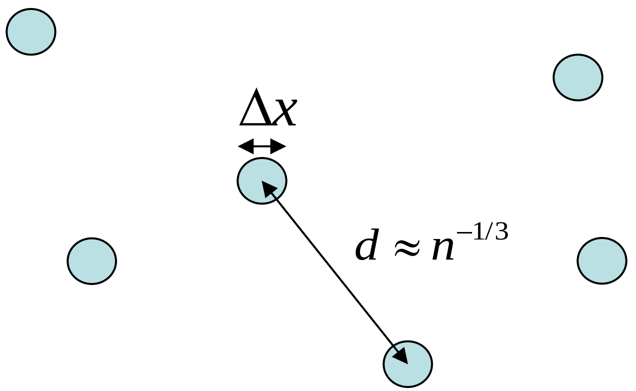


Typical scattering cross-section
between cold atoms

$$\Delta x . \Delta p \approx \hbar \qquad \frac{\Delta p^2}{2m} \approx \frac{\hbar^2}{m(\Delta x)^2} \approx \frac{C_6}{(\Delta x)^6}$$

Potential range >> size of an atom
100 a_0 a_0

Large collision cross-sections
between cold atoms



Potential range << distance between atoms
5 nm 100 nm

$$n(\Delta x)^3 << 1$$

- Polarized fermions do not interact
(→ use mixtures of spin states)

Interactions « de contact », équation de Gross-Pitaevskii

$$V(R) = -\frac{C_6}{R^6} \longrightarrow \boxed{V(R) = \frac{4\pi\hbar^2}{m} a(B)\delta(R)} \qquad a \approx \Delta x$$

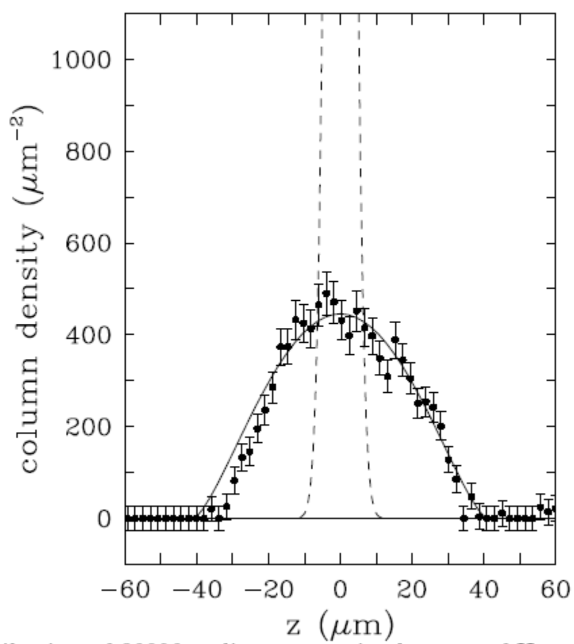
Revient à ne considérer que l'effet des interactions à des distances grandes (devant Δx)

$$\left(-\frac{\hbar^2}{2m}\Delta + V(\vec{R}) + \frac{4\pi\hbar^2 a}{m}\left|\Psi(\vec{R})\right|^2\right)\Psi(\vec{R}) = \mu\Psi(\vec{R}) \qquad na^3 << 1$$

Sans interaction

↓

Profil gaussien



Hau et al., 1998

Avec interaction, régime Thomas Fermi

↓

Profil parabolique

Les interactions dépendent de l'état de spin, la physique des « spinors »

Exemple du spin S=1 (boson)

$$m_S = |-1\rangle, |0\rangle, |1\rangle$$

2 atomes identiques (même état de spin) $\rightarrow$ S=0 ou S=2

$\rightarrow$ deux longueurs de diffusion $a_{S=0}$ ou $a_{S=2}$

Exemple 1 : 2 atomes dans $m_S = |-1\rangle$ $m_{tot} = -2$ $\rightarrow$ S=2 $\rightarrow$ longueur de diffusion a_2

Exemple 2 : 2 atomes dans $m_S = |0\rangle$ $m_{tot} = 0$ $\rightarrow$ S= 0 ou 2

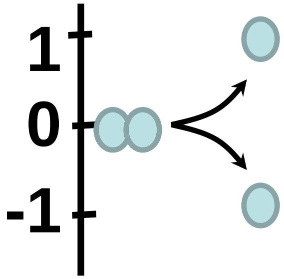
$$|m_S = 0, m_S = 0\rangle = \sqrt{\frac{2}{3}}|S = 2, m_{tot} = 0\rangle - \sqrt{\frac{1}{3}}|S = 0, m_{tot} = 0\rangle$$

Pas un état propre à deux corps !

$$\times \exp\left(i\frac{4\pi\hbar^2 a_2}{m}\frac{t}{\hbar}\right) \qquad \times \exp\left(i\frac{4\pi\hbar^2 a_0}{m}\frac{t}{\hbar}\right)$$

$$|0,0\rangle \leftrightarrow \frac{1}{\sqrt{2}}\left(|1,-1\rangle + |-1,1\rangle\right)$$

$$\hbar\Gamma \propto \left(\frac{4\pi\hbar^2(a_2 - a_0)}{m}\right)$$



“Our” Magnetism

Dipole-dipole interactions

$$V_{dd} = \frac{\mu_0}{4\pi} S^2 \left(g_J \mu_B\right)^2 \left(1 - 3\cos^2(\theta)\right) \frac{1}{R^3}$$

NMR Secular hamiltonian

$$S_{1z} S_{2z} - \frac{1}{4}\left(S_{1+} S_{2-} + S_{1-} S_{2+}\right)$$

Heisenberg hamiltonian

$$S_{1z} S_{2z} + \frac{1}{2}\left(S_{1+} S_{2-} + S_{1-} S_{2+}\right)$$

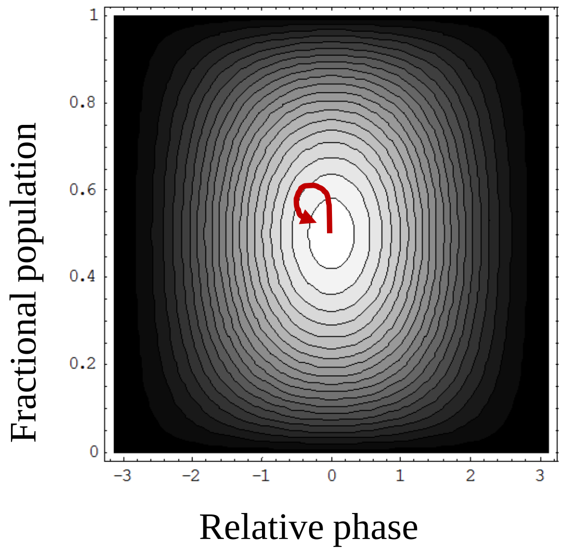
+ Magnetization-changing collisions $S_{1-} S_{2-}$

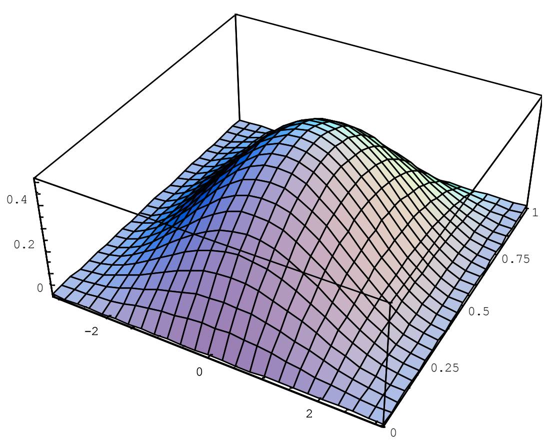
Anisotropy $\left(1 - 3\cos^2\theta\right)$

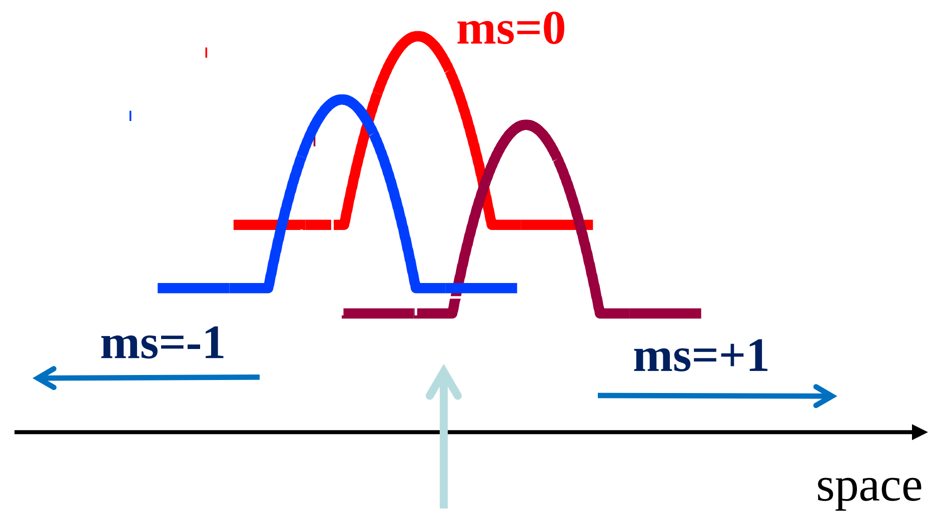
Long Range $1/R^3$

Large Spin

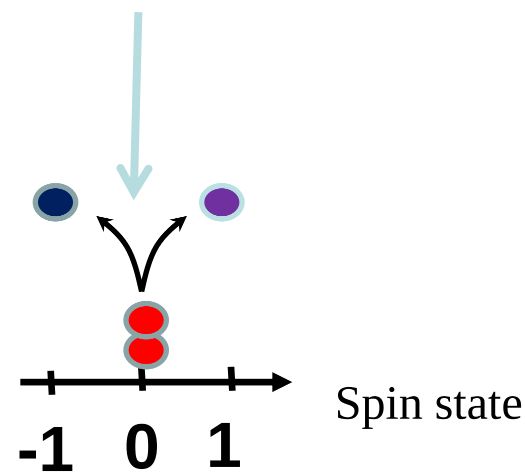
Interpretation: locally, spinor is at a maximum of the interaction energy. Magnetic field gradients cannot change the spinor structure without violating energy conservation







Local structure of spinor
- modified by gradient
- Restaured by spin-exchange

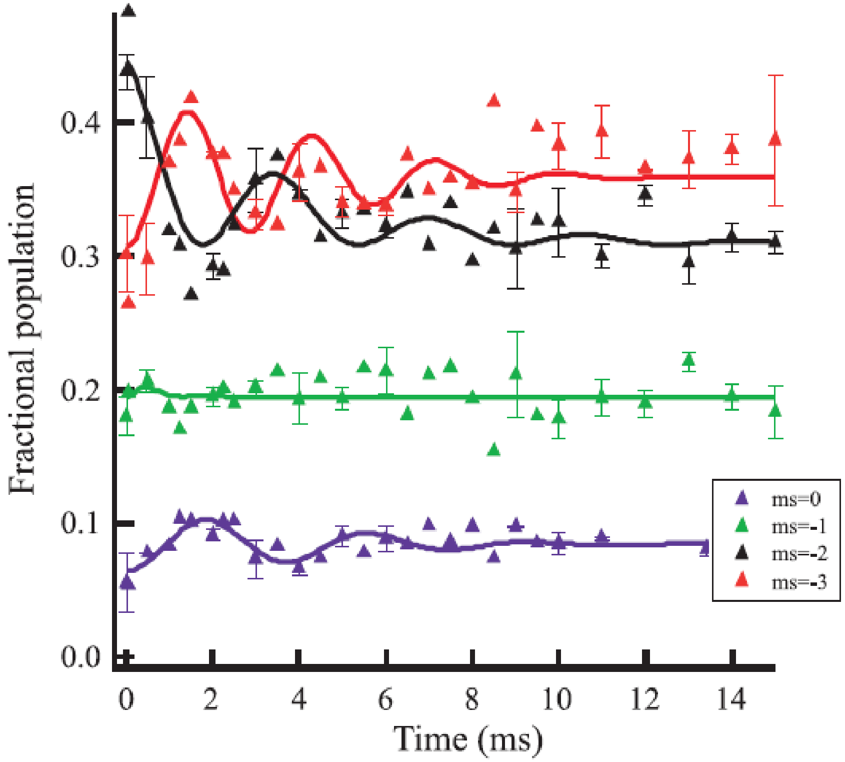


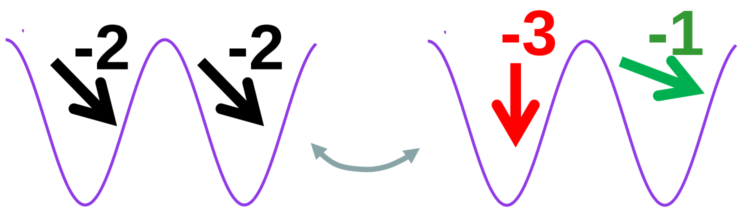
From classical to quantum: dipolar interactions may create correlations

Observation of intersite spin-exchange due to dipolar interactions

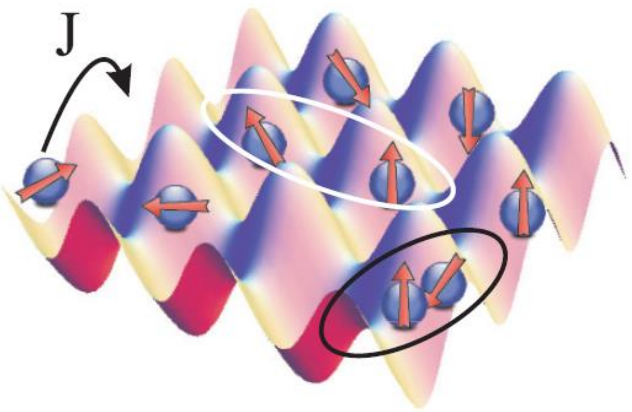
$$\Psi(0) = \left|2,2,\ldots,2,2\right\rangle$$

Start with one atom in each site of a 3D optical lattice in one Zeeman state $m_s=2$

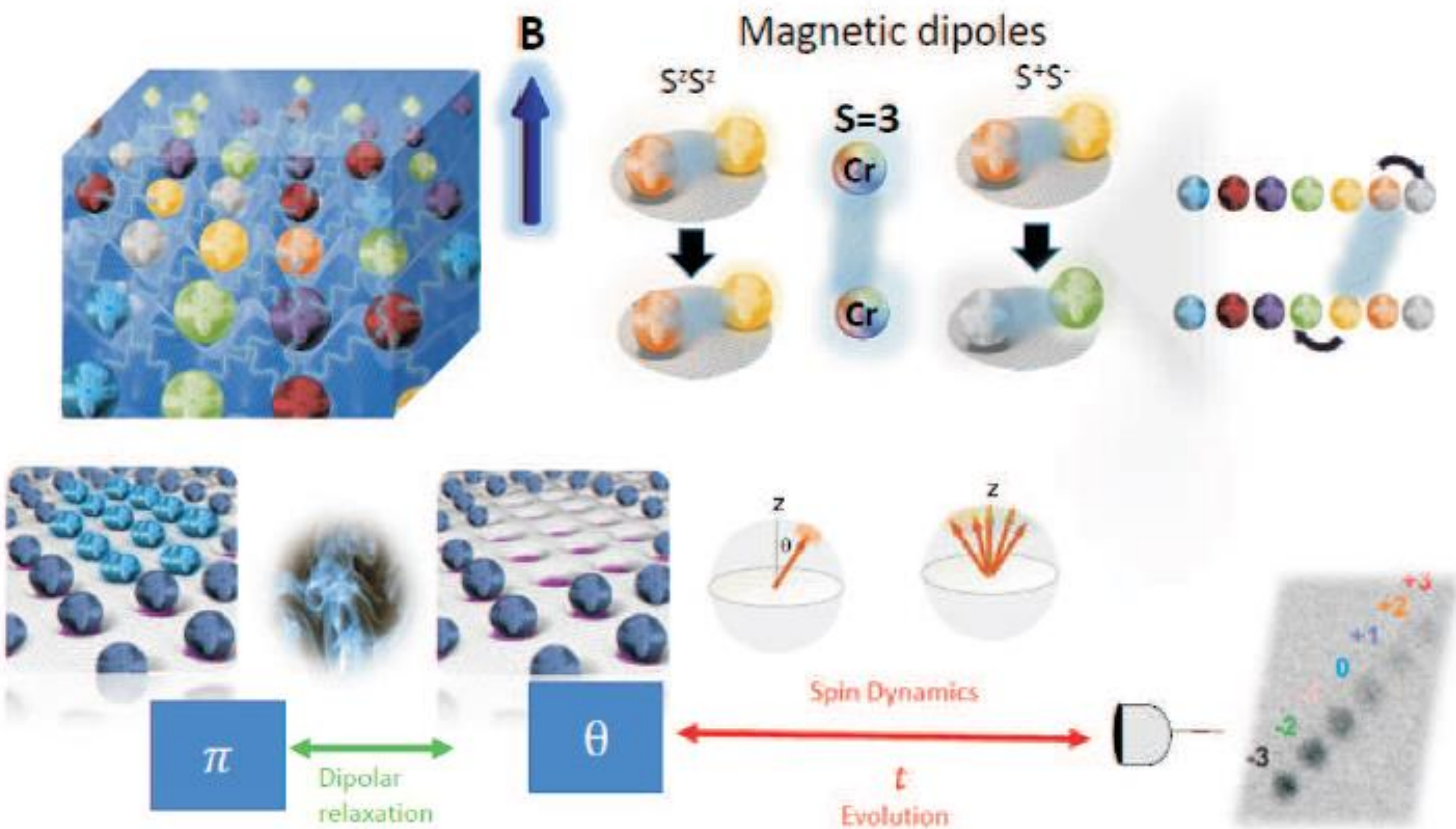




Dynamics inherently many-body
Mean-field theories fails



PRL, 111, 185305 (2013) see also Ye/Jin, Nature 2013

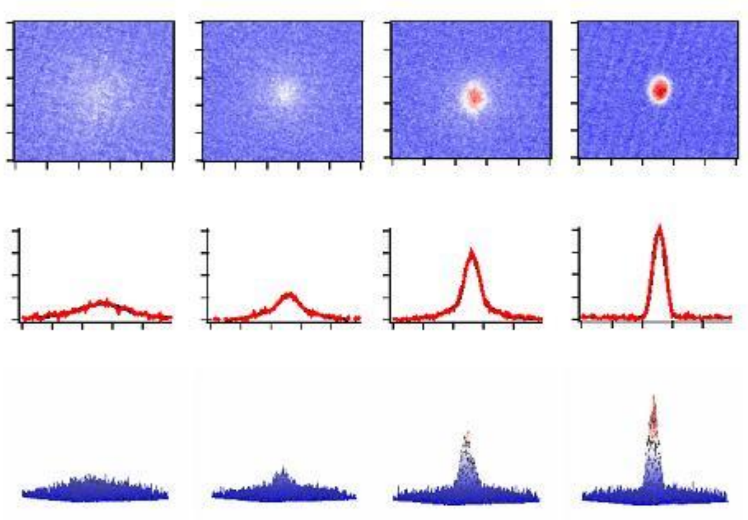


B
Magnetic dipoles
S^zS^z
S^+S^-
S=3
Cr
Cr
z
θ
z
+3
+2
+1
0
-2
-3
Spin Dynamics
π
θ
Dipolar
relaxation
t
Evolution

Experimental system

Nov 2007 : Chromium BEC

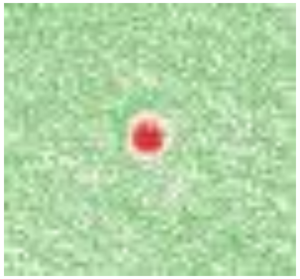
10^4 atoms



S=3

April 2014 : Chromium Fermi sea

10^3 atoms

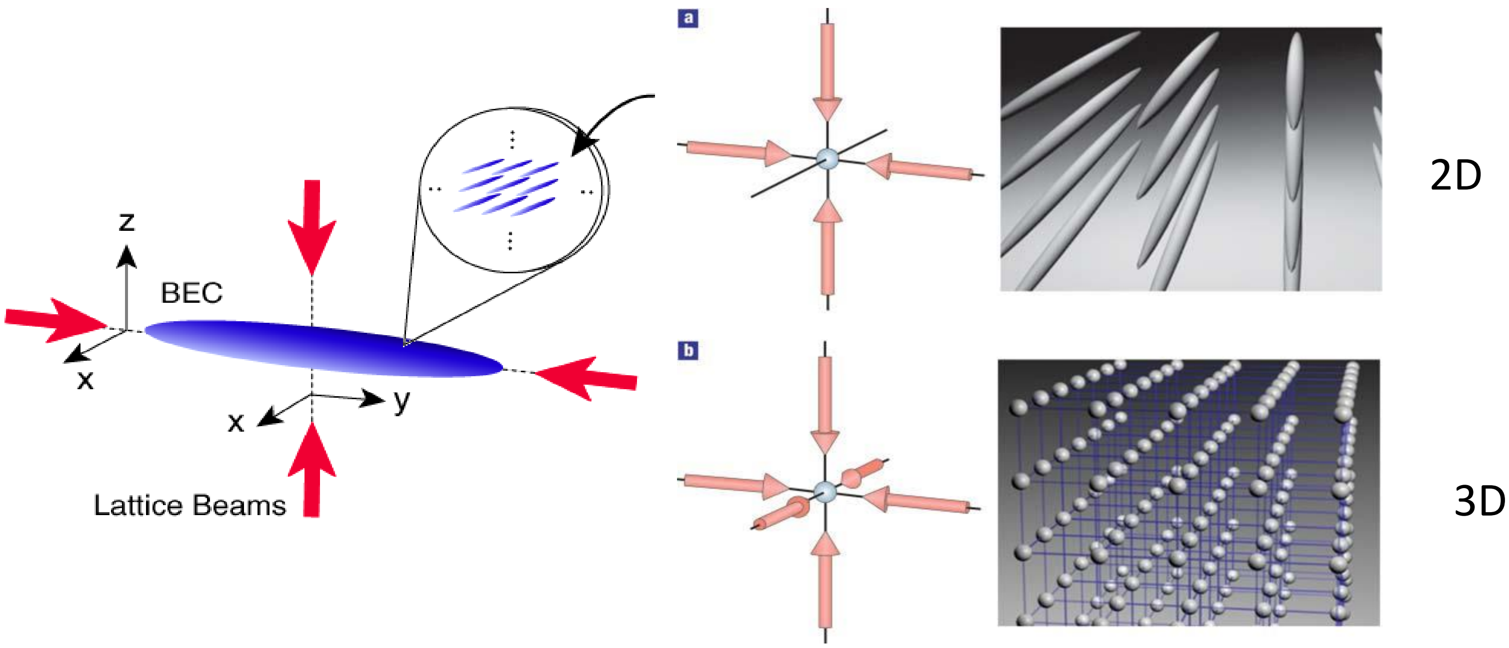


F=9/2

(from only 3.104 atoms in dipole trap !)
Phys. Rev. A 91, 011603(R) (2015)

Change dimensionnality using optical lattices (for instance) :

Periodic potential produced by the AC-Stark shift of a standing wave



1D lattices → 2D Gaz 2D (BKT transition, ENS)

2D lattices → 1D Gaz (fluctuations, correlations, LCFIO)

3D lattices → Strongly correlated systems…